How Collection Cost Structure Drives a Manufacturer’s Reverse Channel Choice

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Abstract
This note discusses the impact of collection cost structure on the optimal reverse channel choice of manufacturers who remanufacture their own products. Using collection cost functions that capture collection rate and collection volume dependency, we show that the optimal reverse channel choice (retailer- versus manufacturer-managed collection) is driven by how the cost structure moderates the manufacturer’s ability to shape the retailer’s sales and collection quantity decisions.

Key words: Closed-Loop Supply Chain; Reverse Logistics; Remanufacturing; Channel Choice

1 Introduction

Many manufacturers undertaking the remanufacturing of their own products are focused on reducing collection costs to increase profitability. In this note, we demonstrate that significant benefits may accrue to manufacturers from first analyzing the structure of their collection costs and shaping their collection strategies accordingly. In particular, we consider manufacturer-, retailer- versus third party-managed collection, where the party who “manages” collection determines the collection amount; this party may then physically undertake collection or subcontract it. We find that whether manufacturer- versus retailer-managed collection of returns maximizes a manufacturer’s profit depends on the collection cost structure, and that third party-managed collection is always dominated.

The following examples based on our industry interactions illustrate how alternative collection cost structures can arise under different operating environments (company names are omitted for confidentiality). Company A is an IT manufacturer that sells print cartridges to end-consumers and provides them with pre-paid envelopes to return their end-of-use cartridges. The company’s contract with the postal network consists of a quantity discount schedule, leading to economies of scale in collection cost (see Drake et al. 2009 for a similar example). Now consider companies B and C; a chemical producer that has industrial customers in the automotive industry, and a consumer electronics manufacturer in Europe, respectively. Each customer of company B generates a given volume of returns, and the company decides whether to collect from each customer. If yes, a subcontractor collects all the quantity available at that customer. The data we obtained from company B reveals that this company collects only from a set of customers that are cheapest to collect from (in this case, cost is based on distance, so these are the closest customers). Company C uses express shipping companies to collect product returns or used products from end-users for remanufacturing. Similarly, this company only collects from a small set of member states in the E.U., which are closer and for which the cost of collection is comparably cheap. Essentially, both companies optimally rank order customers/countries according to increasing collection cost and collect according to that order. In both cases, Companies B and C can increase the collection volume by collecting from progressively more distant (and hence more expensive) sources. Consequently, increasing the collection volume yields a total cost function that exhibits diseconomies of scale in volume.\(^1\)

\(^1\) See also Guide et al. (2003), Guide and Van Wassenhove (2001), Ferguson and Toktay (2006) and Atasu and Souza (2012), who argue that reverse logistics cost can exhibit diseconomies of scale.
In addition to these volume-dependent reverse logistics costs, reverse supply chains may incur costs associated with securing a supply of used products to be collected, e.g., advertisement or investment costs. The effect of such costs on collection channel choice has been modeled and analyzed in Savaşkan et al. (2004), who use an investment cost that is a convex increasing function of the collection rate and conclude that retailer-managed collection is optimal for the manufacturer. In this note, we extend their model by exploring the effect of volume-dependent reverse logistics costs under different operating environments. Using a reverse channel cost structure that has rate-dependent (investment cost) and a volume-dependent (reverse logistics cost) components, where the latter can exhibit either economies or diseconomies of scale, we find that if the collection cost structure is such that the manufacturer can profitably incentivize the retailer to increase its sales volume with retailer-managed collection, retailer-managed collection is preferred by the manufacturer. With purely volume-dependent cost, this happens when there is economies of scale in collection; with diseconomies of scale, the manufacturer prefers to manage collection himself. With purely rate-dependent cost, retailer-managed collection is optimal when the scale effect is strong enough; when it is weak, manufacturer-managed collection is optimal. Third-party-managed collection is never preferred by the manufacturer. The two contributions of our note are to reveal the importance of basing the reverse channel choice on the structure of the collection cost and to describe under what conditions each reverse channel choice should be preferred.

2 Modeling Assumptions

To focus on the effect of cost structure on the collection channel choice, we use a similar model as in Savaşkan et al. (2004) who study the same question; the main differences are that we incorporate a cost component that captures economies or diseconomies of scale in the collection volume, and that we analyze the investment cost case over a broader set of parameter values as we find this analysis to be insightful. We also extend their model for differentiable new and remanufactured products in Appendix B. We briefly summarize the model in Savaşkan et al. (2004) below and refer the reader to the paper for a detailed discussion of the model assumptions. For convenience, Table 5 in the Appendix summarizes the notation used.

The Forward Channel. Undifferentiated new and remanufactured products are sold through the same retailer in a decentralized uncoordinated two-echelon (manufacturer and retailer) supply chain. The archetypal example for undifferentiated products is the Kodak single-use camera where
the customer knows that the company utilizes used parts in the production of some cameras, but does not know whether a specific product contains used parts or not. The manufacturer is the Stackelberg leader and offers a wholesale price contract to the retailer. Since the products are undifferentiated, the manufacturer sells both new and remanufactured products to the retailer at the same wholesale price \( w \), who in turn sells both products at the same price \( p \) on the market. The demand is given by \( q = (1 - p)\phi \), where \( \phi \) is a constant. The cost of producing a new product is \( c_m \) and the cost of producing a remanufactured product is \( c_{rm} \). We assume \( c_m < 1 \) to ensure positive demand at positive margin for new products. Define \( \tau \) as the fraction of demand satisfied by remanufactured units. Then the manufacturer’s profit from selling \( q \) items to the retailer is \( wq - c_m(1 - \tau)q - c_{rm}\tau q \). Defining \( \Delta \equiv c_m - c_{rm} > 0 \) as the cost saving per unit from remanufacturing, we can rewrite this profit as \( (w - c_m + \Delta \tau)q \). Note that with undifferentiated products, it is optimal to remanufacture and sell as many of the returned units as possible; thus \( \tau \) is also the collection rate of products from the previous generation’s sales volume \( q \) (under the implicit assumption that all returns are remanufacturable). The collection volume, \( q_r \), equals \( \tau q \). The feasible range of the collection rate \( \tau \) might be limited in practice. This can be captured by assuming \( 0 \leq \tau \leq \tau_{\text{max}} \). In our analysis, we assume without loss of generality that \( \tau_{\text{max}} = 1 \); with \( \tau_{\text{max}} < 1 \), all results where \( \tau^* = 1 \) will change to \( \tau^* = \tau_{\text{max}} \).

The Reverse Channel. The party that manages collection is defined as the one who determines the collection quantity, regardless of who handles the actual collection operation. This party can be the manufacturer, the retailer or a third party (Figure 1) and incurs a linear (e.g., constant per unit) acquisition cost \( Aq_r \) and collection cost \( C(\tau; q) \). If the retailer or the third party manages collection, he transfers all collected units to the manufacturer and receives a transfer price \( b \) per unit.

The Collection Cost. We model the total cost to collect a fraction \( \tau \) of sales volume \( q \) as \( C(\tau; q) = C_1(\tau) + C_2(q_r) \), with \( q_r = \tau q \).

The first component \( C_1(\tau) \) considers the cost of incentivizing the consumers to return their products, when such action is needed. In particular, we define \( C_1(\tau) = C_L \tau^2 \) as the total cost to achieve a collection rate of \( \tau \) (as in Savaşkan et al. 2004). This cost term captures the investment cost analyzed by Savaşkan et al. (2004). As argued in their paper, promotional/advertising activities may be needed to encourage consumers to return their products. Such investments would be expected to exhibit diminishing returns; the increase in \( \tau = \sqrt{C_1/C_L} \) is slower than the increase in the total investment \( C_1 \).
The second component considers the reverse logistics cost, i.e., the cost of physically collecting the product, $C_2(q_r) = \eta q_r^k$, defined in terms of the collection volume $q_r$. The modeling of $C_2(q_r)$ is a novelty of our model, which originates from our experiences with practice. In this model, $k < 1$ captures economies of scale, while $k > 1$ captures diseconomies of scale in the operational cost of collection, which can both be observed in practice.

To see how a concave cost structure with $k < 1$ can emerge, consider companies who prefer using a drop-off strategy, under which consumers are provided the means to drop off the used product to specified locations. For instance, Cycleon, a European reverse logistics company we worked with, collects used products at European post offices. Cycleon contracts volume-dependent prices (e.g., quantity discounts) with the European postal networks, and charges their customers volume-dependent prices as well. Thus, for a company using Cycleon’s service, the marginal cost of collecting an additional product is decreasing in the collection volume, such that one would observe economies of scale.

To see how a convex cost structure with $k > 1$ emerges in practice, consider the two examples briefly mentioned in §1. Company B, a chemical producer that mainly supplies to automotive industry, collects a certain quantity of used products (which is a fraction of their sales) from their customers and can reuse these materials in the production process. However, the company does not collect from all sources that have available reusable supply. While there are 140 customers, who all have reusable supplies available, company B collects only from 39 closer and cheaper to collect customers. For instance, the company has chosen not to collect from China,
Korea, South America or some sources in Mexico, due to the fact that the reverse logistics cost of collecting from these resources would outweigh the benefits of reusing the collected items. Figure 2a illustrates the collection cost structure of the company based on the fact that the company collects first from cheaper resources. This figure plots the cumulative amount collected from the firm’s customers versus the total collection cost, which exhibits the convex cost structure posited. It is straightforward to fit a simple linear regression model to this data set, i.e., $C(\tau, q) = \eta(\tau q)^k \iff \log(C(\tau, q)) = \log(\eta) + k\log(\tau q)$. For this model, we obtain $\eta = 0.0035161$, and $k = 1.112447$, where $p < 0.001$ and $R^2 = 0.988$ suggests that the regression model accurately represents the data.

![Figure 2a](image-a.png)  
![Figure 2b](image-b.png)

Figure 2: Total cost of collection versus the cumulative quantity collected.

Our second example, Company C, is a European Branch of a global consumer electronics producer that remanufactures certain products and uses express shipping companies to collect product returns or used products from end-users for remanufacturing. This company collects only from 12 countries (out of 27) in the E.U. The company has chosen to collect from closer countries where the cost of collection was comparably cheap. Figure 2b illustrates the collection cost structure of the company, where the company collects first from countries that have lower per unit collection costs charged by the express company. The figure plots the total cost of collection as a function of the cumulative amount collected from different countries over two months. The data is presented by country, in order of increasing average collection cost. This data clearly exhibits the convex cost structure we posit as well. A regression analysis similar to that for the
first data set results in $\eta = 0.007619$ and $k = 1.572939$, where $p < 0.001$ and $R^2 = 0.968$ suggests that the regression model accurately represents this data set as well.

The rest of the paper is organized as follows: We first focus on the operational cost $C_2(q_r) = \eta q_r^k$ in Section 3.1. Then, we focus on a generalized version of the investment cost $C_1(\tau)$ of Savas¸kan et al. (2004) in Section 3.2. Section 3.3 discusses the general model that combines the two costs. We conclude in Section 4.

3 Analysis

In this section, we compare the optimal solutions under manufacturer, retailer, and third party-managed collection and determine which reverse channel choice is preferred by the manufacturer. We let $\Pi_x^y$ denote the profits of party $x$ when party $y$ manages collection, where $x, y \in \{M, R, 3P\}$. Here $M$ denotes manufacturer, $R$ denotes retailer and $3P$ denotes third party.

Manufacturer-managed Collection: When the manufacturer manages collection, the retailer determines his sales price $p$ to maximize $\Pi_M^R(p) = (p - w)q(p)$, given the wholesale price $w$ quoted by the manufacturer. Let $p(w)$ denote the retailer’s best response and $q(w)$ the corresponding sales quantity. The manufacturer’s objective is to maximize $\Pi_M^M(w, \tau) = (w - c_m + \tau(\Delta - A))q(w) - C(\tau; q(w))$.

Retailer-managed Collection: When the retailer manages collection, he determines his sales price $p$ and collection rate $\tau$ to maximize $\Pi_R^R(p, \tau) = (p - w)q(p) + (b - A)\tau q(p) - C(\tau; q(p))$, where $b$ is the payment the retailer receives from the manufacturer per collected unit. Let $p(w, b)$ and $\tau(w, b)$ denote the retailer’s best response and $q(w, b)$ denote the corresponding sales quantity. The manufacturer’s objective is to maximize $\Pi_M^R(w, b) = (w - c_m + \Delta \tau(w, b))q(w, b) - b\tau q(w, b)$ by choosing $w$ and $b$.

Third Party-managed Collection: Here, the retailer determines his sales price $p$ to maximize $\Pi_R^3P(p) = (p - w)q(p)$ given the wholesale price $w$ quoted by the manufacturer. Let $p(w)$ denote the retailer’s best response and $q(w)$ the corresponding sales quantity. For any sales quantity $q$, the third party determines his collection rate $\tau$ to maximize $\Pi_{3P}^{3P}(\tau) = (b - A)\tau q - C(\tau; q)$, where $b$ is the payment the third party receives from the manufacturer per collected unit. Let $\tau(b, q)$ denote the $3P$’s best response. The manufacturer anticipates the retailer’s and the $3P$’s reactions to his choice of $w$ and $b$. His objective is to maximize $\Pi_M^3P(w, b) = (w - c_m + \Delta \tau(b, q(w)))q(w) - b\tau(b, q(w))q(w)$ by choosing $w$ and $b$. 
3.1 Reverse Logistics Cost

In this section, we focus on the physical cost of collection $C_2(q_r)$ only. This analysis can be done directly in terms of the collection volume $q_r$ subject to $q_r \leq q$, but for consistency with the next subsection, we use the notation $C_2(\tau; q) = \eta(\tau q)^k$, where $\tau = \frac{q_r}{q}$ and $\tau \leq 1$.

3.1.1 Scale Economies in the Collection Volume

**Proposition 1** Under economies of scale in collection ($k < 1$), the optimal solution with manufacturer, retailer or third party collection is at one of the two boundaries $\tau^* = 0$ or $\tau^* = 1$.

*Proof:* All proofs are provided in Appendix A.

This result is intuitive: The collection cost exhibits economies of scale in the collection volume, while the revenue from collection increases linearly in the collection volume. Thus, the profit in the reverse channel is convex increasing in the collection volume. As long as the party that manages collection finds collection profitable for any collected quantity, he is better off by choosing a larger collection volume, so that $\tau^* = 1$; otherwise $\tau^* = 0$. Unfortunately, a solution to the embedded optimization problem cannot be obtained in closed form for general $k < 1$ values because the best response function of the party that manages collection needs to be derived from the optimization of a $k^{th}$ order polynomial and cannot be characterized in closed form. However, it is possible to do so numerically. Our numerical study$^2$ (from which a representative sample is presented in Table 1) reveals the following.

**Observation 1** When the retailer finds it profitable to collect (i.e., $\tau_R = 1$) under economies of scale ($k < 1$), manufacturer profits are maximized under retailer-managed collection. The manufacturer may find retailer-managed collection so profitable that $b$ may be larger than $\Delta$ in equilibrium. The sales price is the lowest with retailer-managed collection, and consequently the sales quantity is maximized with retailer-managed collection.

By Proposition 1, when the retailer manages the collection, he finds it optimal to collect all possible units as return channel profits are (convex) increasing in the collection volume. An increase in the transfer price $b$ incentivizes the retailer to sell more so as to collect more and increase profits on the reverse channel. For the manufacturer, the resulting increase in the sales volume is so high as to dominate the margin lost from increasing the transfer price. The

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$^2$The parameter levels chosen are $c_m = \{0.2, 0.5, 0.8\}$, $\Delta = \{0.2c_m, 0.5c_m, 0.8c_m\}$, $A = \{0.2\Delta, 0.5\Delta, 0.8\Delta\}$, $\eta = \{0.01, 0.1, 1\}$, $\phi = \{0.5, 1, 5\}$ and $k = \{0.2, 0.5, 0.8\}$, and cover a broad range of possible outcomes.
manufacturer capitalizes on this property, so much so that he may pass the entire remanufacturing savings or more to the retailer ($b^*$ may exceed $\Delta$).

When the manufacturer manages the collection, he benefits from the scale economies in collection, so he would like to encourage a high sales volume in order to collect more. However, his only mechanism to influence the retailer is to decrease the wholesale price, which is not very effective, especially in the absence of scale economies accruing to the retailer. Despite the higher wholesale price observed in the retailer collection scenario, the combined effect of the high wholesale price and the high transfer price is a lower effective wholesale price $u^* - b^*$ and higher demand $q^*$ than with manufacturer-managed collection, yielding higher profits for the manufacturer with retailer-managed collection.

With third party-managed collection, the manufacturer is even worse off than manufacturer-managed collection, because not only does he have only one lever to compel the retailer to increase the collection volume, but he also shares the reverse channel profits with the third party.

### 3.1.2 Scale Diseconomies in the Collection Volume

Under scale diseconomies, there exists a threshold $\bar{\eta} = \frac{\Delta - A}{(\phi(1-c_m)/4)\kappa}$ such that $\tau^* < 1$ for $\eta > \bar{\eta}$ regardless of who manages collection. We solve the problem analytically in this case and resort to numerical analysis for $\eta < \bar{\eta}$.
Proposition 2 Let $k > 1$ and $\eta > \bar{\eta}$. The manufacturer makes more profit when he manages collection. With manufacturer-managed collection, the collection rate is higher than with retailer or third party-managed collection, but the equilibrium wholesale price, sales price and consequently the sales quantity are the same in all cases.

In the parameter range where $\tau^* < 1$ (i.e. $q_r^* < q^*$) regardless of who manages collection, the economics of the forward and reverse channels are decoupled (in equilibrium). This explains why the equilibrium wholesale price (and hence the retail price and sales volume) are the same regardless of who collects. It is intuitive that the collection rate $\tau^*$ and manufacturer profits are always higher with manufacturer-managed collection: While the manufacturer obtains the full benefit $\Delta$ when he manages collection, he shares it with the other party otherwise, leading the other party to choose a lower collection volume and the manufacturer to make less profit.

Now consider the cost range where any party managing collection would set the collection rate to 1 in equilibrium, i.e., the forward and reverse channels do not decouple regardless of who the collecting agent is. Unfortunately, for general $k > 1$, there is no closed form solution when $\tau^* = 1$. Nevertheless, our numerical analysis (a representative sample of which is presented in Table 2) suggests that the manufacturer again prefers to manage collection. To see why, consider the retailer-managed collection case. The boundary solution $\tau_R^* = 1$ implies that the retailer’s profit function on the return channel is locally (concave) increasing in $q_r$ at $q_r = q$. Thus, in response to an increase in $b$, the retailer would increase the sales volume to increase the collection volume. However, unlike the economies of scale case, her profit on the reverse channel increases at a decreasing rate, creating a moderate increase in sales volume in response to an increase in $b$. In other words, the manufacturer’s instrument $b$ is not as effective at achieving an increase in sales that is sufficient to compensate the manufacturer’s reduced margin from raising $b$. Therefore, the manufacturer prefers to manage collection himself.

3.2 Investment Cost

Now, we focus on the investment cost function $C_1(\tau; q) = C_L \tau^2$, which captures the effect of increasing the collection rate. Our analysis here follows that of Savaşkan et al. (2004), except that for consistency with the analysis in §3.1, we relax the bounds $C_L \geq \frac{\phi((\Delta-A)(1-c_m+\Delta-A))}{8}$ and $b \leq \Delta$ that they use to ensure interior equilibria ($\tau^* < 1$).

Proposition 3 Let $C_1(\tau; q) = C_L \tau^2$. Manufacturer profits are higher with retailer-managed collection when $\frac{\phi((\Delta-A))^2}{8} < C_L < \frac{\phi((\Delta-A)(1-c_m+\Delta-A))}{4}$, and with manufacturer-managed collec-
Table 2: The impact of diseconomies of scale on the manufacturer’s reverse channel choice without decoupling for the following parameter subset: $c_m = 0.5$, $\Delta = 0.2c_m = 0.1$, $A = \{0.2, 0.5, 0.8\} \Delta$, $k = \{2, 5, 8\}$, $\phi = \{0.5, 1, 5\}$ and values of $\eta$ such that $\tau^*_R = \tau^*_M = 1$.

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When $C_L < \frac{\phi(\Delta-A)^2}{8}$; third-party-managed collection is always dominated. When $C_L > \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$, the retailer does not participate, and manufacturer-managed collection is optimal.

We find that the retailer’s collection rate is either 1 or 0 in equilibrium. $\tau^*_R$ is 0 when $C_L \geq \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$, in which case retailer-managed collection is not an option for the manufacturer, and is 1 otherwise. This result shows that this cost structure fundamentally behaves in the same way as the economies of scale case analyzed in §3.1.1. It also complements Savaşkan et al. (2004), who show that there is a range ($C_L \geq \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{8}$ when $b \leq \Delta$) where retailer-managed collection is preferred by the manufacturer, and the sales volume and the collection rate are highest with retailer-managed collection. Our result shows that with no bound on $b$, the region where retailer-managed collection is more profitable becomes $\frac{\phi(\Delta-A)^2}{8} \leq C_L \leq \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$, and the retailer’s collection rate need not be strictly higher than that under manufacturer or the third-party-managed collection in equilibrium. The effectiveness of retailer-managed collection in this case is due to what Savaşkan et al. (2004) dub the “scale effect” (the average collection cost $C(\tau; q)/q_r = C_L \tau^2/q_r$ decreases in the sales volume for a given
collection rate $\tau$), which parallels the effect of economies of scale discussed in Section 3.1.1: The manufacturer offers the retailer a high transfer price (higher than $\Delta$ for large enough $C_L$). Due to the scale effect, the retailer’s response is to raise the sales volume, which has a sufficiently high positive effect to compensate the manufacturer’s margin erosion from the high transfer price.

Interestingly, when $C_L \leq \frac{\beta(\Delta-A)^2}{8}$, the manufacturer prefers to manage collection himself. In this range, the retailer’s sales response to an increase in $b$ is weaker, so using the transfer price tool is not as effective for the manufacturer. He prefers to use only the wholesale price tool to affect retail sales, and not incur losses due to double marginalization in the return channel. This result exhibits the distinction between the scale effect in the investment cost and the economies of scale effect in the reverse logistics cost: the existence of the investment cost “scale effect” alone is not enough to make retailer-managed collection optimal; it has to be strong enough. However, if $C_L$ is too high ($C_L \geq \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$), then the absolute cost of collection is too large for managing collection to be profitable for the retailer. Thus, the manufacturer manages collection.

### 3.3 The General Model

The central result emerging from the preceding analysis is that scale effects are key in determining the optimal reverse channel choice: With economies of scale in the reverse logistics cost, or in the presence of a scale effect in the investment cost that sufficiently incentivizes the retailer to sell more, the manufacturer prefers to have the retailer manage collection. With diseconomies of scale in the reverse logistics cost, or a weak scale effect in the investment cost, he prefers to manage collection. The third-party-managed collection option is never preferred by the manufacturer.

Our analysis treated the reverse logistics costs and investment costs separately. Due to its complicated structure, the combined cost formulation $C(\tau; q) = C_L\tau^2 + \eta(\tau q)^k$ with general $k$ does not lend itself to closed-form solutions. Therefore, we carry out a numerical study that compares manufacturer- and retailer-managed collection; the third-party-managed collection option is always dominated by these alternatives. Not surprisingly, the solution to the general case is parameter dependent; the optimal reverse channel choice depends on the values that $\eta$, $C_L$, and $k$ take. Nevertheless, as may be expected, the optimal channel choice is driven by which of the two previously discussed effects dominates. Consider the example presented in Figure 3. In this example, the investment cost has parameters $0.3 \leq C_L \leq 1$. The reverse logistics cost exhibits scale diseconomies with $k = 3$ and $\eta = 0.3, 0.7, 1$. For these parameters, retailer and manufacturer-managed collection, respectively, would be optimal if the reverse channel cost consisted only of
investment cost and reverse logistics cost, respectively. The optimal collection channel with the two costs combined then depends on which effect dominates. As seen in the figure, when the reverse logistics component is small ($\eta$ is low), the scale effect arising from the investment cost dominates and retailer-managed collection is optimal ($\Pi^R_M > \Pi^M_M$). As $\eta$ increases, the reverse logistics cost starts dominating, and manufacturer-managed collection becomes optimal for an increasingly large range of $C_L$ values.

Figure 3: $\Pi^M_M - \Pi^R_M$ with $k = 3$, $l = 2$, $c_m = 0.5$, $A = 0.05$, $\Delta = 0.25$, and $\phi = 1$.

Our analysis identifies three key factors that determine the optimal reverse channel choice of the manufacturer: (i) whether there are scale economies or diseconomies in the reverse logistics cost (whether $k < 1$ or $> 1$); (ii) the magnitude of the reverse logistics cost (how large $\eta$ is); and (iii) the magnitude of the investment cost (how large $C_L$ is). Table 3 summarizes their typical combined impact in a simple informal framework:

- When the reverse logistics cost exhibits scale economies ($k < 1$), the impact of the reverse logistics cost is aligned with the typical impact of the investment cost. The two effects reinforce each other, creating a strong scale effect such that retailer-managed collection is preferred.

Recall that with a weak enough scale effect in investment cost (low enough $C_L$), manufacturer-managed collection would be optimal if the reverse channel cost consisted only of the invest-
ment cost. With the combined cost structure, the reverse logistics cost has to be insignificant ($\eta$ very low) for this effect to persist, so manufacturer-managed collection is excluded from the $k < 1$ column as being an atypical outcome.

- When the reverse logistics cost exhibits scale diseconomies ($k > 1$), the investment cost and the reverse logistics cost have contrasting impacts on the manufacturer’s decision, so their respective levels play a more important role. For any $C_L$ value, when the reverse logistics cost component $\eta$ is small enough, the scale diseconomies in the reverse logistics cost is dominated by the scale effect inherent in the investment cost, and retailer-managed collection is optimal. As $\eta$ increases, the scale diseconomies in the reverse logistics cost starts dominating and manufacturer-managed collection becomes optimal above a threshold.

As seen in Figure 3, the Retailer-to-Manufacturer transition happens at lower $\eta$ values when $C_L$ is higher since an increase in the investment cost decreases the profitability of the reverse channel and makes it more difficult for the manufacturer to profitably incentivize the retailer to manage collection. We informally express this directional result in Table 3 by including the Manufacturer outcome in the lower middle cell to contrast it to the Retailer only upper middle cell.

<table>
<thead>
<tr>
<th></th>
<th>$k &lt; 1$</th>
<th>$k &gt; 1$, Low $\eta$</th>
<th>$k &gt; 1$, High $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $C_L$</td>
<td>Retailer</td>
<td>Retailer</td>
<td>Manufacturer</td>
</tr>
<tr>
<td>High $C_L$</td>
<td>Retailer</td>
<td>Retailer/Manufacturer</td>
<td>Manufacturer</td>
</tr>
</tbody>
</table>

Table 3: The Manufacturer’s Typical Collection Channel Preference under the General Model

4 Conclusion

The theoretical and practical contribution of our research is to show that the structure of the collection cost should play a crucial role in the manufacturer’s reverse channel strategy in decentralized uncoordinated supply chains. In particular, if the cost structure is such that the manufacturer can incentivize the retailer to sufficiently increase new product sales, then the manufacturer may profit from having the retailer manage collection despite sharing reverse channel profits with him. Thus, with our volume-dependent reverse logistics cost model, retailer-managed collection is optimal when there are economies of scale, while manufacturer-managed collection is optimal when there are diseconomies of scale. With the rate-dependent investment cost model
(modeled by Sava¸skan et al. 2004), if the scale effect is strong enough, retailer-managed collection is optimal, while manufacturer-managed collection is optimal otherwise.

The efficiency of the transfer price instrument, determined by the collection cost structure, is a key driver of these results. With economies of scale in reverse logistics cost or a strong scale effect in investment cost, an increase in the transfer price gives a strong incentive to the retailer to stimulate demand and increase the collection volume; the transfer price is a powerful instrument. The manufacturer capitalizes on this fully, even passing all remanufacturing savings or more to the retailer in some cases. The net increase in demand dominates the manufacturer’s loss from compensating the retailer, and the manufacturer prefers the retailer to manage collection. In contrast, with diseconomies of scale, the transfer price is unable to induce the retailer to sell sufficiently more than under manufacturer-managed collection. Consequently, the manufacturer has no incentive to share the reverse channel profits with the retailer and chooses to manage collection himself. While it may not be surprising that costs impact the optimal solution, our contribution is to show which feature of the cost structure is key (the scale effect, defined under both rate- and volume-based costs in our paper), and to develop simple rules of thumb that provide practical guidance.

It is important to note that our analysis assumed that all parties face the same collection cost structure, no matter who manages collection. This assumption can hold in practice, because any party can outsource the reverse logistics activity, and receive similar quotes from such providers. Nevertheless, it is possible that one party utilizes a different collection method and has a cost advantage over others in managing collection, particularly when utilizing her own resources for reverse logistics (e.g., a retailer who has proximity to consumers or a transporter who manages the reverse channel). If a party has a cost advantage, the scales can tip in favor of having that party manage collection. Proposition 4 in Appendix A shows that the manufacturer’s equilibrium profits decrease in \( A, C_L, k \) and \( \eta \) in all three reverse channel configurations \((M, R, 3P)\). Hence, one can find thresholds on differentials between these parameters in different channels such that one channel is preferred over the other.

<table>
<thead>
<tr>
<th>Table 4: The impact of cost differentials on the reverse channel choice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scale Economies</strong> when ( M ) manages</td>
</tr>
<tr>
<td>( \mathbf{R \ dominates} ) unless ( \eta_R &gt;&gt; \eta_M ) or ( k_R &lt;&lt; k_M )</td>
</tr>
<tr>
<td><strong>Scale Discomonies</strong> when ( M ) manages</td>
</tr>
<tr>
<td>( \mathbf{M \ dominates} ) unless ( \eta_M &gt;&gt; \eta_R ) or ( k_M &lt;&lt; k_R )</td>
</tr>
</tbody>
</table>
Table 4 summarizes the implications of such possible cost structure differentials on the optimal reverse channel choice. The bold results in Table 4 follow our analysis using equal costs. The results in italics follow Proposition 4 and are intuitive: If retailer management dominates at equal cost, then clearly, if the scale effect under manufacturer management is not much stronger, is weaker or even exhibits diseconomies of scale, then retailer management will continue to dominate. The 3P-managed case is not included because it is always dominated by manufacturer-managed collection at equal cost, and it is obvious that the result may be reversed if the 3P-managed case exhibits a sufficient cost advantage. However, a mere cost differential, be it with respect to collection or investment costs, is not sufficient to ensure such reversals. The value of the strategic levers that a manufacturer can utilize to induce the retailers to sell more and the consequences of reverse channel revenue sharing requirements play an important role in the optimal reverse channel choice. Thus, focusing on unit cost differentials in reverse channel costs is an efficiency trap that may result in manufacturers foregoing the strategic benefits of utilizing the right reverse channel choice to maximize their profits.

This research has important managerial implications. It shows that manufacturers can indeed pass the reverse channel decision rights to retailers in decentralized reverse supply chains to improve the profitability of their businesses (even if the retailer has no cost advantage in collection). The profitability of this option depends on the structure of the collection cost, which is driven by the characteristics of their operating environment and their products. For instance, consider a consumer electronics manufacturer that sells small electronic devices. Due to their size, these products can be collected through postal networks (as in the Cycleon example of Drake et al 2009). Where the postal contract specifies quantity discounts, our results suggest that this manufacturer should have the retailers deal with the collection channel. On the other hand, a TV producer, due to the size or volume of its products, would have to employ a different collection strategy, e.g., the collecting agent would have to collect products directly from end-users. In this case, it is likely that reverse logistics costs exhibit scale diseconomies, since collecting a higher volume of used products may require reaching more remote locations. If so, our results suggest that the manufacturer would be better off managing collection himself (unless of course the collection cost of the retailer is sufficiently lower than that of the manufacturer). In sum, while remanufacturing by itself can be an attractive business proposition, carefully managing the reverse channel (by having retailers take responsibility and using the transfer price effectively) can increase sales volume and manufacturer profitability.
An important assumption of our model is that new and remanufactured products are perfectly substitutable. In practice, many remanufactured products are differentiated from their new counterparts. In Appendix B, we model the case of differentiated remanufactured products. Our results suggest that the fundamental insight we uncover regarding the importance of the scale effect in determining the appropriate reverse channel structure continues to hold. Another assumption is that a wholesale-price contract is in place for the forward chain. While the wholesale price contract is a common business model in many supply chains (Lariviere and Porteus 2001, Perakis and Roels 2007), it would be interesting to investigate whether other contractual arrangements in the forward chain would change the optimal reverse channel choice for the manufacturer.

References


Appendix A: Undifferentiated New and Remanufactured Products

Proof of Proposition 1

Manufacturer-Managed Collection

When the manufacturer manages collection, the retailer determines his sales price $p$ to maximize

$$\Pi^M_R(p) = (p - w)q(p)$$

given the wholesale price $w$ quoted by the manufacturer. Let $p(w)$ denote the retailer’s best response and $q(w)$ the corresponding sales quantity. It is straightforward to show that $p(w) = (1 + w)/2$ and $q(w) = \phi(1 - w)/2$.

The manufacturer’s objective is to maximize

$$\Pi^M_M(w, \tau) = (w - c_m + \tau \Delta)q(w) - \eta(\tau q(w))^k - Arq(w).$$

The unique solution to the first-order conditions of the manufacturer’s profit function is $w = (1 + c_m)/2$ and $\tau = \frac{4\Delta}{\phi(1-c_m)}$. However, one can show that when $k < 1$, the determinant of the Hessian is negative at this solution. Thus, this stationary point cannot be a maximizer of the manufacturer’s problem, and there are only two possible solutions to the manufacturer’s problem: either collect nothing or collect everything.
Retailer-Managed Collection

When the retailer manages collection, he determines his sales price \( p \) and collection rate \( \tau \) to maximize

\[
\Pi^R_M(p, \tau) = (p - w)q(p) - (A - b)\tau q(p) - \eta(\tau q(p))^k,
\]

where \( b \) is the payment the retailer receives from the manufacturer per collected unit. Let \( p(w, b) \) and \( \tau(w, b) \) denote the retailer’s best response and \( q(w, b) \) the corresponding sales quantity.

Similar to the manufacturer-managed collection scenario, there exists a unique solution to the first-order conditions at \( p(w, b) = (1 + w)/2 \) (with \( q(w, b) = \phi(1 - w)/2 \)) and \( \tau(w, b) = \frac{2(\frac{b - A}{\phi(1 - w)})^{1/(k - 1)}}{\phi(1 - w)} \). One can show that the determinant of the Hessian is negative at this solution. Thus, the two candidates for optimality are at the boundaries \( \tau = 0 \) or 1 as in the manufacturer collection case.

3P-Managed Collection

When the third party manages collection, he determines the collection rate \( \tau \) to maximize

\[
\Pi^{3P}_{3P}(\tau) = (b - A)q_*(\tau) - \eta(q_*)^k,
\]

where \( b \) is the payment the third party collector receives from the manufacturer per collected unit. Similar to the manufacturer-managed collection scenario, there exists a unique solution to the first-order condition to the third party’s objective at \( \tau(b) = \frac{2(\frac{b - A}{\phi(1 - w)})^{1/(k - 1)}}{\phi(1 - w)} \). One can show that the local determinant of the Hessian is negative at this solution. Thus, the two candidates for optimality are at the boundaries \( \tau = 0 \) or 1 as in the manufacturer-managed collection case.

Proof of Proposition 2

Manufacturer-Managed Collection

As in Proposition 1, it is straightforward to show that \( p(w) = (1 + w)/2 \) and \( q(w) = \phi(1 - w)/2 \). The manufacturer’s objective is to maximize

\[
\Pi^M_M(w, \tau) = (w - c_m + \tau\Delta)q(w) - \eta(\tau q(w))^k - Ar\tau q(w).
\]

Solving the first-order conditions of the manufacturer’s objective results in the unique solution \( w^* = (1 + c_m)/2 \) and \( \tau^* = \frac{4(\frac{b - A}{\phi(1 - c_m)})^{1/(k - 1)}}{\phi(1 - c_m)} \). With the assumption \( \eta > \bar{\eta} \), \( \tau^* < 1 \). At this point, the Hessian is negative definite, meaning that this point is the only interior maximizer. The optimal profit is \( \Pi^M_M = \frac{1}{8}\phi(c_m - 1)^2 + \eta(k - 1)\left(\frac{\Delta}{\eta k}\right)^{k-1} \).

Retailer-Managed Collection:

Let \( \eta > \bar{\eta}_R(w, b) = \frac{b - A}{k(\phi(1 - w)/2)^2} \). In this case, the unique optimal solution to the retailer’s problem is obtained at \( p(w, b) = (1 + w)/2 \) (with \( q(w, b) = \phi(1 - w)/2 \)) and \( \tau(w, b) = \frac{2(\frac{b - A}{\phi(1 - w)})^{1/(k - 1)}}{\phi(1 - w)} \), where \( \tau(w, b) < 1 \).
The manufacturer anticipates the retailer’s reaction to his choice of \( w \) and \( b \) and his objective is to maximize

\[
\Pi_M^R(w,b) = (w - c_m + \Delta \tau(w,b))q(w,b) - b\tau(w,b)q(w,b)
\]

by choosing \( w \) and \( b \). It is straightforward to show that the manufacturer profit is maximized at \( w^* = (1 + c_m)/2 \) and \( b^* = \frac{A(k-1) + \Delta}{k} \). Substituting this solution into \( \bar{\eta}_R(w,b) \), we require \( \eta > \bar{\eta}_R = \frac{k^2(\phi(1-e^{-c_m})/2)^{k-1}}{k^2(\phi(1-e^{-c_m})/2)^{k-1}} \) to guarantee that \( \tau^* < 1 \). This condition is satisfied when \( \eta > \bar{\eta} \) since \( k > 1 \). At this solution, the optimal profit is \( \Pi_M^R = \frac{1}{8}\phi(1 - c_m)^2 + \frac{(k-1)(\Delta-A)(\Delta-A)}{k} \frac{1}{x^1} \).

### 3P-Managed Collection:

Let \( \eta > \bar{\eta}_{3P}(b) = \frac{b-A}{k(\phi(1-p))} \). Then the unique optimal solution to the third party’s problem is \( \tau(b) = \frac{(\Delta-A)}{\phi(1-p)} \), where \( \tau(b) < 1 \). It is straightforward to show that the retailer chooses \( p(w) = (1 + w)/2 \) and \( q(w) = \phi(1 - w)/2 \). The manufacturer anticipates the retailer’s and the 3P’s reactions to his choice of \( w \) and \( b \) and his objective is to maximize

\[
\Pi_{3P}^M(w,b) = (w - c_m + \Delta \tau(w,b))q(w,b) - b\tau(w,b)q(w,b)
\]

by choosing \( w \) and \( b \). It is again straightforward to show that the manufacturer profit is maximized at \( w^* = (1 + c_m)/2 \) and \( b^* = \frac{A(k-1) + \Delta}{k} \). Substituting this solution into \( \bar{\eta}_{3P}(b) \), we require \( \eta > \bar{\eta}_{3P} = \frac{k^2(\phi(1-e^{-c_m})/2)^{k-1}}{k^2(\phi(1-e^{-c_m})/2)^{k-1}} \) to guarantee that \( \tau^* < 1 \). This condition is satisfied when \( \eta > \bar{\eta} \) since \( k > 1 \). At this solution, the optimal profit is \( \Pi_{3P}^M = \frac{1}{8}\phi(1 - c_m)^2 + \frac{(k-1)(\Delta-A)(\Delta-A)}{k} \frac{1}{x^1} \).

### Comparison

A summary of results for the three scenarios is provided in Tables 6 and 7, from which we see that the optimal prices are the same in all collection channels and that \( \tau_M^* \geq \tau_R^* = \tau_{3P}^* \).

<table>
<thead>
<tr>
<th>Manufacturer-Managed Collection</th>
<th>Retailer-Managed Collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^* = \frac{3+2c_m}{2} )</td>
<td>( p^* = \frac{3+2c_m}{2} )</td>
</tr>
<tr>
<td>( w^* = \frac{1+c_m}{2} )</td>
<td>( w^* = \frac{1+c_m}{2} )</td>
</tr>
<tr>
<td>( \tau^* = \frac{4(\Delta-A)}{\phi(1-c_m)(k-1)} )</td>
<td>( \tau^* = \frac{k^2(\phi(1-e^{-c_m})/2)^{k-1}}{k^2(\phi(1-e^{-c_m})/2)^{k-1}} )</td>
</tr>
<tr>
<td>( b^* = \frac{A(k-1)+\Delta}{k} )</td>
<td>( b^* = \frac{A(k-1)+\Delta}{k} )</td>
</tr>
<tr>
<td>( \Pi_{M}^R = \frac{1}{8}\phi(1 - c_m)^2 + \eta(k-1)\left(\frac{\Delta-A}{\eta k}\right)^{\frac{1}{k-1}} )</td>
<td>( \Pi_{M}^R = \frac{1}{8}\phi(1 - c_m)^2 + \frac{(k-1)(\Delta-A)(\Delta-A)}{k} \frac{1}{x^1} )</td>
</tr>
<tr>
<td>( \Pi_{R}^M = \frac{1}{16}\phi(1 - c_m)^2 )</td>
<td>( \Pi_{R}^M = \frac{1}{16}\phi(1 - c_m)^2 )</td>
</tr>
</tbody>
</table>

Table 6: Manufacturer- and Retailer-Managed Collection with Diseconomies of Scale in Reverse Logistics Cost

We note that the manufacturer profit at the third party-managed collection solution is equal to the retailer-managed collection solution. It is straightforward to show that the ratio of the second term of \( \Pi_M^R \)
When the third party manages collection, the retailer’s best response to the manufacturer’s wholesale price is $p^r = \frac{3+cm}{4}$. Given the retailer’s best response, the manufacturer’s objective is to maximize $\Pi^M = \phi \left( \frac{1}{2}(w + 1) \right) (\Delta - A) - cm + w) - CLr^2$. Substituting and solving the first-order conditions gives the unique stationary point $\tau^*_M = -\left(\frac{cm-1}{\phi(A-\Delta)}\right)$ and $w^*_M = \frac{\phi(A-\Delta)^2 - 4CL(c_m + 1)}{\phi(A-\Delta)^2 - 8CL}$. 

Note that $\tau^*_M$ is decreasing in $CL$ and the condition $8CL > \phi(A - \Delta)(A + c_m - \Delta - 1)$ guarantees that $\tau^*_M < 1$. Also, $\frac{d^2 \Pi^M}{d\tau^2} < 0$, and the determinant of the Hessian is $2CL\phi - \frac{1}{4}\phi^2(A - \Delta)^2$, which is always positive when $8CL > \phi(A - \Delta)(A + c_m - \Delta - 1)$. Thus, the unique stationary point maximizes manufacturer profits when $8CL > \phi(A - \Delta)(A + c_m - \Delta - 1)$. The equilibrium profits of the manufacturer and the retailer are $\Pi^*_M = \frac{CL(c_m - 1)^2\phi}{8CL - \phi(A-\Delta)^2}$ and $\Pi^*_R = \frac{4CL(c_m - 1)^2\phi}{\phi(A-\Delta)^2 - 8CL}^2$.

When $8CL \leq \phi(A - \Delta)(A + c_m - \Delta - 1)$, the interior solution obtained from the manufacturer’s first order conditions is not a maximizer. In this case, the manufacturer’s objective will be obtained at the boundaries. It is straightforward to show that the manufacturer’s profit is maximized at $\tau^*_M = 1$. When $\tau_M = 1$, the manufacturer sets $w = \frac{1+c_m + A - \Delta}{2}$. At this solution, the manufacturer obtains $\Pi^*_M = \frac{1}{8}\phi(A + c_m - \Delta - 1)^2 - CL$ and the retailer obtains $\Pi^*_R = \frac{1}{16}\phi(A + c_m - \Delta - 1)^2$. These results are summarized in Table 8.

### Third Party-Managed Collection

<table>
<thead>
<tr>
<th>Third Party-Managed Collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p^*_T = \frac{3+cm}{4}$</td>
</tr>
<tr>
<td>$w^*_T = \frac{1+c_m}{2}$</td>
</tr>
<tr>
<td>$\tau^*_T = \frac{4(\frac{k-1}{nk})^{1/(k-1)}}{\phi(1-c_m)}$</td>
</tr>
<tr>
<td>$b^*_T = \frac{2k(k-1)+\Delta}{\phi(1-c_m)}$</td>
</tr>
</tbody>
</table>

$\Pi^T_M = \frac{1}{8}\phi(1-c_m)^2 + \frac{k(1-c_m)(\Delta-A)}{\phi(A-\Delta)^2 - 8CL}$

$\Pi^T_R = \frac{1}{8}\phi(1-c_m)^2$

Table 7: 3P-Managed Collection with Diseconomies of Scale in Reverse Logistics Cost

over the second term of $\Pi^R_M$ is equal to $k^{1/(k-1)}$. The natural log of this term is nonnegative for $k > 1$, thus this term is always large than one. Thus, manufacturer collection dominates.

### Proof of Proposition 3

The proofs in this section follow Savaskan et al. (2004), with the exception that we do not impose the condition $b \leq \Delta$ and we relax the lower bound on $C_L$.

#### Manufacturer-Managed Collection

When the manufacturer manages collection, it is straightforward to show that the retailer’s best response is $p(w) = \frac{1+w}{2}$. Given the retailer’s best response, the manufacturer’s objective is to maximize $\Pi^M = \phi \left( \frac{1}{2}(w + 1) \right) (\Delta - A) - cm + w) - CLr^2$. Substituting and solving the first-order conditions gives the unique stationary point $\tau^*_M = -\left(\frac{cm-1}{\phi(A-\Delta)}\right)$ and $w^*_M = \frac{\phi(A-\Delta)^2 - 4CL(c_m + 1)}{\phi(A-\Delta)^2 - 8CL}$. 

The proofs in this section follow Savaskan et al. (2004), with the exception that we do not impose the condition $b \leq \Delta$ and we relax the lower bound on $C_L$.
When the retailer manages collection, given the wholesale price boundaries as well. Based on the third party’s best response, it is easy to see that:

\[
\Pi^*_M = \frac{1}{8} \phi(A - \Delta - 1)^2 - C_L \quad \Pi^*_R = \frac{1}{16} \phi(A - \Delta - 1)^2
\]

\[
\tau^*_M = 1 \quad \tau^*_R < 1
\]

Table 8: Manufacturer-Managed Collection with Investment Cost

Given the retailer’s and the third party’s best responses, the manufacturer’s objective is to maximize

\[
\Pi^*_M = \phi \left( \frac{1}{2}(-w + 1) \right) (\Delta - A - c_m + w) \quad \Pi^*_R = \frac{1}{16} \phi(A + c_m - \Delta - 1)^2
\]

Substituting and solving the first-order conditions gives the unique stationary point \( b = \frac{A+\Delta}{2} \) and \( w = \frac{\phi(A-\Delta)^2 - 8C_L(c_m+1)}{\phi(A-\Delta)^2 - 16C_L} \).

When \( 16C_L > \phi(A - \Delta)(A + c_m - \Delta - 1) \), \( \tau^*_M < 1 \). Also, \( \frac{\partial^2 \Pi^*_M}{\partial w^2} < 0 \), and the determinant of the Hessian, \( \frac{\phi(w-1)^2(4C_L - \phi(A^2 + (A-\Delta)^2 + 3\phi^2 - 3\phi^2 \Delta + \Delta^2))}{16C_L^2} \), is positive at this stationary point. Thus the only interior maximizer of manufacturer profit is obtained at \( b = \frac{A+\Delta}{2} \) and \( w = \frac{\phi(A-\Delta)^2 - 8C_L(c_m+1)}{\phi(A-\Delta)^2 - 16C_L} \) when \( 16C_L > \phi(A - \Delta)(A + c_m - \Delta - 1) \). The equilibrium profits of the manufacturer and the retailer at this point are \( \Pi^*_M = \frac{2C_L(c_m-1)^2}{16C_L - \phi(A-\Delta)^2} \) and \( \Pi^*_R = \frac{16C_L(c_m-1)^2}{(\phi(A-\Delta)^2 - 16C_L)^2} \).

Although the stationary solution obtained above is a local maximizer, the joint concavity of the manufacturer’s objective with respect to \( b \) and \( w \) cannot be guaranteed, thus this is not necessarily a global maximizer. For the sake of completeness, we need to check the manufacturer’s objective at the boundaries as well. Based on the third party’s best response, it is easy to see that:

- \( \tau^*_M = 0 \) when \( b = A \). In this case \( \Pi^*_M = \frac{1}{8} \phi(1 - c_m)^2 \) and \( \Pi^*_R = \frac{1}{16} \phi(1 - c_m)^2 \)
- \( \tau^*_M = 1 \) when \( b = A + \frac{8C_L}{\phi(1 - c_m + \Delta - A)} \). In this case, \( \Pi^*_M = \frac{1}{8} \phi(A + c_m - \Delta - 1)^2 - 2C_L \) and \( \Pi^*_R = \frac{1}{16} \phi(A + c_m - \Delta - 1)^2 \)

By comparing the manufacturer’s interior maximizer with his objective at the boundary solutions, we observe that the interior solution dominates the boundary solutions when \( 16C_L > \phi(A - \Delta)(A + c_m - \Delta - 1) \). Otherwise, the boundary solution with \( \tau^*_M = 1 \) dominates. These results are summarized in Table 9.

Table 9: Third Party-Managed Collection with Investment Cost

Retailer-Managed Collection

When the retailer manages collection, given the wholesale price \( w \) and the transfer price \( b \), the retailer maximizes \( \Pi^*_R = (1 - p)\phi(\tau_R(b - A) + p - w) - C_L\tau^2_R \). Solving the first-order conditions yields the stationary best response \( \tau(w, b) = -\frac{\phi(w-1)(A-b)}{\phi(A-b)^2 - 4C_L} \) and \( p(w, b) = \frac{\phi(A-b)^2 - 2C_L(w+1)}{\phi(A-b)^2 - 4C_L} \). Also \( \frac{\partial^2 \Pi^*_R}{\partial p^2} < 0 \) and the
determinant of retailer’s Hessian is \( \phi \left( 4C_L - \phi(A - b)^2 \right) \). Therefore, the objective is not necessarily jointly concave in \( p \) and \( \tau_R \). In what follows, we prove that the retailer’s stationary best response will be forced to the boundary, i.e., \( \tau_R = 1 \), by the manufacturer.

Let us assume that the retailer’s unique stationary solution is a local maximizer and investigate the manufacturer’s choice of \( w \) and \( b \). Replacing the retailer’s best response function in the manufacturer’s objective, we obtain:

\[
\Pi^R_M = -\frac{2C_L(\phi(A-b)\left(A(c_m - w) + b(-c_m) + b + \Delta(w-1)\right) + 4C_L(w - c_m))}{2(\phi(A-b)^2 - 4C_L)^2}
\]

In a similar spirit to Savaškan et al. (2004), we start by investigating the behavior of this function with respect to \( w \). Assume that there is a local interior maximizer pair \((w_1, b_1)\). This pair would have to satisfy the first-order condition

\[
\frac{\partial \Pi^R_M}{\partial w} = -\frac{2C_L(\phi(A-b)\left(A(c_m - w) + b(-c_m) + b + \Delta(w-1)\right) + 4C_L(c_m - w+1))}{(\phi(A-b)^2 - 4C_L)^2} = 0.
\]

Thus, a maximizer should satisfy

\[
w(b) = \frac{4C_L(c_m+1) - \phi(A-b)(A(m+c_m+b+\Delta(w-1)) - 4C_L(c_m-2w+1))}{8C_L - 2\phi(A-b)(A-\Delta) - 2\phi(1/b)\phi(A-b) - 4C_L}.
\]

Replacing this in the manufacturer’s objective, we obtain

\[
\Pi^R_M(b) = \frac{C_L(c_m-1)^2 \phi}{4C_L - 2\phi(A-b)(A-\Delta)}.
\]

It is easy to see that this function is increasing in \( b \). Thus, at the retailer’s stationary best response, the manufacturer chooses the highest \( b \) possible. Now note that \( \tau(w, b) = -\frac{\phi(w-1)(A-b)}{\phi(A-b)^2 - 4C_L} \) is also increasing in \( b \). Therefore, at the equilibrium, the value of \( b \) will be such that \( \tau_R = 1 \). This is achieved when

\[
b^* = \frac{4C_L(c_m+1) - \phi(A-b)(A(m+c_m+b+\Delta(w-1)) - 4C_L(c_m-2w+1))}{8C_L - 2\phi(A-b)(A-\Delta) - 2\phi(1/b)\phi(A-b) - 4C_L}.
\]

This point is a maximizer of the manufacturer’s objective, because

\[
\frac{\partial^2 \Pi^R_M}{\partial w^2} = -\frac{(\phi(2A-c_m+2\Delta-1))^3}{(\phi(2A-c_m+2\Delta-1)^2 - 16C_L)^2} < 0.
\]

In this case, \( w(b^*) = \frac{1+c_m}{2} + b^* - \Delta \). Similarly, this is a maximizer of the retailer’s objective since \( \frac{\partial^2 \Pi^R_M}{\partial p^2} < 0 \) on the \( \tau_R = 1 \) boundary. In this case, the manufacturer’s profit will be \( \Pi^R_M = \frac{1}{8}(c_m-1)\phi(2A+c_m-2\Delta-1) \) and the retailer’s profit will be

\[
\Pi^R_R = \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2 - C_L.
\]

Note also that in order for the equilibrium characterized above to take place, the retailer needs to have non-negative profits. In other words, it has to be true that \( \Pi^R_R = \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2 - C_L > 0 \). This means that when \( C_L > \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2 \) the retailer is better off not selling nor collecting anything. Thus, when \( C_L > \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2 \) the retailer will not participate and both parties will obtain zero profits.

Since there is no equilibrium with interior \( \tau_R \), we need to check the retailer’s potential equilibria at the boundaries for \( \tau_R \). The above analysis characterizes the case of \( \tau_R = 1 \). Therefore, we consider an equilibrium at \( \tau_R = 0 \) next. When the manufacturer chooses \( b = A \) and \( w = (1 + c_m)/2 \) (i.e., the forward channel only equilibrium), the retailer will choose not to collect but will be selling for profit from the forward channel. In this case \( \Pi^R_M = \frac{1}{8}(1 - c_m)^2 \) and \( \Pi^R_R = \frac{1}{16}\phi(1 - c_m)^2 \).

Comparing the manufacturer profits at the two boundaries, we can fully characterize the manufacturer’s decisions and the equilibrium as follows. When \( 4C_L > \phi(A-\Delta)(A+c_m-\Delta-1) \), \( \Pi^R_M(\tau_R = 0) = \frac{1}{8}(1 - c_m)^2 > \Pi^R_M(\tau_R = 1) = \frac{1}{8}(c_m-1)\phi(2A+c_m-2\Delta-1) \), and the equilibrium will take place at \( \tau_R = 0 \). (Note that this condition also guarantees that the retailer will have positive profit at the equilibrium.) Otherwise, the equilibrium will take place at \( \tau_R = 1 \). These results are summarized in
Manufacturer profit is decreasing in Proposition 4.

Comparison of Manufacturer Profits

Let us define \( C_L \leq \frac{\phi(A-\Delta)(A+\epsilon_m-\Delta-1)}{4} \), \( C_M \leq \frac{\phi(A-\Delta)(A+\epsilon_m-\Delta-1)}{8} \), and \( C^P_L \leq \frac{\phi(A-\Delta)(A+\epsilon_m-\Delta-1)}{16} \). With some algebraic manipulation of the data in Tables 8, 9 and 10, Table 11 provides the ranking of manufacturer profits under manufacturer- and retailer-managed collection for different parameter levels. The third party-managed collection channel is always dominated and therefore is not presented in this table.

<table>
<thead>
<tr>
<th>( C_L \leq \frac{\phi(A-\Delta)^2}{8} )</th>
<th>( \Pi_M^M &gt; \Pi_M^R )</th>
<th>( \Pi_M^M &gt; \Pi_M^R )</th>
<th>( \Pi_M^R &gt; \Pi_M^M )</th>
<th>( \Pi_M^R &gt; \Pi_M^M )</th>
<th>( \Pi_M^M &gt; \Pi_M^R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_M^* = 1 )</td>
<td>( \tau_M^* = 1 )</td>
<td>( \tau_M^* &lt; 1 )</td>
<td>( \tau_M^* &lt; 1 )</td>
<td>( \tau_M^* &lt; 1 )</td>
<td>( \tau_M^* &lt; 1 )</td>
</tr>
</tbody>
</table>

Table 11: Comparison of Manufacturer Profits

Proposition 4

Manufacturer profit is decreasing in \( A, C_L, \eta, \) and \( k \) for all channel configurations. Hence, there exist thresholds \( A_{M,R,3P}, C_{L,M,R,3P}, k_{M,R,3P} \) and \( \eta_{M,R,3P} \) for all channel configurations, below which the manufacturer is better off with the lower cost parameter party managing the reverse channel.

Proof of Proposition 4

By the envelope theorem, \( \frac{\partial \Pi_M^M(q, \tau)}{\partial y} |_{q^*, \tau^*} = \frac{\partial \Pi_M^R(q, \tau)}{\partial y} |_{q^*, \tau^*} \), where \( x = \{ M, R, 3P \} \) and \( y \) is any parameter in the model. Furthermore, \( \frac{\partial \Pi_M^M(q, \tau)}{\partial y} |_{q^*, \tau^*} = -\frac{\partial C(q, \tau)}{\partial y} |_{q^*, \tau^*} \) for \( y = \{ A, C_L, \eta, k \} \) because these parameters only come in via \( C(q, \tau) \). It is straightforward to show that \( \frac{\partial C(q, \tau)}{\partial \Pi_M^M} |_{q^*, \tau^*} = \tau^* \geq 0 \) since \( \tau \in [0, 1] \). Similarly, \( \frac{\partial C(q, \tau)}{\partial \Pi_M^R} |_{q^*, \tau^*} = q^* k \geq 0 \) since \( q^* \geq 0 \), \( \frac{\partial C(q, \tau)}{\partial \Pi_M^L} |_{q^*, \tau^*} = q^* \geq 0 \) since \( q^* \geq 0 \), and \( \frac{\partial C(q, \tau)}{\partial \eta} |_{q^*, \tau^*} = k q^*(k-1) \geq 0 \) since \( q^* \geq 0 \) and \( k > 0 \). Therefore, \( \forall x \in \{ M, R, 3P \} \) and \( \forall y \in \{ A, C_L, \eta, k \} \), \( \frac{\partial \Pi_M^M}{\partial y} |_{q^*, \tau^*} \leq 0 \). In other words, for any collection channel, the manufacturer’s equilibrium profits are decreasing in \( A, C_L, \eta \) and \( k \).

This result clearly indicates that when a party other than the manufacturer has a sufficiently large advantage in collection, manufacturer-managed collection is no longer optimal even if it was so at equal cost. As an example, consider a scenario with scale diseconomies where manufacturer-managed collection is optimal at equal costs, but some of the manufacturer’s cost parameters are larger than that of the retailer. In this case, \( \exists y^M > y^R \) such that \( \Pi_M^M(y^M) > \Pi_M^R(y^R) \). In other words, there exists a sufficiently large cost differential between the costs of the manufacturer and the retailer, such that manufacturer preferences...
would be reversed. However, it is important to note that this result does not hold for any $y^M > y^R$, a sufficiently large difference $y^M - y^R$ is required for the reversal.

Appendix B: Differentiable New and Remanufactured Products

In this appendix, we explore whether the fundamental insight we obtain from the undifferentiated product model continues to hold when new and remanufactured products are differentiated. To do so, we make a number of additional assumptions to represent a market with differentiable new and remanufactured products. In this setting, the manufacturer charges different wholesale prices to the retailer for new and remanufactured products, $w_n$ and $w_r$, respectively. The retailer sells new and remanufactured products at prices $p_n$ and $p_r$, respectively. We assume that consumer valuations ($\theta$) for new products are uniformly distributed between 0 and 1, and consumers’ valuations for the remanufactured product are lower. If a consumer values the new product at $\theta$, she values the remanufactured product at $\delta\theta$, where $\delta < 1$. Under these assumptions, the market can be characterized by the following inverse demand functions derived from consumer utilities:

\[ p_n = 1 - q_n - \delta q_r \]
\[ p_r = \delta(1 - q_n - q_r) \]

Here, $q_n$ ($q_r$) is the sales quantity of the new (remanufactured) product (see Ferguson and Toktay (2006) for derivations).

We assume that only a certain proportion ($\rho \leq 1$) of used products are accessible, i.e., if $q_n$ new products are sold, the maximum amount of remanufactured products that can be sold is $\rho q_n$ ($q_r \leq \rho q_n$). As before, the party that manages collection incurs cost $C(\tau, q_n)$ for collecting a fraction $\tau$ of available used products $\rho q_n$, and the cost to acquire $q_r = \tau \rho q_n$ units is $Aq_r$. We focus on a comparison between two special but representative cost structures exhibiting scale economies and diseconomies for tractability. We analyze the impact of scale economies by focusing on the investment cost and assume that $C(\tau, q_n) = C_L \tau^2$. The cost structure under diseconomies of scale is assumed to be $C(\tau, q_n) = \eta q_r^2 = \eta(\tau \rho q_n)^2$, i.e., $k = 2$, for tractability. This specification is representative of the general scale diseconomies case (see Atasu and Souza (2012) for an empirical validation). We assume that $c_m < 1$ and $\delta c_m > A + c_{rm}$, so that positive margins can be obtained from selling both new and remanufactured products. We also assume that the collection cost coefficients $\eta$ and $C_L$ are sufficiently high such that $\tau^* \leq 1$ and the desired scale effects are present. When the retailer manages collection, with differentiated wholesale prices, there is no need for an additional transfer price instrument (such as the transfer price $b$ in the base model). The equilibrium collected quantity matches the remanufactured quantity, and the wholesale price internalizes the transfer price.

In what follows, we focus on the manufacturer- and retailer-managed reverse channels as the third-party option will be dominated by the other channel configurations. We analytically prove that manufacturer collection is optimal under scale diseconomies. Under economies of scale, we resort to numerical analysis due to the complexity of the analytical expressions, and demonstrate that retailer collection is optimal. Thus, the results and intuition from the undifferentiable remanufactured products case hold when new and
remanufactured products are differentiated. These results reinforce the finding that the scale effect is the key element that determines the optimal reverse channel choice.

Diseconomies of Scale
Manufacturer-Managed Collection

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer’s sales quantities for any given \((w_n, w_r)\) pair that maximize \(\Pi_M^R = (p_n - w_n)q_n + (p_r - w_r)q_r\).

The problem is jointly concave in \(q_n\) and \(q_r\). The first-order conditions can be written as \(\frac{\partial \Pi_M^R}{\partial q_n} = 1 - 2q_n - 2\delta q_r - w_n = 0\), and \(\frac{\partial \Pi_M^R}{\partial q_r} = -2\delta q_n - \delta (1 + q_r) - \delta q_r - w_r = 0\). Solving the first-order conditions gives \(q_n(w_n, w_r) = \frac{-1 - \delta + w_n + w_r}{2(1 + \delta)}\) and \(q_r(w_n, w_r) = \frac{-\delta w_n - w_r}{2(1 + \delta)}\). Given the retailer’s anticipated reaction to \(w_n\) and \(w_r\), the manufacturer solves:

\[
\max_{w_n, w_r} (w_n - c_m)q_n + (w_r - c_r m - A)q_r - \eta q_r^2 \quad \text{subject to} \quad q_r \leq pq_n.
\]

The manufacturer’s constrained Lagrangian objective can be written as

\[
L(w_n, w_r, \lambda) = \Pi_M^R + \lambda (pq_n - q_r) = q_n (-c_m + \lambda \rho + w_n) - q_r (A + c_r m + \lambda + \eta q_r - w_r).
\]

Replacing the anticipated retailer sales quantities in the Lagrangian we obtain

\[
L(w_n, w_r, \lambda) = (\lambda \rho - w_n) (-1 + \delta + w_n - w_r) + \frac{(\delta w_n - w_r) (A + c_r m + \lambda - w_r + \frac{\lambda (-\delta w_n - w_r)}{\lambda + 1 + \delta})}{2 (1 + \delta)}.
\]

The problem is jointly concave in \(w_n\) and \(w_r\). The KKT conditions can be written as

\[
\frac{\partial \Pi_M^R}{\partial w_n} = \frac{1 + A - c_m + c_r m + \delta + \lambda + \lambda \rho + 2w_n - \frac{\lambda (-\delta w_n - w_r)}{2(1 + \delta)} + 2w_n - \frac{\lambda (-\delta w_n - w_r)}{2(1 + \delta)}}{2(1 + \delta)} = 0,
\]

\[
\frac{\partial \Pi_M^R}{\partial w_r} = \frac{c_m - \lambda \rho - w_n + \frac{\lambda (-\delta w_n - w_r)}{2(1 + \delta)} + A + c_r m + \lambda - w_r + \frac{\lambda (-\delta w_n - w_r)}{2(1 + \delta)}}{2(1 + \delta)} = 0,
\]

and

\[
\lambda (\rho - \frac{\delta w_n - w_r}{2(1 + \delta)} + \frac{\lambda (-\delta w_n - w_r)}{2(1 + \delta)}) = 0.
\]

**Case 1: \(\lambda > 0\), Constrained Problem**

The KKT conditions give

\[
w_n = \frac{1 + c_m + (A + c_r m + 2c_m + \delta) \rho + ((2 - c_r m + \delta + \eta) \rho^2)}{2 + \eta \rho^2 + 2 \delta \rho (2 + \rho)},
\]

\[
w_r = \frac{\delta (1 + c_m + (1 - 1 + \lambda + A + c_r m + 3 \delta) \rho + (A + c_r m + \delta + \eta) \rho^2)}{2 + \eta \rho^2 + 2 \delta \rho (2 + \rho)},
\]

and

\[
\lambda = \frac{\lambda (1 + c_m + (1 - 1 + \lambda + A + c_r m + 3 \delta) \rho + (A + c_r m + \delta + \eta) \rho^2)}{2 + \eta \rho^2 + 2 \delta \rho (2 + \rho)}.
\]

The resulting sales quantities are \(q_n = \frac{-1 + c_m + (A + c_r m - \delta) \rho}{4 + 2 \eta \rho^2 + 4 \delta \rho (2 + \rho)}\) and \(q_r = \frac{-2 (A + c_r m - c_m - \delta)}{2 + \eta \rho^2 + 2 \delta \rho (2 + \rho)}\).

It is easy to see that both \(q_n\) and \(q_r\) are positive when both margins are positive, i.e., \(\delta > A + c_r m\) and \(c_m < 1\). The condition \(\lambda > 0\) requires that \(\rho < \rho^M = \frac{-2 (A + c_r m - c_m - \delta)}{2 + A + c_r m + c_m - \delta + \eta - c_m \eta} \). The manufacturer’s profit when the manufacturer manages collection is \(\Pi_M^M = \frac{2 (A - c_r m - c_m - \delta)}{8 + 4 \eta \rho^2 + 8 \delta \rho (2 + \rho)}\).

**Case 2: \(\lambda = 0\), Unconstrained Problem**

The KKT conditions give \(w_n = \frac{1 + c_m}{2}\) and \(w_r = \frac{\delta (2 - 2 c_r m - 2 \delta + 2 A + 2 c_r m - 2 \delta - \eta - c_m \eta)}{2 (2 + 2 \eta \rho^2 - \eta)}\). The resulting sales quantities are \(q_n = \frac{\delta (2 - 1 + A + c_r m + \delta + (1 + c_m) \eta)}{8 (1 + \delta) \delta - 2 \eta}\) and \(q_r = \frac{A + c_r m - c_m - \delta}{4 (1 + \delta) \delta - 2 \eta}\). It is easy to see that both \(q_n\) and \(q_r\) are positive when \(1 - c_m > \delta - (c_r m + A)\), i.e. when the margin from manufacturing is higher...
than the margin from remanufacturing, and when both margins are positive, i.e., $\delta > A + c_{rm}$ and $c_{m} < 1$. The manufacturer’s optimal profit when the manufacturer collects is $\Pi^M_M = \frac{(-1 + c_m)^2 + 2(1 + c_m + c_{rm} - \delta)^2}{8}$. 

**Retailer-Managed Collection**

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer’s sales quantities for any given $(w_n, w_r)$ pair that maximize $\Pi^R_R = (p_n - w_n)q_n + (p_r - w_r - A)q_r - \eta q_r^2 = -q_n^2 - q_n (-1 + 2\delta q_r + w_r) - q_r (A - \delta + (\delta + \eta) q_r + w_r)$. The problem is jointly concave in $q_n$ and $q_r$. The first-order conditions can be written as $\frac{\partial \Pi^R_R}{\partial q_n} = 1 - 2q_n - 2\delta q_r - w_n = 0$, and $\frac{\partial \Pi^R_R}{\partial q_r} = -A + \delta - 2\delta q_n - 2(\delta + \eta) q_r - w_r = 0$. Solving the first-order conditions gives $q_n(w_n, w_r) = \frac{\delta^2 + \eta(-1 + w_n) - \delta(1 + A - w_n + w_r)}{2((-1 + \delta)\delta - \eta)}$ and $q_r(w_n, w_r) = \frac{A - \delta w_n + w_r}{(-1 + \delta)\delta - \eta}$.

Given the retailer’s anticipated reaction to $w_n$ and $w_r$, the manufacturer’s constrained Lagrangian objective can be written as $L(w_n, w_r, \lambda) = \Pi^R_R + \lambda(pq_n - q_r) = q_n (-c_m + \lambda \rho + w_n) - q_r (c_{rm} + \lambda - w_r)$. Replacing the anticipated retailer sales quantities in the Lagrangian we obtain

$L = \frac{(-c_{rm} + \lambda - w_r)}{(A - \delta w_n + w_r)} + \frac{(-c_m + \lambda \rho + w_n)}{(-\delta^2 + \eta(-1 + w_n))} \frac{(1 + A - w_n + w_r)}{2((-1 + \delta)\delta - \eta)}$.

The problem is jointly concave in $w_n$ and $w_r$. The KKT conditions can be written as

$\frac{\partial \Pi^R_R}{\partial w_n} = \frac{-(-\delta^2 + \eta(-1 + w_n) - \delta(-c_{rm} + \lambda - w_r) + (1 + A - w_n + w_r))}{2((-1 + \delta)\delta - \eta)} = 0,$

$\frac{\partial \Pi^R_R}{\partial w_r} = \frac{-(-A + c_{rm} + \lambda + \delta w_n + \delta(-c_m + \lambda \rho + w_n) - 2w_r)}{2((-1 + \delta)\delta - \eta)} = 0$, and

$\frac{\partial \Pi^R_R}{\partial \lambda} = \frac{-A - \delta w_n + w_r + \rho(-\delta^2 + \eta w_n + \delta(1 + A - w_n + w_r))}{2((-1 + \delta)\delta - \eta)} = 0$.

**Case 1: $\lambda > 0$, Constrained Problem**

Solving the KKT conditions gives $w_n = \frac{1 + c_m + c_{rm} \delta + \rho(A + c_{rm} + (A + c_{rm} - \delta^2 + \eta(1 + \lambda)\rho) A + c_{rm} - \delta^2 + \eta(1 + \lambda)\rho)}{2 + 4 \delta + 2(\delta + \eta)\rho^2}$, and $w_r = \frac{-2 A + \delta + c_{rm} \delta + (\delta - 1 - 3 A + c_{rm} + c_{rm} - \delta)(\delta - 1 - 3 A + c_{rm} + c_{rm} - \delta)(\delta - 1 - 3 A + c_{rm} + c_{rm} - \delta)(\delta - 1 - 3 A + c_{rm} + c_{rm} - \delta)}{4 + 8 \delta + 4(\delta + \eta)\rho^2}$. The resulting sales quantities are $q_n = \frac{-1 + c_m + (A + c_{rm} - \delta)^2}{4 + 8 \delta + 4(\delta + \eta)\rho^2}$, and $q_r = \frac{-\rho(-1 + c_m + (A + c_{rm} - \delta)^2)}{4 + 8 \delta + 4(\delta + \eta)\rho^2}$. It is easy to see that both $q_n$ and $q_r$ are positive when both margins are positive, i.e., $\delta > A + c_{rm}$ and $c_{m} < 1$. The condition $\lambda > 0$ requires that $\rho < \rho^R = \frac{A + c_{rm} - c_m \delta}{(-1 - A + c_m - c_{rm} - \delta^2 + (1 + c_m)\eta)}$. The manufacturer profit is $\Pi^R_R = \frac{(-1 + c_m + (A + c_{rm} - \delta)^2)}{8 + 8 \rho(\eta(1 + \delta(2 + \rho)))}$.

**Case 2: $\lambda = 0$, Unconstrained Problem**

Solving the KKT conditions at $\lambda = 0$ gives $w_n = \frac{1 + c_m + c_{rm} \delta}{2 + 4 \delta + 2(\delta + \eta)\rho^2}$ and $w_r = \frac{-A + c_{rm} - \delta^2 + (1 + c_m)\eta}{4(-1 + \delta)\delta - 4 \eta}$. The resulting sales quantities are $q_n = \frac{-1 - A + c_{rm} - c_m \delta + \delta^2 + (1 + c_m)\eta}{4(-1 + \delta)\delta - 4 \eta}$ and $q_r = \frac{A + c_{rm} - c_m \delta}{4(-1 + \delta)\delta - 4 \eta}$. It is easy to see that both $q_n$ and $q_r$ are positive when $c_{m} < 1$ and $A + c_{rm} < \delta c_{m}$. The manufacturer profit is $\Pi^R_R = \frac{(-1 + c_m)^2 + 2(1 + c_m + c_{rm} - \delta)^2}{8}$. 

**Comparison of Alternatives**

We would like to know when the manufacturer makes more profit by managing collection himself. To compare the profits we need to compare $\rho^R$ and $\rho^M$ first. It is easy to see that

$\frac{\rho^R}{\rho^M} = \frac{2}{2} \frac{(1 + A - c_m + c_{rm} - \delta)(\delta + \eta - c_m \eta)}{(1 + A - c_m + c_{rm} - \delta)} > 1$

Thus, we can compare the profits depending on the value of $\rho$ as follows:

- **When $\rho \geq \rho^M$, we should compare the unconstrained profits. In this case,**
\[ \Pi^M_M - \Pi^R_M = \frac{(A + c_{cm} - c_m \delta)^2 \eta}{8 (2 (1 - \delta) \rho^2 - (1 - \delta) \delta \eta + \eta^2)}. \]

This term is always positive. Thus the manufacturer is always better off when he manages collection.

- When \( \rho < \rho^R \), we should compare the constrained profits. In this case,
  \[ \Pi^M_M - \Pi^R_M = (-1 + c_m + (A + c_{rm} - \delta) \rho)^2 \left( \frac{1}{8 + 4 \eta \rho^2 + 8 \eta \rho (2 + \rho)} - \frac{1}{8 + 8 \rho (\eta \rho + 3 + 2 \rho)^2} \right). \]
  Note that the term on the left hand side is always positive. The term on the right hand side within parentheses is also nonnegative since \((8 + 8 \rho (\eta \rho + (2 + \rho))) - (8 + 4 \eta \rho^2 + 8 \delta \rho (2 + \rho)) = 4 \eta \rho^2 \geq 0\). Thus, the manufacturer is always better off managing collection himself when \( \rho \leq \rho^R \).

- When \( \rho^R \leq \rho < \rho^M \), the comparison of profits should be done between the constrained profit under manufacturer-managed collection and the unconstrained profit under retailer-managed collection.

It is straightforward to see that the constrained manufacturer profit is increasing in \( \rho \) when the constraint is binding and constant otherwise. Accordingly, when \( \rho^R \leq \rho < \rho^M \), \( \Pi^M_M \) is increasing in \( \rho \) while \( \Pi^R_M \) is constant. We know from the previous case that at the lower bound of this range, i.e., when \( \rho = \rho^R \), \( \Pi^M_M \geq \Pi^R_M \). Thus, for any value of \( \rho \) in this range \( \Pi^M_M > \Pi^R_M \) continues to hold since \( \Pi^M_M \) is increasing and \( \Pi^R_M \) is constant in \( \rho \). This completes the proof that the manufacturer is better of managing the collection himself for any range of \( \rho \).

**Economies of Scale**

In this case, since neither party would optimally collect a higher volume than that would be remanufactured, the inequality that \( q_r \leq \rho r q_m \) will be binding. Thus, we set \( q_r = \tau q_m \rho \) in the following analysis.

**Manufacturer-Managed Collection**

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer’s sales quantities for any given \((w_r, w_r)\) pair that maximize \( \Pi^R_M = (p_n - w_n) q_n + (p_r - w_r) q_r \). The problem is jointly concave in \( q_n \) and \( q_r \). The first-order conditions can be written as \( \frac{\partial \Pi^M_M}{\partial q_n} = 1 - 2 q_n - 2 \delta q_r - w_n = 0 \), and \( \frac{\partial \Pi^M_M}{\partial q_r} = -2 \delta q_n - \delta (-1 + q_r) - \delta q_r - w_r = 0 \). Solving the first-order conditions gives \( q_n(w_n, w_r) = \frac{-(1-\delta w_n + w_r)}{2(1-\delta)\rho} \) and \( q_r(w_n, w_r) = \frac{\delta w_n - w_r}{2(1-\delta)\rho} \). Replacing the retailer’s anticipated reaction in the manufacturer’s objective \( \Pi^M_M = (w_n - c_m) q_n + (w_r - c_r - A) q_r - C_L \tau^2 \), we obtain

\[ \Pi^M_M = \frac{\delta}{2 (1 - \delta) \rho} \left( -2 (-1 - \delta) C_L \tau^2 + w_n (-1 + A + c_{rm} + \delta + w_n) - c_m \delta (1 + \delta + w_n - w_r) - (A + c_{rm} + 2 \delta w_n) w_r + w_r^2 \right). \]

Thus, the Lagrangian is written as \( L(w_n, w_r, \tau, \lambda) = \Pi^M_M - \lambda (q_r(w_n, w_r) - q_n(w_n, w_r)) \rho \tau \). The first-order conditions are

\[ \frac{\partial \Pi^M_M}{\partial w_n} = \frac{\delta (1 + c_{rm} + A + 1) + \lambda + \lambda \tau + 2 w_n - 1 - 2 \delta w_n}{2(1-\delta)\rho} = 0, \]
\[ \frac{\partial \Pi^M_M}{\partial w_r} = \frac{-c_m - A - \delta - \lambda \tau - 2 w_n - \delta - 2 w_n - \lambda + 2 w_n}{2(1-\delta)\rho} = 0, \]
\[ \frac{\partial \Pi^M_M}{\partial \lambda} = \frac{\delta (1 - \delta) \rho \tau + (\rho + 1) w_n - (\rho \tau + 1) w_r}{2(1-\delta)\rho} = 0, \text{ and} \]
\[ \frac{\partial \Pi^M_M}{\partial \tau} = \frac{\delta (1 - \delta) (4 C_L \tau - \lambda + 1) w_n - 3 \delta \rho w_n}{2(1-\delta)\rho} = 0. \]
The joint solution to these first-order conditions is not feasible in closed form. Multiple stationary points exist and finding the maximizer requires solving a fifth-order polynomial with respect to $\tau$. Nevertheless, a numerical solution can be obtained, which we use later to compare the two scenarios.

**Retailer-Managed Collection**

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer’s sales quantities for a $(w_n, w_r)$ pair that maximize $\Pi^R_M = (p_n - w_n)q_n + (p_r - w_r - A)q_r - C_L \tau^2$. The first-order conditions on the retailer’s objective are $\frac{\partial \Pi^R_M}{\partial q_n} = -2q_n - 2\delta q_r + \lambda \rho \tau - w_n + 1 = 0$, $\frac{\partial \Pi^R_M}{\partial q_r} = -A - \lambda - 2\delta q_n - \delta(q_r - 1) - \delta q_r - w_r = 0$, $\frac{\partial \Pi^R_M}{\partial \lambda} = q_n \rho \tau - q_r = 0$, and $\frac{\partial \Pi^R_M}{\partial \tau} = \lambda q_n \rho - 2C_L \tau = 0$. Unfortunately, the joint solution to these first-order conditions cannot be obtained in closed form and is not unique. Nevertheless, the retailer’s best response functions are unique for any $\tau$. Thus, the manufacturer can anticipate the retailer’s best response functions for any $\tau$. Solving the first three first-order conditions, we obtain $q_n(w_n, w_r|\tau) = -\frac{w_n + \rho \tau(A - \delta + w_r)}{2\rho \tau(\rho + 2) + 2}$, $q_r(w_n, w_r|\tau) = -\frac{\rho \tau(w_n + \rho \tau(A - \delta + w_r - 1))}{2\rho \tau(\rho + 2) + 2}$, and $\lambda(w_n, w_r|\tau) = -\frac{\delta \rho \tau(A + \delta + (\delta(w_n + \rho \tau(A - \delta + w_r - 1)) - w_r))}{\rho \tau(\rho + 2) + 2}$. Anticipating the retailer’s decision for a given $\tau$, the manufacturer’s objective is $\Pi^M_M(w_n, w_r|\tau) = \frac{(c_m - w_n + \rho \tau(c_m - w_r))(w_n + \rho \tau(A - \delta + w_r - 1))}{2\rho \tau(\rho + 2) + 2}$. It is easy to see that the manufacturer’s objective $\Pi^M_M(w_n, w_r|\tau)$ can be written as a function of $w_n + w_r \tau^2$, i.e., $w_n$ and $w_r$ are linearly dependent in the manufacturer’s objective. Thus, any $w_n, w_r$ pair that satisfies $w_n = \frac{1 + c_m + \rho \tau(c_m - w_r) + (\delta - w_r)}{2}$ characterizes the manufacturer’s best response. Consequently, the manufacturer announces wholesale prices as a function of the collection rate $w_n(\tau) = \frac{1 + c_m + \rho \tau(c_m - w_r) + (\delta - w_r)}{2}$. (Note that this is equivalent to the effective wholesale price $(w - \tau b)$ in the perfect substitution scenario.) This induces the retailer to sell $q_n(\tau) = \frac{c_m + (A - c_m + \delta) \rho \tau + 1}{4\rho \tau(\rho + 2) + 4}$ and $q_r(\tau) = \frac{\rho \tau(c_m + A - c_m + \delta) \rho \tau + 1}{4\rho \tau(\rho + 2) + 4}$.

As in the manufacturer managed collection scenario, a closed-form characterization of the optimal $\tau$ is not possible, since it requires solving a fifth-order polynomial. Nevertheless, numerically this task is not as challenging as it is analytically.

**Comparison of Alternatives**

A numerical analysis reveals that collection at the retailer dominates under economies of scale. Tables 12 and 13 illustrate this finding with a full factorial analysis for $c_m = 0.4, 0.2$, $\delta = 0.2, 0.5, 0.6, 0.8$, $C_L = 0.008, 0.005$, $\rho = 0.8, 0.4$ and $\gamma = 0.8, 0.6$, where $c_{rm} = c_m \delta \gamma$. These parameter settings are chosen to ensure that both the manufacturer and the retailer choose a positive collection rate $\tau$, new product sales $q_n$ and remanufactured product sales $q_r$. 

28
Table 12: Comparison of manufacturer- and retailer- managed collection scenarios under economies of scale for differentiable products for intermediate $\delta$ values

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Table 13: Comparison of manufacturer- and retailer-managed collection scenarios under economies of scale for differentiable products for high and low $\delta$ values

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