Carbon leakage: The impact of asymmetric emission regulation on technology and capacity investments

Kristel M. R. Hoen
Quintiq, 5201 AG’s-Hertogenbosch, The Netherlands, kristel.alons@quintiq.com

Ximin (Natalie) Huang
Scheller College of Business, Georgia Institute of Technology, Atlanta, Georgia 30308, ximin.huang@scheller.gatech.edu

Tarkan Tan
Industrial Engineering and Innovation Sciences, Eindhoven University of Technology, 5612 AZ Eindhoven, The Netherlands, t.tan@tue.nl

L. Beril Toktay
Scheller College of Business, Georgia Institute of Technology, Atlanta, Georgia 30308, beril.toktay@scheller.gatech.edu

In the absence of global emission regulation, production cost differences between regions are typically accentuated due to different emission charges, which may render offshore production in an unregulated region more attractive and result in “carbon leakage.” We study three anti-leakage policies including Border Tax (BT), Output-based Allocation (OB) and Grandfathering (GF). We investigate the problem from the perspective of an energy-intensive good producer subject to geographically asymmetric emission regulation and uncertainty in future emission cost. The producer has two ex-ante investment opportunities to mitigate its emission cost: investing in clean production technology in the regulated region and building production capacity in the unregulated region. It determines its production quantities in the two regions ex-post, after emission price uncertainty is resolved. We start from a Baseline case without an anti-leakage policy and then study the above three anti-leakage policies. We compare the effectiveness of the policies by exploring how they influence the producer decisions in both an analytical model and a numerical study with a full factorial design based on the European cement industry. We show that when off-shore production is feasible (a carbon leakage threat exists), a moderate rather than a very high emission price with low volatility helps encourage more technology improvement and less offshore production. The BT and the OB policies are both effective in reducing carbon leakage. Interestingly, the effect of the GF policy significantly depends on the allowance level chosen, which, if set improperly, may even lead to more severe carbon leakage than in the absence of an anti-leakage policy.

Key words: Carbon leakage, Anti-leakage policies, Technology choice, Production capacity choice.

1. Introduction
Carbon dioxide (CO\textsubscript{2}) emissions account for approximately three-quarters of the worldwide greenhouse gas (GHG) emissions, which are widely believed to be the main cause of global warming and the resulting threats to human health and social well-being (EPA 2014). In response, the development and implementation of carbon emission regulation have shown rapid progress in the recent decades. Carbon emission regulation may adopt different approaches: Carbon cap-and-trade
regulation (Tietenburg 2010) sets a cap for the overall emissions, and allows the trading of emission permits. The European Union Emission Trading Scheme (EU-ETS) has built the largest GHG trading scheme in the world (European Commission 2010). Other similar schemes have been established in Tokyo (Business Green 2010), New Zealand (The New Zealand Emissions Trading Scheme 2011), nine northeastern states in the US under the Regional Greenhouse Gas Initiative (RGGI 2011), recently in South Korea (ICAP 2015b), and China plans to introduce a national trading scheme in 2016 (ICAP 2015a). Alternatively, carbon taxes can be used. Examples of carbon tax regulation have been seen in the UK under the Carbon Reduction Commitment Energy Scheme (DCLG 2010) and in California (California Environmental Protection Agency 2013). Carbon emission regulation has a wide scope of influence: The EU-ETS alone covers 11,000 installations and 40% of the total GHG emissions in the EU; the 17 emission trading systems across the globe cover jurisdictions that produce about 40% of the global GDP (ICAP 2015).

Carbon taxes and carbon trading schemes, although they differ in multiple aspects, effectively impose a unit price on carbon emissions. Such a carbon price poses challenges for producers, especially those in energy-intensive sectors such as the cement, electricity and steel industries. First, the carbon price may considerably erode operating profit margins in the regulated region. For example, under the EU-ETS, a cement producer in Europe can be charged with an emission price of up to 25% of the sale price (Drake et al. 2010, Lafarge 2010). The second challenge is that producers have to deal with uncertainty in the emission price and plan accordingly, where uncertainty may arise in the regulation process or the market dynamics of the emission trading system. For example, the carbon tax scheme launched in 2012 in Australia was repealed in 2014 (ICAP 2015a), and the emission trading price in EU-ETS has varied between €1/ton and €32/ton.

The compliance costs and the associated uncertainty raise the risk of inducing producers to shift production to a region in which carbon emission charges are absent or lower. This is referred to as “carbon leakage.” As estimated by the European Commission, in Phase III of the EU-ETS, 60% of the covered industries are subject to the carbon leakage risk, accounting for 95% of all covered emissions (De Bruyn et al. 2013). Carbon leakage is environmentally undesired. First, it increases total emissions: As production is shifted to a region with weaker regulatory requirements, and hence is likely be conducted with less up-to-date technologies, it results in higher production emission. Moreover, transporting products back to the sales market in the regulated region generates additional transportation emissions. These are Scope III emissions that are generally out of the scope of regulation, which adds to the severity of the problem. Second, carbon leakage weakens or even mutes the effectiveness of emission regulation in incentivizing clean production technology in the regulated region, because it diminishes the returns on the technology investment (as the technology applies to a smaller volume of production).
To deal with the carbon leakage problem, “anti-leakage” policies have been proposed. These policies adopt either one of the two approaches: imposing a “border adjustment (BA)” to imported products, or providing producers with emission rights in the regulated region. BA can take diverse forms such as requiring importers to purchase emission permits (Bingaman-Specter Bill) or to surrender emission allowances from the “International Reserve Allowance Program” (Waxman-Markey Bill). One of the major objections to BA is the concern that it sets trade barriers and causes discrimination concerns that violate the WTO rules. An approach to mitigate this concern is to use a border tax based on the “best available technology” (BAT) in the same sector (Ismer and Neuhoff 2007). Moreover, the BAT approach may facilitate easier implementation as only one technological process needs to be accessed in contrast to having to evaluate the technology used by each producer. Therefore, we assume the BAT approach is used in the specification of the border tax. Such a border tax deters carbon leakage by effectively diminishing profit margins from offshore production, and hence incentivizes producers to keep production in the regulated region. It may also drive the unregulated regions to adopt certain regulations to avoid the border tax.

There are a number of ways to provide producers with emission rights for production in the regulated region, such as a free emission allowance under a cap-and-trade system, or a tax credit under a carbon tax system (hereafter uniformly referred to as free allowance for ease of exposition). Contrary to the “stick” approach adopted by a border tax that penalizes production in the unregulated region, the allowance allocation is a “carrot” approach that keeps production in the regulated region by exempting some of the emission charges. Grandfathering is a popular method of free allowance allocation, which calculates the allowance based on historical production figures (Demailly and Quirion 2006). Although it exhibits higher feasibility and simplicity in implementation, Grandfathering raises the concern that it may unfairly favor producers that have produced more and generated more emissions in the past. Output-based allocation remedies this problem by granting an allowance based on current production figures. Allowance allocation in reality can fall somewhere between these two cases due to updates of allocation based on recent production figures as in the EU-ETS.

In this paper, we analyze the operational decisions of a producer under emission regulation that is asymmetric across regions and uncertain in the future. Specifically, we study the producer decisions on investments in clean production technologies in the regulated region and production capacity in the unregulated region, as well as the respective production volumes, in a Baseline case without an anti-leakage policy. Then we extend the analysis to the cases with different anti-leakage policies. By studying these cases, we compare the effectiveness of the anti-leakage policies in restoring the effectiveness of emission regulation that is undermined by carbon leakage, in (i) reducing overall emissions, and (ii) motivating clean production technology improvement. In
particular, we consider three anti-leakage policies including Border Tax, Output-based Allowance Allocation, and Grandfathering.

We study the problem analytically, and also conduct a numerical study based on data from the cement industry in Europe, which is one of the most influenced sectors under the EU-ETS regulation. Our analysis demonstrates the underlying mechanisms through which the border tax and the allowance mitigate carbon leakage. We show that the producer tends to invest less in emission abatement technologies when the emission price is more volatile. The Border Tax and the Output-based policies are more efficient in inducing more investment in cleaner production technology in the regulated region and reducing overall production emissions. Another interesting finding is that the outcomes under the Grandfathering policy are quite sensitive to the policy specifications, and may lead to rebound effects with more severe carbon leakage consequences.

The rest of the paper is organized as follows: In §2, we position our work in the existing body of literature. We present the model in §3 and analyze the problem in §4. In a numerical study presented in §5, we apply the analysis to a case study. §6 concludes the paper.

2. Literature

As we consider a setting with multiple aspects including technology choice, capacity investment, emission cost uncertainty, carbon leakage, and alternative anti-leakage policies, our paper is related to a wide range of research fields within the operations management and environmental economics literature. In what follows, we highlight research that draws from the intersections of two or more of these aspects and position our work in the literature.

Within the environmental economics literature, there exists a large body of research that focuses on the impact of environmental regulation. See e.g., Jaffe et al. (2003) for a review on the inter-relation between technological changes and environmental issues and policies; Smale et al. (2006) for the influence of emission trading on firm competitiveness in five energy-intensive industries including cement, newsprint, steel, aluminum and petroleum; and Heutel (2011) and Fan et al. (2010) for the impact of uncertain emission regulation in the power industry. The latter two papers show that under certain conditions, emission regulation may adversely increase investments in the dirty as opposed to the clean technology, which is in line with our conclusions. Bushnell and Chen (2012) study the influence of different emission permit allocations on the permit price and the total emissions in a cap-and-trade system for the US electricity market. Demailly and Quirion (2006) and Fischer and Fox (2009) consider alternative anti-leakage policies and compare their impact and effectiveness. Demailly and Quirion (2006) consider producers that choose an abatement level and a production quantity in a Cournot competition model, and then compare Grandfathering with Output-based Allocation and investigate their impacts on competitiveness of the European
cement industry using simulations. Fischer and Fox (2009) use a computable general equilibrium model and simulations to investigate the impact of asymmetric emission regulation on certain emission-intensive sectors, where foreign firms may enter to supply to the home market. The main difference between our work and all of the papers mentioned above is that the environmental economics literature approaches the problem from the perspective of a social planner, whereas we take an operational point of view from the producer and explore how the producer (i) invests in clean production technology in the regulated region and offshore production capacity in the unregulated region to hedge the uncertainty in future emission regulation, and (ii) optimizes the production volumes when the uncertainty is resolved. As such, we uncover richer results, for example, how the expected value and volatility of uncertain future emission price influence the producer decisions, and that grandfathering may lead to even more severe carbon leakage under certain policy parameter choices.

The impacts of emission regulation and anti-leakage policies are also investigated in the operations management literature. Sunar and Plambeck (2015) study an interesting problem of comparing different rules of allocating total emissions among coproducts: They explore how the border emissions tax imposed on the primary product influences the total emissions and welfare under each of the allocation rules, and show that the border tax may increase emissions under certain allocations. Another stream of research shares our focus on the technology investment decisions under emission regulation. Yalabik and Fairchild (2011) address this problem in monopolistic and duopolistic settings without considering the capacity investment option and anti-leakage policies. Krass et al. (2013) study how the carbon tax rate influences a producer’s adoption of green production technology and pricing decisions. Some research along the same line also considers the uncertainty in regulation. Zhao (2003) for example, looks at abatement technology choices for competing producers with uncertain emission charges. In a particularly relevant work, Drake et al. (2015) consider a capacity investment problem for a number of available technology types under emission cost uncertainty. They consider an exogenous sales price for a profit-maximizing company and compare the impacts of carbon tax and cap-and-trade regulation on technology investments and company profit. We consider investments in both the technology and offshore production capacity simultaneously in a price-dependent demand setting, and address the carbon leakage problem together with anti-leakage policies. Drake (2012) and Islegen et al. (2015) analyze carbon leakage when firms can shift production or use an alternative technology. Drake (2012) considers a set of domestic firms and a set of foreign firms in a Cournot competition with a fixed emissions price where domestic firms can produce offshore, and each company decides how much to produce with each technology in the absence of uncertainty. Islegen et al. (2015) consider a similar setting where
firms determine how much to produce in a domestic and a foreign region under emission cost uncertainty. Our work is distinguished from both of them as we simultaneously consider the investments in local production technology and offshore capacity building, as well as the associated investment costs simultaneously. Moreover, we compare the effectiveness of three anti-leakage policies from aspects including producer profitability, overall emission reduction, and technology advancement.

In related work outside our environmental focus, an extensive overview of the operations management literature that considers capacity investment decisions under uncertainty is given by Van Mieghem (2003). In our model, the producer is facing an operating margin that is influenced by the uncertain emission cost, and it hedges against the risk of a low margin from a high emission price by choosing offshore production, which causes carbon leakage. In Ding et al. (2007) that considers a producer with production and sales in two regions, the uncertainty that influences the operating margins emerges from the exchange-rate, and the producer hedges by investing in capacity and/or buying financial options on the exchange-rate. Similar to offshore production, outsourcing is another typical strategy used to deal with uncertainties regarding demand and/or production capacity, see e.g., Van Mieghem (1999), Yang et al. (2005) and Wang et al. (2011). In particular, Wang et al. (2011) study the sourcing decision subject to non-tariff barriers (NTBs) that work similarly to a carbon border tax. They consider several supply chain strategies that differ in the procurement quantities from different locations, and study how the preferences of the strategies are influenced by the mean and variance of the NTB price. In comparison, our analysis comprehensively considers the technology and capacity investments under possible offshore production.

3. The Model

We consider a monopolist currently producing and selling a single type of product in a regulated region. The regulation imposes an emission price on the producer for each unit of emission from production, and the future emission price is uncertain. The producer has two options to deal with the economic burden associated with the emission price in the future. The first option is to reduce the amount of emissions in the regulated region by investing in emission abatement technology in production facilities in the regulated region. The second option is to reduce the volume of production subject to emission price by producing offshore in an unregulated region where emission regulation is absent. This option results in carbon leakage. Both alternatives require an upfront capital investment and significant lead time to become operational.

We study this problem in a two-stage framework. To capture the uncertainty regarding the emission price, \( e \) is a random variable in Stage 1 whose value is realized in Stage 2. At the beginning of Stage 1, the producer accounts for the uncertainty regarding the emission price, and makes the investment decisions on both (i) how much to invest in the cleaner technology for production in
the regulated region, and (ii) how much production capacity to build in the unregulated region. At the beginning of Stage 2, both the cleaner technology and the production capacity become available, and the actual value of the emission price is revealed. The producer then determines the production sizes in both the regulated and unregulated regions.

We first study a Baseline case with no anti-leakage policy. It reveals the optimal investment and production decisions for the producer under the asymmetric emission regulation with uncertain future emission price, and demonstrates the carbon leakage problem. Building on this case, we then compare three anti-leakage policies: the Border Tax policy, the Output-based policy, and the Grandfathering policy.

3.1. Model description

At the beginning of Stage 1, the emission intensity, defined as the emission amount from each unit of production, is $\gamma^+$ for the producer. The best available technology for the product, representing a lower bound of the feasible emission intensity, is denoted by $\gamma^-$. In Stage 1, the producer determines the emission intensity $\gamma$ ($\gamma^- \leq \gamma \leq \gamma^+$) to be achieved in Stage 2 in the regulated region. This results in an emission abatement of $\gamma^+ - \gamma$ and incurs an immediate technology investment cost of $\beta_r (\gamma^+ - \gamma)^2$, with $\beta_r (\beta_r > 0)$ denoting the abatement cost factor. The quadratic form of the cost reflects the general notion that it becomes increasingly more difficult and expensive to reduce production emissions. Moreover, since all the current production is taking place in the regulated region, we assume the production capacity in the regulated region is sufficiently large and does not constitute a constraint in Stage 2, which simplifies the analysis.

Another Stage 1 decision is the production capacity $K$ to be built in the unregulated region. We assume the associated immediate investment cost to be linear in the capacity with coefficient $\beta_u$ ($\beta_u > 0$), which will be $\beta_u K$.

In Stage 2, after the emission price is revealed, the producer determines production volumes $q_j$ ($q_j \geq 0$), where we use the subscript $j$ ($j \in \{r, u\}$) to represent the quantities in the regulated region (with $r$) and the unregulated region (with $u$), respectively. We assume a downward sloping demand curve, so the inverse demand function is $p(q_r, q_u) = \tilde{p} - \frac{1}{\epsilon} (q_r + q_u)$ where $\epsilon$ is the demand elasticity and $\tilde{p} = \frac{Q}{\epsilon}$ with $Q$ being the maximum market size. The price is determined by the aggregate production since all products are sold in the regulated region. Production in each region incurs a base unit production cost $c_j$ (with $c_j > 0$ and $j \in \{r, u\}$), which includes material, labor and other variable costs; specifically, $c_u$ also includes the additional transport cost for shipping the product from the unregulated region to the regulated region. Moreover, for production in the regulated region, we allow for an incremental unit production cost $\nu (\gamma^+ - \gamma)$ with $\nu > 0$ to reflect that production with the cleaner technology may be more expensive due to added complexity. For
example, in the case of cement, the production cost of the current technology was 43.6 €/ton in 2010 whereas the production cost of the best available technology was 55.0 €/ton (Drake et al. 2015). An emission price \( e \) is also charged for each unit of emissions from production. Therefore, the unit production cost in the regulated and the unregulated regions will be \( c_r + \nu(\gamma^+ - \gamma) + \gamma e \) and \( c_u \), respectively.

For production in the unregulated region in Stage 2, we assume that the original technology used in the regulated market applies and hence the emission intensity is \( \gamma^+ \). Moreover, shipping the products back to the regulated market incurs additional transportation emissions (i.e., Scope III emissions), which we denote by \( \gamma_t^u \) (\( \gamma_t^u > 0 \)). Therefore, the total emissions attributable to one unit of production in the regulated and the unregulated regions are \( \gamma \) and \( \gamma^+ + \gamma_t^u \), respectively.

Table 1 provides an overview of the model parameters.

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>Emission intensity in the regulated market</td>
</tr>
<tr>
<td>( K )</td>
<td>Production capacity in the unregulated region</td>
</tr>
<tr>
<td>( q_{ij} )</td>
<td>Stage 2 production quantity in market ( j ), ( j \in {r, u} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_j )</td>
<td>Base unit production cost, ( j \in {r, u} )</td>
</tr>
<tr>
<td>( \beta_u )</td>
<td>Capacity investment cost factor in the unregulated region</td>
</tr>
<tr>
<td>( \beta_r )</td>
<td>Emission abatement cost factor in the regulated market</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Unit production cost change due to emission intensity reduction</td>
</tr>
<tr>
<td>( \gamma^+ )</td>
<td>Initial emission intensity</td>
</tr>
<tr>
<td>( \gamma^- )</td>
<td>Emission intensity of the best available technology</td>
</tr>
<tr>
<td>( \gamma_t^u )</td>
<td>Unit transportation emission associated with production in the unregulated region</td>
</tr>
<tr>
<td>( e )</td>
<td>Emission cost (random variable), value realized in Stage 2</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Demand elasticity</td>
</tr>
</tbody>
</table>

### 3.2. Problem formulation

We first formulate the problem in the Baseline case in the absence of any anti-carbon leakage policy. Due to the sequential nature of the problem, we solve it by backward induction. In Stage 2, given the investment decisions on \( K \) and \( \gamma \) and the realization of \( e \), the Stage 2 profit \( \Pi(q_r, q_u|e, \gamma, K) \) as a function of \( q_r \) and \( q_u \) is:

\[
\Pi(q_r, q_u|e, \gamma, K) = p(q_r, q_u)(q_r + q_u) - q_r(c_r + \nu(\gamma^+ - \gamma) + e\gamma) - q_u c_u
\]

\[
= -\frac{1}{\epsilon}(q_r + q_u)^2 + q_r(\tilde{p} - c_r - \nu(\gamma^+ - \gamma) - \gamma e) + q_u(\tilde{p} - c_u) = -\frac{1}{\epsilon}(q_r + q_u)^2 + q_r(m_r - e\gamma) + q_u(m_u),
\]

where we denote \( m_r = \tilde{p} - c_r - \nu(\gamma^+ - \gamma) \) and \( m_u = \tilde{p} - c_u \) related to the profit margins in the regulated and unregulated regions, respectively. In the following analysis, we assume \( c_u > c_r \) such that before Stage 2, the producer only produces in the regulated region and carbon leakage does not occur.

The Stage 2 profit optimization problem is:

\[
(P_2) \max_{q_r, q_u} \Pi(q_r, q_u|e, \gamma, K) \quad \text{s.t.} \quad q_r \geq 0, \quad 0 \leq q_u \leq K
\]
Let $\Pi^*(e, \gamma, K)$ denote the maximum Stage 2 profit given the investment quantities $\gamma$ and $K$ for a realized emission cost $e$, and let $E_e[\Pi^*(e, \gamma, K)]$ be the expected maximum Stage 2 profit that accounts for the distribution of the emission price $e$. We omit the profit from production and sales in Stage 1 because it has no influence on the investment and production decisions under consideration. Let the two-stage profit, as a function of the Stage 1 decisions $\gamma$ and $K$, be denoted by $Z$. Then

$$ Z(\gamma, K) = E_e[\Pi^*(e, \gamma, K)] - \beta_r(\gamma^+ - \gamma)^2 - \beta_uK. \tag{1} $$

The two-stage problem is:

$$ (P) \max_{\gamma, K} Z(\gamma, K) \quad \text{s. t.} \quad K \geq 0 \quad \gamma^- \leq \gamma \leq \gamma^+ $$

The total expected Stage 2 emission is denoted by $\Gamma$ and $\Gamma = E[\Gamma_r] + E[\Gamma_u]$ where $E[\Gamma_r] = \gamma E_e[q_r]$ and $E[\Gamma_u] = (\gamma^+ + \gamma_u) E_e[q_u]$.  

4. Analysis

In this section, we first analyze the Baseline case. Then we study and compare the anti-leakage policies: the Border Tax policy, the Output-Based Allocation policy, and the Grandfathering policy in §4.2, §4.3, and §4.4, respectively. All proofs can be found in the Appendix.

4.1. The Baseline Case

We first solve the Stage 2 production quantity problem ($P_2$) and denote the optimal production quantities as $(q^*_r, q^*_u)$. Note that $K \leq \frac{1}{2}m_u$ always holds, because it is straightforward to prove that the optimal $q_u$ that is unconstrained by the production capacity is $\frac{1}{2}m_u$, and building any costly production capacity in excess will be suboptimal. Let $\bar{\gamma} = \frac{e_u - e_r - \gamma^+}{e_r - e_u}$ denote the emission intensity at which the profit margins in the two regions are equal.

**Lemma 1.** Given $e, \gamma, K \geq 0$, the profit-maximizing Stage 2 production quantities are:

**Case (i)** When $e \geq \nu$:

$$ (q^*_r, q^*_u) = \begin{cases} 
(\frac{1}{2}(m_r - e\gamma), 0) & \text{if } \gamma \leq \bar{\gamma}, \\
(\frac{1}{2}(m_r - e\gamma) - K, K) & \text{if } \bar{\gamma} \leq \gamma \quad \text{and} \quad K \leq \frac{1}{2}(m_r - e\gamma) \\
(0, K) & \text{if } \bar{\gamma} \geq \gamma \quad \text{and} \quad \frac{1}{2}(m_r - e\gamma) \leq K
\end{cases} \tag{2} $$

**Case (ii)** When $0 < e < \nu$:

$$ (q^*_r, q^*_u) = \begin{cases} 
(\frac{1}{2}(m_r - e\gamma), 0) & \text{if } \bar{\gamma} \geq \gamma, \\
(\frac{1}{2}(m_r - e\gamma) - K, K) & \text{if } \gamma \geq \gamma^+ \quad \text{and} \quad K \leq \frac{1}{2}(m_r - e\gamma) \\
(0, K) & \text{if } \gamma \geq \gamma \quad \text{and} \quad \frac{1}{2}(m_r - e\gamma) \leq K
\end{cases} \tag{3} $$

with $\gamma^- \leq \gamma \leq \gamma^+$ and $\frac{1}{2}m_u \geq K$.  

Improved production technology (i.e., lower emission intensity $\gamma$) has the benefit of reducing the emission cost of each unit of production, but it also incurs incremental unit production cost. In Case(i), the emission price is high, so the benefit of lowering $\gamma$ in reducing the emission charges outweighs the associated increase in production cost. Therefore, when $\gamma$ is smaller, it leads to a higher profit margin in the regulated region, and hence the producer optimally only produce in the regulated region with $q^*_r = \frac{e}{2}(m_r - e\gamma)$. If $\gamma$ is large, then it is optimal to shift as much production offshore as possible, and hence $q^*_u = K$, and $q^*_r = \left(\frac{e}{2}(m_r - e\gamma) - K\right)^+$. In Case (ii) where $e < \nu$, a small emission intensity works against production in the regulated market because the cost effect dominates. In sum, Lemma 1 identifies three actions that can be taken in Stage 2. (1) Regulated (R): production is only conducted in the regulated market; (2) Both (B): production is conducted in both markets; and (3) Unregulated (U): production is only conducted in the unregulated region.

In the remainder of the paper, to keep the analysis concise while providing main insights, we assume the emission price $e$ to be a random variable that follows a two-point distribution; i.e, $e$ can only take one of the two values in Stage 2: $e = e_l$ with probability $\rho$ and $e = e_h$ with probability $1 - \rho$, with $0 \leq e_l \leq e_h$ and $0 \leq \rho \leq 1$. A pair $(s_l, s_h)$ with $s_l, s_h \in \{R, B, U\}$ specifies the contingent production strategies in Stage 2 where the producer adopts action $s_i$ if $e = e_i$ for $i = l, h$. For example, $(R, U)$ means the producer adopts the $R$ action if the low emission cost is realized and the $U$ action if the high emission cost is realized.

As it is intractable to characterize the second-stage expected profit in closed-form for the optimization of $\gamma$ and $K$, we take the following approach: First, we identify the (four) undominated production strategies in Stage 2 with which the optimal Stage 2 profit must be associated. Then, assuming Strategy $i$ is optimal in Stage 2, we find $(\gamma^*_i, K^*_i)$ that optimizes the two-stage profit. We check for self-consistency that under $\gamma^*_i$ and $K^*_i$, Strategy $i$ is indeed optimal in Stage 2 (based on the Lemma 1 results). Finally, we identify the $(\gamma^*, K^*)$ from the strategy that yields the highest two-stage profit.

The following lemma summarizes the undominated profit-maximizing production strategies.

** Lemma 2.** The optimal profit is achieved by one of the following four contingent production strategies: $(R, R), (R, U), (B, U), (U, U)$.

The $(R, R)$ strategy produces in the regulated market regardless of the emission price realization, and hence will be referred to as the *Local* strategy. The $(R, U)$ strategy produces only in one market, depending on the realization of the emission price, and hence is called the *Dual* strategy. Finally, the $(B, U)$ strategy is referred to as *Hybrid* because it always produces in the unregulated region and produces also in the regulated market only when the emission price is
low. The \((U, U)\) strategy produces only in the unregulated region and hence is named the Foreign strategy. We use the subindex \(i = 1, 2, 3, 4\) to represent the Local, Dual, Hybrid and Foreign \((\text{L}, \text{L}), (\text{D}, \text{L}), (\text{H}, \text{L}), (\text{L}, \text{L})\) strategies, respectively, and \(Z_i(\gamma, K)\) refers to the two-stage profit when the producer uses Strategy \(i\).

Let \(\gamma_i^*\) and \(K_i^*\) denote the profit-maximizing quantities associated with \(Z_i(\gamma, K)\). To ensure that \(Z_i(\gamma, K)\) is concave in \(\gamma\) \(\forall i\), we assume that \(\mathbb{E}(e - \nu)^2 < \frac{1}{e} \beta_r\) and \(\rho(e_1 - \nu)^2 < \frac{(1 - \rho)}{e} \beta_r\) (These two assumptions suggest that the upfront investment in clean technology as reflected by \(\beta_r\) is significant, which appear not to be stringent in our numerical study).

**Proposition 1.** The profit-maximizing solutions for the two-stage profit functions associated with Strategy \(i\) \((Z_i(\gamma, K))\) are as follows, with \(\tilde{m}_r = m_r - \nu\gamma\):

<table>
<thead>
<tr>
<th>Strategy (i)</th>
<th>(\gamma_i^*)</th>
<th>(K_i^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\text{L}, \text{L})</td>
<td>(\frac{\beta r \rho}{\nu} - \frac{1}{2} \frac{\beta r \rho}{\nu} (\nu m_r - \nu \gamma))</td>
<td>(K_1^* = 0)</td>
</tr>
<tr>
<td>2. (\text{D}, \text{L})</td>
<td>(\frac{\beta r (1 - \rho) \gamma^*}{\nu} + \frac{1}{2} \frac{\beta r (1 - \rho)}{\nu} (\nu m_r - \nu \gamma))</td>
<td>(K_2^* = \frac{e m_u - \rho \beta u}{2(1 - \rho) \beta u})</td>
</tr>
<tr>
<td>3. (\text{H}, \text{L})</td>
<td>(\frac{\beta r (1 - \rho) \gamma^* + \frac{1}{2} \frac{\beta r (1 - \rho)}{\nu} (\nu m_r - \nu \gamma)}{\nu \beta u})</td>
<td>(K_3^* = \frac{e}{2} (m_u - \beta u) + \frac{e}{2} \beta u m_u - \beta u (m_r - \nu \gamma))</td>
</tr>
<tr>
<td>4. (\text{L}, \text{L})</td>
<td>(\gamma^*)</td>
<td>(K_4^* = \frac{e}{2} (m_u - \beta u))</td>
</tr>
</tbody>
</table>

Based on these results, we derive the following structural results that hold for all Strategy \(i\).

**Proposition 2.** \(\gamma_i^*\) \((q_{*,i}^*\) is weakly decreasing (increasing) in \(\tilde{p} - c_r\), and weakly increasing (decreasing) in \(\beta_r\) and \(\gamma^+\). \(K_i^*\) is weakly increasing in \(m_u\) and weakly decreasing in \(\beta_u\).

The proposition shows that factors such as a higher profit margin potential (indicated by \(\tilde{p} - c_r\)), a lower cost required for emission abatement investment (\(\beta_r\)) and a better foundation for clean technology development (\(\gamma^+\) is lower) encourage the producer to reduce the emission intensity and hence potentially keep more production in the regulated region. On the other hand, a higher profit margin \(m_u\) or a lower investment required for capacity build-up \(\beta_u\) in the unregulated region induces high production capacity build-up and potentially more offshore production.

Next, we compare the investment decisions associated with the different strategies.

**Proposition 3.** \(\gamma_1^* \leq \gamma_2^* \leq \gamma_3^* \leq \gamma_4^* = \gamma^+\). Moreover, \(K_2^* \leq K_4^*\) and \(K_3^* \leq K_4^*\); and if \(Z_3^* \geq Z_2^*\), then \(K_3^* \geq K_2^*\).

To better understand the rationale behind these conclusions, we take a closer look at the four strategies. Strategy 1 \((\text{L}, \text{L})\) leads to production only in the regulated region regardless of the actual emission price. Hence we expect the production quantity \(q_{*,1}^*\) to be high related to other strategies. This then translates into a higher marginal benefit from making production cleaner, because it reduces the emission charges for a higher volume of production. This explains why \(\gamma_1^*\) is the lowest, i.e., with the largest technology improvement of \(\gamma^+ - \gamma_1^*\). Strategy 2 \((\text{D}, \text{L})\) hedges the emission uncertainty by producing in the regulated market only when \(e\) is low and shifting
production offshore otherwise. As a result, the expected returns on the technology improvement is lower since it does not generate positive payoff when \(e\) is high. Therefore \(\gamma_2^* \geq \gamma_1^*\). Strategy 3 (\(B, U\)) is similar to Strategy 2 in that production in either market is possible. The difference is that the hedge of the emission price by offshore production is more aggressive for Strategy 3 such that even when \(e\) is low, only part of the production is kept in the regulated region, after exhausting the production capacity built offshore. Correspondingly, the incentive to introduce cleaner technology is further weakened, leading to \(\gamma_3^* \geq \gamma_2^*\). Finally, Strategy 4 (\(U, U\)) responds to the future emission regulation by completely moving production offshore, and therefore gives up on the option of making production cleaner in the regulated market while establishing a large production capacity in the unregulated region.

While analytically characterizing the parameter conditions under which each strategy emerges as optimal is not tractable, we develop insights through an illustrative example. The example is constructed based on reported data from a European factory in the cement industry, which is one of the most vulnerable industries under emission regulation (Drake et al. 2010). We specifically focus on the emission price uncertainty.

### 4.1.1. Example

Based on the data and calculation from the cement factory, which we will describe in more detail in §5, we derive the following parameters: \(m_r = 343.63 + 18.24\gamma, m_u = 349.49, \beta_u = 10.1, \beta_r = 36.6 \times 10^6, \gamma^+ = 0.7, \gamma^- = 0.075\). We present the optimal production strategy in Figure (1a) and the optimal technology profile in Figure (1b) for a range of emission prices \(e_l\) and \(e_h\), assuming the probability \(\rho = 0.5\).

In Figure (1a), the vertical axis is \(e_h - e_l\). When this value is low, it means the emission price is less volatile. Then strategies with production in only one market (i.e., \((R, R)\) and \((U, U)\)) will be preferred, because preparing for production in both markets (e.g., improving the local production technology and building offshore production capacity) to hedge against the emission cost uncertainty is not worthwhile. Consequently, production will be conducted in the regulated region when
the expected emission price is low, and in the unregulated region when it is high. The \((B,U)\) strategy, where production takes place in the regulated region only after capacity in the unregulated market is exhausted, is optimal at intermediate values of \(e_l\) that does not justify investing in a large offshore production capacity. In this case, the producer builds limited production capacity offshore since it will always be utilized.

On the other hand, when the emission price is volatile (with high \(e_h - e_l\)), hedging by preparing for production in both regions becomes more favorable. Consequently, \((R,U)\) replaces \((R,R)\) to be the optimal strategy when \(e_l\) is low, such that although producer still produces only in the regulated area when the emission price is low, it shifts all production offshore when the emission price is high.

Figure 1 also demonstrates the carbon leakage problem: As the emission price or its volatility increases, production starts to be shifted offshore. Such carbon leakage may lead to an increase in total emissions in two ways. (i) Each unit of offshore production is associated with higher emissions because it uses less clean technology and incurs additional emissions from transportation. (ii) As less production is conducted in the regulated region, clean technology investment has a lower marginal return and hence is discouraged and cannot contribute to the total emission reduction. The latter effect is evident from Figure (1b), which shows the changes of \(\gamma^*\), given a certain value of \(e_h - e_l\). As \(e_l\) goes up (\(e_h\) also rises accordingly since \(e_h - e_l\) is kept constant), \(\gamma^*\) starts from being \(\gamma^+\), then decreases in a range of the emission prices. Beyond that range, however, \(\gamma^*\) then jumps back to \(\gamma^+\). This is because when the emission price is originally low, it is not strong enough to influence the technology choice \(\gamma^*\). When the emission price increases, it generates sufficient incentive for the producer to invest in emission abatement in order to lower the emission charges. However, this favorable effect vanishes as soon as production is shifted offshore (i.e., the carbon leakage emerges) in response to the emission price increase.

These observations from investigating the producer’s operational decisions on production and investment motivate our study of various anti-leakage policies below.

4.2. The Border Tax Policy

One anti-leakage approach is the Border Tax policy, which imposes a border tax in Stage 2 to each product produced in the unregulated region upon importation to the regulated market for sale. As suggested by Ismer and Neuhoff (2007), we study a border tax that is related to the emission price currently charged in the regulated market based on the emission level of the best available technology (we briefly discuss an alternative form of border tax in §5.4). Then the border tax is of the form \(\beta_b e \gamma^-\) with \(\beta_b\) being the relevant policy parameter. Without loss of generality, we assume
$\beta_b = 1$ in the analysis. The Stage 2 profit function under this policy, given the values of $K$, $\gamma$ and $e$, is shown below. A superscript $b$ indicates that the quantity is derived under the Border Tax policy.

$$\Pi^b(q_r, q_u | e, \gamma, K) = -\frac{1}{\epsilon} (q_r + q_u)^2 + q_r (m_r - e\gamma) + q_u (m_u - e\gamma^2).$$ (4)

When the border tax is imposed, the operating margin in the unregulated region is now dependent on the emission cost: $m^b_u = m_u - \gamma e$. The optimization is similar to the baseline case. The optimal production quantities are similar to the Baseline scenario as described in Proposition 1, with $m_u$ replaced by $m^b_u$.

We next examine the impact of a Border Tax policy by comparing the optimal investment and production decisions under the Baseline case and the Border Tax case.

**Proposition 4.** Under a Border Tax policy, $K^b, \gamma^b, q^b_r \geq q^* r$ and $q^b_u \geq q^*_u$. In addition, when the producer’s production strategy remains unchanged, then $\Gamma^b_i \leq \Gamma^*_i$.

This proposition shows the effect of the Border Tax policy in reducing carbon-leakage. The border tax lowers the profit margin from offshore production. This disincentivizes building capacity and shifting production offshore (as reflected by lower $K^b, \gamma^b, q^b_r$ and $q^b_u$) and consequently reduces emissions in the unregulated region. On the other hand, more market demand will be met by production in the regulated region. This encourages the producer to invest more in the emission abatement technology as it applies to a larger production volume and hence has a higher return. Given the higher production volume but lower emission intensity, the resulting change in the emissions in the regulated region may not be directly clear. However, when the producer optimally adopts the same production strategy, then the total emissions decrease under the Border Tax policy. We illustrate the behavior we see in other cases (i.e., the producer changes the production strategy after an anti-leakage policy) by continuing our numerical example in §4.1.1. To facilitate discussion, we introduce the strategy pair notation $(i, i')$, where Strategy $i$ and $i'$ are the optimal strategy that maximize the total (two-stage) profit in the Baseline case and in the case with an anti-leakage policy, respectively.

**4.2.1. Example - continued** We show the effect of the Border Tax policy by comparing the optimal production strategy and the production technology investment with a border tax and those in the Baseline case. We exhibit the results in Figure 2. They reveal the anti-leakage effects of the border tax.

First, Figure (2a) is illustrative of what we find in our numerical investigation: When the Border Tax policy induces a change in the production strategy, the change is towards strategies that favor more production in the regulated region and less offshore production (the set of all strategy changes in this example is $\{(2, 1), (3, 2), (3, 1), (4, 3), (4, 1)\}$).
Figure 2  The Border Tax Scenario

Figure (2b) also illustrates how the border tax helps to restore the effectiveness of the emission regulation in promoting clean technology. Under the border tax, the range of emission prices within which the emission intensity decreases expands, and $\gamma^*$ is also lower, compared to the Baseline case. These positive impacts are rooted in the capability of the border tax to essentially diminish the profit margin of offshore production and mitigate carbon leakage.

4.3. Output-based Allocation Scenario

In contrast to the Border Tax policy that imposes a carbon tax on offshore production, we study the Output-based policy that grants a free emission allowance for production in the region in this section: While the former approach deters the producer from moving production out of the regulated region by “penalizing” offshore production, the latter approach “rewards” production in the regulated region. Therefore, by studying the producer’s investment and production decisions under the two types of policies, we derive insights in comparing the “stick” versus the “carrot” policy on anti-leakage.

The Output-based policy allocates a free emission allowance to the producer based on three factors: (i) the current production figure; (ii) a product-specific benchmark; and (iii) a factor that reflects emission reduction targets set by authorities. This structure is in line with proposed Output-based policies discussed in the literature (Demailly and Quirion 2006, Fischer and Fox 2009); moreover, an allowance allocation based on current production levels is also reflected in Phase III of the EU ETS as it updates the allocation every five years (European Commission 2011). We assume the product-specific benchmark is reflected by the best available technology benchmark $\gamma^-$; the emission reduction factor is modeled by $\alpha$ ($\alpha \in [0, 1]$); and the allowance applies for all the production in the regulated region; i.e., the total free allowance amounts to $\gamma^- \alpha q_o^p$, where we use a superscript o for the quantities in the Output-based case. This results in an increase of $\varepsilon \alpha \gamma^-$ in the
operating margin in the regulated region (due to exemption of charges for the emissions covered by the allowance). The resulting Stage 2 profit function is:

\[ \Pi_o(q_r, q_u|e, \gamma, K) = -\frac{1}{\epsilon} (q_r + q_u)^2 + q_r (m_r - e\gamma + e\alpha\gamma^-) + q_u(m_u). \]  

(5)

Since the output-based allocation of allowances boils down to increasing the profit margin in the regulated region as shown in Equation (5), the optimal production quantities have the same structure with those in the Baseline scenario described in Proposition 1, with the change of \( m_r - e\gamma \) to \( m_r - e\gamma + e\alpha\gamma^- \). We next examine how the Output-based allowance changes the producer’s optimal investment and production decisions compared to the Baseline case.

**Proposition 5.** Under an Output-based Allowance Allocation policy, \( \gamma_o^* \leq \gamma^* \), \( K_o^* \leq K^* \), \( q_o^* \geq q_r^* \) and \( q_o^* \leq q_u^* \).

The results suggest that the Output-Based policy mitigates the carbon leakage problem: It reduces the offshore production, and also promotes emission abatement for local production. Notably, although these effects are similar to those of the Border Tax policy, the underlying mechanism is very different. The Output-Based allowance increases the profit margin of the local production and hence directly stimulates higher local production volume. This also increases the returns on the technology investment as it now applies to a larger production volume, and hence leads to a lower \( \gamma^* \). As a result, the need for offshore production is reduced, resulting in less carbon leakage.

**Example - continued** We continue with the numerical example to further illustrate the Output-Based case, by comparing it to the Baseline case.

Figure 3 displays the impact of Output-Based allocation on the production strategy choice (for \( \alpha = 1 \)). Since Output-based allocation only impacts the operating margin for production in the
regulated market, the structure of the regions is similar but shifted compared to the Baseline case. The figures demonstrate the carbon leakage mitigation effects of the Output-based policy: When it induces a change in the production strategy, it changes towards the strategy that favors more local production (the set of all strategy change pairs in this example is \{(2, 1), (3, 2), (3, 1), (4, 3), (4, 1)\}), as shown by Figure(3a). Moreover, the Output-based policy leads to an expansion of the emission price range within which there is emission abatement, as well as a reduction in \(\gamma^*\). They suggest while the emission abatement incentives of carbon emission regulation are compromised by carbon leakage, some of them are restored by the Output-based policy.

**4.4. Grandfathering Scenario**

The Grandfathering policy is another anti-leakage policy with allowance allocation. It differs from the Output-based policy in that it grants free emission allowances to producer based on historical past, i.e. before Stage 1, which we will investigate in more detail in the numerical study. We use a superscript \(g\) for the quantities in the Grandfathering case. The Stage 2 profit function in this scenario is:

\[
\Pi^g(q_r, q_u | e, \gamma, K) = -\frac{1}{\epsilon} (q_r + q_u)^2 + q_r(m_r - \gamma e) + \gamma e \left(\frac{\Omega}{\gamma} - \left(\frac{\Omega}{\gamma} - q_r\right)^+\right) + q_u(m_u). \tag{6}
\]

We first derive the optimal production decision under Grandfathering, by following a similar solution procedure for Lemma 1.

**Lemma 3.** Given \(e, \gamma, K \geq 0\), the profit-maximizing Stage 2 production quantities under the Grandfathering policy are:

\[
(q_r^*, q_u^*) = \begin{cases} 
(R') \{ (\frac{\Omega}{\gamma} + \gamma e, 0) & \text{when } 0 \leq \frac{\Omega}{\gamma} < \frac{\Omega}{2} \text{ and } \frac{\epsilon}{\gamma} m_u \leq \frac{\Omega}{\gamma} \\
(\frac{\Omega}{\gamma}, 0) & \text{when } \frac{\Omega}{\gamma} \leq \frac{\Omega}{2} < \frac{\Omega}{\gamma} + re \text{ and } \frac{\epsilon}{\gamma} m_u \leq \frac{\Omega}{\gamma} \\
(\frac{\Omega}{\gamma}(m_r - \gamma e), 0) & \text{when } \frac{\Omega}{\gamma} + \gamma e \leq \frac{\Omega}{2} < \frac{\gamma}{\epsilon} m_u \text{ and } m_u \leq m_r - \gamma e \\
(B_2) \{ (\frac{\Omega}{\gamma}, \frac{1}{2}m_u - \frac{\Omega}{2}) & \text{when } \frac{\Omega}{\gamma} \leq \frac{1}{2}m_r \leq \frac{\Omega}{2} + K + \gamma e \text{ and } \text{max}[\frac{\Omega}{2}(m_r - \gamma e), \frac{\Omega}{2}] \leq \frac{1}{2}m_u \leq \text{min}[\frac{1}{2}m_r, \frac{\Omega}{\gamma} + K] \\
(B_1) \{ (\frac{\Omega}{\gamma}, K) & \text{when } \frac{\Omega}{\gamma} + K \leq \frac{1}{2}m_u \text{ and } \frac{\Omega}{\gamma} \leq \frac{1}{2}m_r \leq \frac{\Omega}{2} + K + \gamma e \\
(B) \{ (\frac{\Omega}{\gamma}(m_r - \gamma e) - K, K) & \text{when } m_r - \gamma e \leq m_u \text{ and } \frac{\Omega}{\gamma} + K + \gamma e \leq \frac{1}{2}m_r
\end{cases}
\]

We name the production actions as \(R', B_2, B_1\) and \(B\) as indicated in the lemma. \(R'\) represents a production action that produces only in the regulated region, with an amount depending on the effective operating margin. \(B_2\) refers to producing in the regulated region at an amount that maxes out the emission allowance, and then filling the remaining market demand by offshore production.
$B_1$ refers to producing in the regulated region at an amount that maxes out the emission allowance, and also producing in the unregulated region at an amount that exhausts the production capacity. Finally, $B$ refers to producing in the unregulated region at an amount that exhausts the production capacity, and then meeting the remaining market demand by producing in the regulated region.

Comparing Lemma 3 to Lemma 1 in the Baseline case reveals the effects of the Grandfathering policy and suggests that Grandfathering may lead to adverse carbon leakage consequences. In particular, the producer profit function under Grandfathering (6) can be rewritten as

$$
\Pi^g(q_r, q_u|e, \gamma, K) = \begin{cases} 
-\frac{1}{2} (q_r + q_u)^2 + q_r (m_r) + q_u (m_u), & \text{for } 0 \leq q_r \leq \frac{\Omega}{\gamma} \\
-\frac{1}{2} (q_r + q_u)^2 + q_r (m_r - \gamma e) + \Omega e + q_u (m_u), & \text{for } q_r > \frac{\Omega}{\gamma}.
\end{cases}
$$

As seen in (7), Grandfathering defines a threshold $\Omega/\gamma$ for $q_r$: When production is beyond the threshold (high-production case), Grandfathering only offers a fixed lump-sum exemption from emission charges. Therefore, the marginal profit with respect to $q_r$ is unaffected, and Lemma 3 confirms that this results in an optimal production $q_r^*$ that is the same as the Baseline case, as illustrated on the right-most ranges in Figure 4a and 4b (both constructed based on Lemma 3 and Lemma 1). Therefore, Grandfathering fails to have an anti-leakage effect although the policy maker gives up the charges from $\Omega$ amount of emissions.

On the other hand, when production is within the threshold $\Omega/\gamma$ (low-production case), all emissions become totally free because they are fully covered by the allowances. The associated implications are not straightforward in this case. First, Grandfathering increases the profit margin in the regulated region, and Lemma 3 confirms that this encourages more production in the regulated region than the Baseline case, as illustrated in the left ranges in Figure 4a and 4b, which incentivizes more investment in clean production technology improvement. This favorable effect acts in a similar way as the Output-based policy. Second, Grandfathering may generate additional incentives for technology improvement because a lower $\gamma^*$ increases the production quantity threshold $\Omega/\gamma$ within which emissions in the regulated region are covered by allowances. Therefore, it appears that Grandfathering is a strong tool for promoting clean technology, but this view ignores the following unfavorable effect of Grandfathering: When the profit margin is low and hence the emissions associated with the optimal production volume in the regulated region (in the Baseline case) do not exceed $\Omega$, the producer cannot take full advantage of the emission allowance provided by Grandfathering unless it chooses a less clean production technology than under the Baseline case. Consequently, Grandfathering potentially risks resulting in lower technology investment as its net effect is jointly determined by all these three effects. In addition, in the low-production case, the profit margin in the regulated region is higher than in the Baseline case, so the producer may choose to increase its sales by expanding its production capacity in the unregulated region,
creating a more severe carbon leakage problem. We demonstrate this risk in the examples and the numerical study.

Next, we summarize all the non-dominated production strategies in the Grandfathering case.

**Proposition 6.** The profit-maximizing production strategy is one out of: \((B_1, B_1), (B, B_1), (R', B_1), (R', R')\).

To understand the differences between Proposition 6 and Lemma 2 that specifies the four production strategies in the Baseline case, except that all the \(U\) actions are replaced by \(B_1\) under Grandfathering: \(B_1\) is in fact very similar to \(U\). The only difference is that with the grandfathered allowance, the producer keeps \(\Omega/\gamma\) amount of production in the regulated region, because this production is fully covered by the allowance. Based on this rationale, we number the strategies as below: Strategy 1’ refers to \((R', R')\), Strategy 2’ to \((R', B_1)\), Strategy 3’ to \((B, B_1)\), and Strategy 4’ to \((B_1, B_1)\), for comparing the Grandfathering case to the Baseline case.

4.4.1. Example - continued We continue with the numerical example to investigate the effect of the emission allowance grandfathered. Since the allowance from Grandfathering is based on the past production volume, it can be rewritten as \(\Omega = \eta \gamma^+ q_0\), with \(\eta\) being the policy parameter, and \(q_0\) being the historical production figure (which will be the optimal production quantity in the regulated region given the starting emission price; the derivation is provided in §5).

In this example we examine the impacts of two values of \(\eta\), to be 0.33 and 1; i.e., \(\Omega = 0.33 q_0 \gamma^+\) and \(\Omega = q_0 \gamma^+\). The two cases help highlight the importance and significance of the parameter choice for the Grandfathering policy. The results are presented in Figure 5 and 6.

Figure (5a) and (6a) show that, contrary to other anti-leakage policies, when Grandfathering induces a change in the production strategy, it does not always change to the one that focuses more production in the regulated region, e.g. \((2, 4')\) in Figure (5a) and \((3, 4')\) in Figure (6a). This is
because the aforementioned adverse effect of Grandfathering in the low-production case dominates other anti-leakage effects, which induces more offshore production. The adverse effect also shows up in the left ranges in both figures, leading to less (or even no) technology improvement compared to the Baseline case.

Another revealing insight is that depending on the specification of Grandfathering, as represented by the policy parameter $\eta$, Grandfathering may influence the incentive in improving technology differently, as shown by comparing the right-most ranges of Figure 5b and 6b. In these ranges, the strategy pair is $(4, 4')$, which means Grandfathering changes the production quantity in the regulated region from $q^*_r = 0$ in the Baseline case (as the production strategy $4(U, U)$ is used) to $q^*_r = \frac{\Omega}{\gamma}$ (as the production strategy $4'(B_1, B_1)$ is used). When $\eta = 1$, $q^*_r$ is higher and hence induces the producer to improve the technology; whereas when $\eta = 0.3$, this incentive is not strong enough, resulting in no technology investment. Our numerical study below will further demonstrate the significance of setting an appropriate allowance level for Grandfathering to avoid the risk of a more severe carbon leakage problem.

**Figure 5** The Grandfathering Scenario with $\eta = 1$

**Figure 6** The Grandfathering Scenario with $\eta = 0.3$
5. Numerical study

In this section we apply our model and analysis to a case study based on a cement producer in Europe to gain an in-depth understanding of how the three anti-leakage policies compare from different aspects. We describe the data and the corresponding full factorial design in §5.1. Then we present the main insights.

5.1. Data set and the full factorial design

We choose cement production in Europe for the study because it is one of the sectors that is sensitive to emission regulation. The data, unless noted otherwise, is based on the case study in Drake et al. (2015) about a European cement producer. We consider carbon capture and storage (CCS) as the best available production technology in the regulated region, and Egypt as the offshore location. Details of the data and derivation of parameter values are provided in the Appendix.

We describe a full factorial design to investigate the influence of several factors. While keeping other parameters constant, we consider the variations of four product-specific and three emission cost-related factors, all summarized in Table 2. The technology improvement potential \((\gamma^+ - \gamma^-)\) takes values \{0.313, 0.625\}. The technology investment cost factor \((\beta_r)\) takes values \{36.6 \cdot 10^6, 73.2 \cdot 10^6\}. We consider two production facility locations in the regulated region, the farther away (from the offshore location) one is in western Europe (e.g., Rotterdam) and the nearer one is in the south-east of France (e.g., Marseille). The respective transportation distances \((\delta)\) and the associated transportation emissions of one unit of product \((\gamma^t_u)\), taken from ECOTransIT (2011), will be \((\delta, \gamma^t_u) \in \{(2, 773, 0.0271), (5, 900, 0.0563)\}\). The demand factors for cement \((Q, \epsilon)\) take values from \{(2.5 \cdot 10^6, 6.25 \cdot 10^3), (4 \cdot 10^6, 2.22 \cdot 10^3), (5 \cdot 10^6, 3.5 \cdot 10^3)\}.

Next, we look at the emission price. We let the probability of realizing a low emission price \((\rho)\) take values \{0.1, 0.5, 0.9\}. The low emission price \((e_l)\) takes values \{15, 30\}. We then assume the difference between the high and low prices \((e_h - e_l)\) to be 20 or 50. We choose an emission price range that is higher than the current price because carbon emission regulation is expected to become more stringent over time (e.g., EU-ETS will expand its coverage and reduce the emission cap during Phase III, which is effective until 2020). Moreover, the European economy is recovering and surplus emission permits built up from earlier years (e.g., during the financial crisis) are being used up. These factors may well lead to a rise in the carbon emission price in the long run. Moreover, studying these value ranges where the carbon leakage problem is more pronounced helps to highlight and compare the influence of the different anti-leakage policies.

Following our previous analysis, we study 4 cases: the Baseline (BL) case without an anti-leakage policy; the case with the Border Tax policy (BT); the case with the Output-Based Allocation policy (OB); and the case with the Grandfathering policy (GT). For the GT case, we further investigate
three policy parameter values $\eta = 0.33, 0.66$ and 1, so the grandfathered emission allowance will be $\Omega = \eta q_0 \gamma^-$ with $q_0$ denoting the historical production figure. $q_0$ is calculated as the optimal production quantity under the emission price in the past with $e_0 = 15$; i.e., $q_0 = \frac{x}{Z}(m_r - e_0 \gamma^+)$. We refer to Grandfathering with $\eta = 0.33$ (GF0.3) and $\eta = 0.66$ (GF0.6) as partial Grandfathering and otherwise full Grandfathering ($\eta = 1$, denoted as GF1). Therefore, we study 6 scenarios, each corresponding to one policy choice. In each of these scenarios, we consider 288 ($2^5 \times 3^2$) instances (in each of the instances, we vary the value of one factor while keeping all other factors unchanged).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^+ - \gamma^-$</td>
<td>0.313, 0.625</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$36.6 \cdot 10^6, 73.2 \cdot 10^6$</td>
</tr>
<tr>
<td>$(\delta, \gamma_t)$</td>
<td>(2.773, 0.0271), (5, 900, 0.0563)</td>
</tr>
<tr>
<td>$(Q, \epsilon)$</td>
<td>$(2.5 \cdot 10^6, 6.25 \cdot 10^3), (4 \cdot 10^6, 2.22 \cdot 10^3), (5 \cdot 10^6, 3.5 \cdot 10^3)$</td>
</tr>
<tr>
<td>$e_l$</td>
<td>15, 30</td>
</tr>
<tr>
<td>$e_h - e_l$</td>
<td>20, 50</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.1, 0.5, 0.9</td>
</tr>
</tbody>
</table>

**5.2. Optimal producer decisions in absence of an anti-leakage policy**

We first focus on the BL scenario to investigate under what conditions carbon leakage is a risk (for the case study) and how it is impacted by the product characteristics and the uncertain future emission price. To this end, we study how the factor values influence the producer investment and production decisions. As discussed earlier, deriving closed-form solutions for these decisions is not possible, so we conduct a numerical analysis using the full factorial design in the following way: To study the the influence of a factor, for example the technology improvement potential ($\gamma^+ - \gamma^-$) that takes a low value (0.313) and a high value (0.625), we separate the 288 instances into two groups, with high and low $\gamma^+ - \gamma^-$ values, respectively. Then instances within each group have the same $\gamma^+ - \gamma^-$ value, while the values of other factors vary. For each instance, we apply our earlier results to compute $\gamma^+$, $K^*$ and $E(q^*_r)$. Next, we calculate the average values of these decisions for the two groups, and denote them by $(\gamma^*_L, K^*_L, E(q^*_r)_L)$ for the group with low (high) $\gamma^+ - \gamma^-$, based on which we can compare the two groups.

To present the results, note that the emission regulation aims to induce more technology improvement, reduce offshore production and keep more production in the regulated region; and realizing these outcomes indicates a lower degree of carbon leakage. We hence present the factor value (e.g., “Low” or “High” for $\gamma^+ - \gamma^-$) that results in a higher $\gamma^+ - \gamma^*$, lower $K^*$ and higher $E(q^*_r)$. The same process is repeated for other factors with results exhibited in Figure 7 (the factor of $\epsilon$ is not listed individually because it changes together with $Q$ in our study). We summarize the varying emission cost-related factors (i.e., $\rho$, $e_l$, and $e_h - e_l$) by calculating the mean ($E(\epsilon)$) and variance ($Var(\epsilon)$).
of the unit emission price for each of the 288 instances. Then we perform a similar comparison procedure, taking $E(e)$ and $Var(e)$ as two factors in place of the original emission cost-related factors.

<table>
<thead>
<tr>
<th></th>
<th>$\gamma^+ - \gamma^-$</th>
<th>$K^*$</th>
<th>$E(q^*_r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma^+ - \gamma^-$</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>$\delta$</td>
<td>High</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>$Q$</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>$E(e)$</td>
<td>Relatively High</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>$Var(e)$</td>
<td>Medium</td>
<td>Low</td>
<td>Low</td>
</tr>
</tbody>
</table>

Figure 7  The factor values that favor a higher $\gamma^+ - \gamma^-$, a lower $K^*$, and a higher $E(q^*_r)$.

Results in Figure 7 provide the following insights: A lower value of $\gamma^+ - \gamma^-$, which essentially indicates a more mature production technology foundation (lower $\gamma^+$) and hence favors more production in the regulated region, suppresses offshore production and carbon leakage. A longer distance (a higher $\delta$) has a similar effect as it decreases the profit margin for offshore production. A higher market demand ($Q$) encourages more production in both of the regions and hence incentivizes more investment in both production technology improvement and production capacity build-up.

The expected emission price $E(e)$, which represents the emission regulation stringency, has a non-monotonic effect with respect to technology improvement: The highest technology improvement occurs under relatively high (but not the highest) value of $E(e)$. At the highest value of $E(e)$, technology investment is lower and offshore capacity build-up is at its highest. These results suggest that in the cement industry, carbon leakage is a prominent risk, where technology improvement incentives will be weakened and production will be shifted offshore as emission regulation stringency rises.

The carbon leakage problem can become more severe when the uncertainty in the emission price increases, as only low $Var(e)$ can lead to lower production capacity built offshore.

Given the interesting observations from $E(e)$ and $Var(e)$, we examine them further: In the following experiment, we group all the 288 instances in the BL scenario depending on the emission price triplet $(e_l, e_{h-l}, \rho)$ and hence derive 12 groups of instances. For each group, we calculate the optimal investment and production decisions for each instance and take the average within the group. Results are summarized in Figure 8a. To see how the optimal production strategy may change due to emission price variations, we construct $(a, b, c, d)$ in the last column, which shows that the Local $(R, R)$ strategy is optimal for a number of instances within the group, and $b, c$ and
respectively counts the number of instances where the Dual \((R, U)\), Hybrid \((B, U)\) and Foreign \((U, U)\) Strategies are optimal.

In Figure 8a, the values of \((e_t, e_h - e_t, \rho)\) are taken from Table 2. In Figure 8b, in order to highlight the impact of the mean and variance of the emission price, we fix \(\rho = 0.5\), let \(E(e)\) to take values \{20, 40, 60\} and \(Var(e)\) take values \{0.15, 0.30, 0.45, 0.60\}. Then given \((E(e), Var(e))\), we calculate \(e_t\) and \(e_h - e_t\), and then derive the optimal investment and production decisions and construct Figure 8b in the same way as Figure 8a. The results from the analysis reveal the following:

### Table 8

<table>
<thead>
<tr>
<th>(E(e))</th>
<th>(Var(e))</th>
<th>(\gamma^+ - \gamma^-)</th>
<th>(K^+) (x 10^3)</th>
<th>(E(q^*_e)) (x 10^3)</th>
<th>Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>36</td>
<td>0</td>
<td>1811</td>
<td>(24, 6, 0, 0)</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>225</td>
<td>0.023</td>
<td>461</td>
<td>(18, 6, 2, 4)</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>100</td>
<td>0.093</td>
<td>1303</td>
<td>(18, 6, 2, 4)</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>36</td>
<td>0.175</td>
<td>13953</td>
<td>(18, 0, 2, 4)</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>36</td>
<td>0.184</td>
<td>13953</td>
<td>(18, 0, 2, 4)</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>225</td>
<td>0.385</td>
<td>13953</td>
<td>(17, 0, 3, 4)</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td>0.225</td>
<td>1266</td>
<td>(16, 5, 4, 4)</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>625</td>
<td>0.185</td>
<td>759</td>
<td>(15, 5, 3, 4)</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>30</td>
<td>0.065</td>
<td>884</td>
<td>(11, 6, 0, 7)</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>225</td>
<td>0.154</td>
<td>985</td>
<td>(10, 5, 6, 8)</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>225</td>
<td>0.556</td>
<td>985</td>
<td>(10, 5, 4, 6)</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>225</td>
<td>0.150</td>
<td>1167</td>
<td>(8, 0, 6, 10)</td>
<td></td>
</tr>
</tbody>
</table>

### Table 8 (b)

<table>
<thead>
<tr>
<th>(E(e))</th>
<th>Coefficient of variance</th>
<th>(\gamma^+ - \gamma^-)</th>
<th>(K^+) (x 10^3)</th>
<th>(E(q^*_e)) (x 10^3)</th>
<th>Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>0.15</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(6, 0, 2, 4)</td>
</tr>
<tr>
<td>20</td>
<td>0.50</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(6, 0, 2, 4)</td>
</tr>
<tr>
<td>29</td>
<td>0.45</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(6, 0, 2, 4)</td>
</tr>
<tr>
<td>30</td>
<td>0.60</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(6, 0, 2, 4)</td>
</tr>
<tr>
<td>39</td>
<td>0.15</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(5, 0, 1, 6)</td>
</tr>
<tr>
<td>30</td>
<td>0.50</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(5, 0, 3, 4)</td>
</tr>
<tr>
<td>40</td>
<td>0.45</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(5, 0, 3, 4)</td>
</tr>
<tr>
<td>49</td>
<td>0.60</td>
<td>0.202</td>
<td>451</td>
<td>451</td>
<td>(2, 0, 4, 4)</td>
</tr>
<tr>
<td>60</td>
<td>0.15</td>
<td>0.202</td>
<td>716</td>
<td>317</td>
<td>(2, 0, 6, 10)</td>
</tr>
<tr>
<td>60</td>
<td>0.50</td>
<td>0.202</td>
<td>716</td>
<td>317</td>
<td>(2, 0, 6, 10)</td>
</tr>
<tr>
<td>65</td>
<td>0.45</td>
<td>0.202</td>
<td>716</td>
<td>317</td>
<td>(2, 0, 7, 8)</td>
</tr>
<tr>
<td>60</td>
<td>0.60</td>
<td>0.202</td>
<td>716</td>
<td>317</td>
<td>(2, 0, 7, 8)</td>
</tr>
</tbody>
</table>

**Figure 8** The impact of emission cost factors on the average expected investment and production quantities and optimal production strategies

The expected emission price has a significant, yet non-monotonic influence on inducing emission abatement: When it first increases, it incentivizes the producer to adopt cleaner technology so as to lower the emission charges. However, when it is too high, lowering the emission charges by investing in technology becomes too expensive, and the producer will be better off shifting production offshore. This is evident by the reduction in the number of instances where the Local strategy is chosen and an increase in the offshore production capacity build-up, resulting in a more severely carbon leakage problem.

High volatility in the emission price typically leads to unfavorable results with lower investment in clean technology and more offshore production build-up, as evident by the changing strategy portfolios.

### 5.3. The impact of anti-leakage policies on producer decisions

We now consider the impact of anti-leakage policies on the optimal producer investment decisions and the expected two-stage profit \((Z^*)\). In each of the six policy scenarios, we derive \(\gamma^+\), \(K^+\), and \(Z^*\) for all of the 288 instances, given the respective anti-leakage policy (or no anti-leakage policy
in the BL scenario), and then take the average. Results are summarized in Figure 9: In each cell of the table, we first specify the anti-leakage policy and then show the corresponding average value in the parenthesis. For each column, we start the list from the more favorable results (i.e., higher values for $\gamma^+ - \gamma^*$ and $Z^*$, and lower values for $K^*$), with the BL scenario values highlighted as benchmarks.

<table>
<thead>
<tr>
<th>$\gamma^+ - \gamma^*$</th>
<th>$K^* (\times 10^3)$</th>
<th>$Z^* (\times 10^9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OB (0.3363)</td>
<td>OB (424.25)</td>
<td>GF1 (1.1764)</td>
</tr>
<tr>
<td>BT (0.3335)</td>
<td>BT (427.76)</td>
<td>OB (1.1763)</td>
</tr>
<tr>
<td>GF1 (0.3275)</td>
<td>GF1 (564.24)</td>
<td>GF0.6 (1.1735)</td>
</tr>
<tr>
<td>BL (0.2968)</td>
<td>BL (635.08)</td>
<td>GF0.3 (1.1731)</td>
</tr>
<tr>
<td>GF0.3 (0.2905)</td>
<td>GF0.3 (661.14)</td>
<td>BL (1.1728)</td>
</tr>
<tr>
<td>GF0.6 (0.2871)</td>
<td>GF0.6 (707.06)</td>
<td>BT (1.1709)</td>
</tr>
</tbody>
</table>

Figure 9 The impact of anti-leakage policies on producer decision and profit

The OB policy results in the most favorable outcome with high technology improvement and low offshore production capacity. Following OB is the BT policy, with a small difference between their influence. However, the producers bear the cost imposed by the border tax under the BT policy, whereas they get a reduction on their financial obligations under the OB policy. Therefore, given the comparably favorable impacts, the availability of both OB and BT may allow policy makers to take into account certain financial considerations. For example, if maintaining market competitiveness of producers is an important concern, OB may be favored.

Interestingly, the influence of the GF policy is not as straightforward, and it significantly depends on the choice of policy parameter ($\eta$). The full grandfathered allowance ($\eta = 1$) achieves its anti-leakage objective as there is more (less) investment in technology improvement (offshore production capacity) than the BL scenario. However, these effects are weaker than the OB and BT policies, although GF1 may be favored by producers as it leads to a higher profit due to the allowance.

At lower $\eta$, surprising effects may occur: GF0.6 and GF0.3 lead to more severe carbon leakage than the BL scenario. Moreover, the effect of $\eta$ is not necessarily monotonic, and in this case study, GF1, GF0.3 and GF0.6 result in less and less favorable outcomes. To explain these observations, recall that Grandfathering has both positive anti-leakage effects (e.g., induces more production in the regulated region by increasing the associated profit margin through granting allowances, and induces more technology improvement because the fixed amount of allowances cover a higher production volume in the regulated region when $\gamma$ is lower) and adverse effects (e.g., when the production exceeds the grandfathered quantity, the “non-covered” part is exposed to the risk of carbon leakage). However, these effects may not be changing in the same direction in $\eta$. For example, when $\eta$ is higher, it is more likely that the production quantity in the regulated region is
lower compared to the allowance level and hence the adverse effect that the producer invests less in technology improvement becomes more pronounced. Meanwhile, the positive effect where more production can be covered by the allowance by investing more in technology improvement becomes stronger (as the magnitude of \( \frac{\partial \Omega}{\partial \gamma} \) is larger). As the net effect of Grandfathering in influencing producer decisions results from the interplay of these effects, carbon leakage can be even worse than in the Baseline case, and the magnitude of this effect can be non-monotonic in the allowance level (\( \eta \)).

### 5.4. The impact of anti-leakage policies on emissions

We next explore the impact of the anti-leakage policies on emissions. For each of the anti-leakage policies, we calculate the changes in the average emission in the regulated and unregulated regions, as well as the overall emissions, denoted by \( \Delta \Gamma_r, \Delta \Gamma_u \) and \( \Delta(\Gamma_r + \Gamma_u) \), where the + and − signs indicate an increase and a decrease in the emissions compared to the BL case, respectively. Results are summarized in Figure 10.

<table>
<thead>
<tr>
<th></th>
<th>BT</th>
<th>OB</th>
<th>GF0.3</th>
<th>GF0.6</th>
<th>GF1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \Gamma_r )</td>
<td>39587</td>
<td>40068</td>
<td>6082</td>
<td>637</td>
<td>-28786</td>
</tr>
<tr>
<td>( \Delta \Gamma_u )</td>
<td>-111530</td>
<td>-114180</td>
<td>9886</td>
<td>32890</td>
<td>-25040</td>
</tr>
<tr>
<td>( \Delta(\Gamma_r + \Gamma_u) )</td>
<td>-71943</td>
<td>-74112</td>
<td>15968</td>
<td>33527</td>
<td>-53826</td>
</tr>
</tbody>
</table>

**Figure 10** The impact of anti-leakage policies on emission

These results strengthen our conclusions about the favorable anti-leakage effects of the BT and OB policies, as they also lead to the highest reduction in total emissions and increase production in the regulated region. Emissions in the regulated region are higher due to a higher production volume despite the improvement in technology. These are dominated by gains from avoiding offshore production and hence a net overall emission reduction is realized. As discussed in Drake (2012) and Grubb and Neuhoff (2006), BT can be implemented by specifying a symmetric border tax (i.e., determined by the emissions from the actual technology used). In Drake (2012), it is shown that under a symmetric border tax and when the producers are competing with foreign firms who produce in the unregulated region and can choose their production technologies, it is possible that the BT policy will induce improvement in production technology by the foreign firms. Under those conditions, BT is even more effective in reducing overall emissions.

For the GF policies, the outcomes critically depend on the policy parameter \( \eta \), which is consistent with the earlier results and discussion on the GF policies: The worse performance of Grandfathering under the more stringent implementations (low \( \eta \)), and the non-monotonicity of the emissions in
η are rooted in the interplay of the positive and adverse effects of GF. Therefore, policy makers should be careful in determining the specification of the GF policies based on a solid understanding of the target industry (e.g., the product and market characteristics, investment and production cost information), and avoid choosing policy specifications that lead to even higher emissions (e.g., GF0.3 and GF0.6 in this study with the emissions highlighted in Figure 10).

6. Conclusions

We study the carbon leakage problem that arises under asymmetric emission regulation with uncertain future stringency from an operational perspective, and we compare three anti-leakage policies as potential remedies. We examine the investment decisions in both building production capacity in the unregulated region and improving clean production technology in the regulated market, before the regulatory uncertainty is resolved. We also investigate the production quantity decisions after the realization of the uncertain emission regulation.

We first observe that, under carbon emission regulation without an anti-leakage policy, when the profit margin in one region is sufficiently higher than the other, the resulting optimal production strategy will be to produce only in the more profitable region regardless of the realized emission price. In contrast, when the comparison between the profit margins strongly depends on the future emission price realization, the production strategy hedges against uncertainty by building capacity offshore and choosing different production locations contingent on the final emission price.

An important theme in this paper is to study the impact of different anti-leakage policies aimed at mitigating carbon leakage. The first one we consider is the Border Tax policy that imposes a carbon tax on offshore produced products imported and sold in the regulated region. We show that this policy always reduces production capacity and hence emissions in the unregulated market, while encouraging more local production and technology improvement in the regulated market. The Output-based policy grants the producer a free emission allowance for production in the regulated region based on the current production level. It alleviates carbon leakage by increasing the operating margin of production in the regulated market and hence reduces offshore production. The third policy we consider is Grandfathering, which grants the producer a free emission allowance for production in the regulated region based on the historical production level. We observe that although Grandfathering may help keep more production in the regulated region and hence mitigate carbon leakage, it may also have an adverse effect and induce the producer to expand offshore production as it increases the total sales volume due to the higher margin that Grandfathering provides in the regulated region. When this happens, carbon leakage in fact becomes more severe than in the absence of an anti-leakage policy.

We then apply our model to a case study based on calibrated data from a cement producer in Europe. The study reveals that high expected value and volatility of carbon price lead to
unfavorable outcomes with low (high) investments in emission abatement (offshore production capacity). Both Border Tax and Output-based policies achieve comparable anti-leakage effects, although they have different implications for the associated financial responsibility of producers. Finally, the performance of Grandfathering is very much dependent on allowance levels. Hence, policy makers should be careful in designing the Grandfathering policies, as inappropriately set policy specifications may lead to even more severe carbon leakage. Our results provide a theoretical foundation to facilitate this process.

Acknowledgments
Acknowledgment here.

References


Appendix

Proof of Lemma 1

We solve Problem ($P_2$):

$$\max_{q_r, q_u} \Pi(q_r, q_u | e, \gamma, K) = -\frac{1}{\epsilon}(q_r + q_u)^2 + (m_r - \gamma e)q_r + m_u q_u, \quad \text{subject to} \quad 0 \leq q_r, \quad 0 \leq q_u \leq K.$$

To maximize profit under the constraints, we form the Lagrangian:

$$L = -\frac{1}{\epsilon}(q_r + q_u)^2 + (m_r - \gamma e)q_r + m_u q_u + \lambda(q_r) + \mu_1(q_u) + \mu_2(K - q_u).$$

From the first-order condition equations $\frac{\partial L}{\partial q_r} = 0$ and $\frac{\partial L}{\partial q_u} = 0$, we have

$$\begin{cases} 
\lambda = \frac{2}{\epsilon}(q_r + q_u) - (m_r - \gamma e) \quad (1) \\
\mu_1 - \mu_2 = \frac{2}{\epsilon}(q_r + q_u) - m_u \quad (2)
\end{cases}$$

- **Case (i):** When $e \geq \nu$.

There are 6 candidate solutions (excluding the inconsistent ones such as those with $q_u = 0$ and $K - q_u = 0$).

1. $q_r = 0$ and $q_u = K$. In this case $\mu_1 = 0$. To ensure $\lambda \geq 0$ and $\mu_2 \geq 0$ for the validity of this candidate solution, it must hold that $\frac{2}{\epsilon}(m_r - \gamma e) \leq K \leq \frac{2}{\epsilon}m_u$.

2. $q_r = 0$ and $q_u = 0$. This leads to the uninteresting case of no production and hence is discarded.

3. $q_r > 0$ and $q_u = K$. In this case $\lambda = \mu_1 = 0$ and $q_r = \frac{2}{\epsilon}(m_r - \gamma e) - K$. To ensure $q_r \geq 0$ and $\mu_2 \geq 0$ for validity of this candidate solution, it must hold that $K \leq \frac{2}{\epsilon}(m_r - \gamma e)$ and $(m_r - \gamma e) \leq m_u$.

4. $q_r > 0$ and $q_u = 0$. In this case $q_r = \frac{2}{\epsilon}(m_r - \gamma e)$ and $\lambda = \mu_2 = 0$, to ensure $\mu_1 \geq 0$ for the validity of this candidate solution, it must hold that $(m_r - \gamma e) \geq m_u$.

5. $q_r = 0$ and $0 < q_u < K$. In this case $\mu_1 = \mu_2 = 0$ and $q_u = \frac{2}{\epsilon}m_u$. To ensure $\lambda \geq 0$ and $0 < q_u < K$ for the validity of this candidate solution, it must hold that $\frac{2}{\epsilon}(m_r - \gamma e) \leq \frac{2}{\epsilon}m_u \leq K$.

6. $q_r > 0$ and $0 < q_u < K$. In this case $\lambda = \mu_1 = \mu_2 = 0$, i.e., $m_r - \gamma e = m_u$, which reduces to a special case of the above cases.

Moreover, in all of the above cases, the Hessian matrix is negative semi-definite, which ensures the solutions are profit-maximizing. Also note that since $m_r - \gamma e$ is linear in $\gamma$, $m_r - \gamma e \geq m_u$ and $m_r - \gamma e \leq m_u$ are equivalent to $\gamma \leq \tilde{\gamma}$ and $\tilde{\gamma} \leq \gamma$ respectively, with $\tilde{\gamma} = \frac{e \gamma - c_r - \gamma^+ \nu}{e - \nu}$.

To summarize:

$$\begin{align*}
(q_r^*, q_u^*) = \begin{cases} 
(R) \left( \frac{2}{\epsilon}(m_r - \gamma e), 0 \right) & \text{when } \gamma^{-} \leq \gamma \leq \tilde{\gamma} \\
(B) \left( \frac{2}{\epsilon}(m_r - \gamma e) - K, K \right) & \text{when } \max\{\gamma^{-}, \tilde{\gamma}\} \leq \gamma \leq \gamma^{+} \quad \text{and} \quad K \leq \frac{2}{\epsilon}(m_r - \gamma e) \\
(U) \left( 0, K \right) & \text{when } \max\{\gamma^{-}, \tilde{\gamma}\} \leq \gamma \leq \gamma^{+} \quad \text{and} \quad \frac{2}{\epsilon}(m_r - \gamma e) \leq K
\end{cases}
\end{align*}$$

- **Case (ii):** When $0 < e < \nu$.

This case can be solved in a similar way as Case (i).
All results are summarized in Lemma 1.

Moreover, given a realized emission price $e$, the second-period profit under each production action will be

\[(R) : \Pi(R) = -\frac{1}{\epsilon} \left( \frac{\epsilon}{2} (m_r - \gamma e)^2 + \frac{\epsilon}{2} (m_r - \gamma e)^2 \right) = \frac{\epsilon}{4} (m_r - \gamma e)^2\]
\[(B) : \Pi(B) = \frac{\epsilon}{4} (m_r - \gamma e)^2 + (m_u - (m_r - \gamma e)K)\]
\[(U) : \Pi(U) = -\frac{1}{\epsilon} K^2 + m_u K\]

**Proof of Lemma 2**

When $e$ follows a two-point distribution, all the possible contingent production strategies are: $(R, R), (R, B), (R, U), (B, R), (B, B), (B, U), (U, R), (U, B), (U, U)$.

First, we rule out $(B, R), (U, R), (U, B)$. For $(B, R)$, the $B$ action is optimal when $e = e_l$, which indicates $m_r - \gamma e_l \leq m_u$. Then when $e = e_h$, it must hold that $m_r - \gamma e_h < m_u$ and hence the $R$ action cannot be optimal. The other two strategies can also be proved to be suboptimal following a similar argument.

Second, we rule out $(R, B)$ and $(B, B)$. With $(R, B)$ the expected two-period profit will be $Z(\gamma, K) = \beta_r (\gamma^+ - \gamma)^2 + \beta_u K + (\rho \Pi(R) + (1 - \rho) \Pi(B))$, which is linear in $K$ and hence $K^*$ will be chosen as high as possible or as low as possible. Consequently, $(R, B)$ will be dominated by $(R, R)$ or $(R, U)$. Similarly, $(B, B)$ is also suboptimal.

To summarize, $(R, R), (R, U), (B, U)$ and $(U, U)$ constitute the set of non-dominated production strategies.

**Proof of Proposition 1**

To solve for $(\gamma^*_i, K^*_i)$ for each of the non-dominated profit-maximizing strategies $(R, R), (R, U), (B, U)$ and $(U, U)$, we solve the Problem (P) below.

\[
\max_{\gamma, K} Z(\gamma, K) = E_e[\Pi^*(e, \gamma, K)] - \beta_r (\gamma^+ - \gamma)^2 - \beta_u K, \quad \text{subject to} \quad 0 \leq K, \ \gamma^- \leq \gamma \leq \gamma^+.
\]

We look into each of the strategies. For ease of exposition, denote $\tilde{m}_r = \tilde{p} - c_v - \nu \gamma^+$.

1. For the strategy $(R, R)$, the profit is

\[
Z_1(\gamma, K) = -\beta_u K - \beta_r (\gamma^+ - \gamma)^2 + \frac{\epsilon}{4} (\tilde{m}_r - \gamma (e_l - \nu))^2 + \frac{\epsilon(1 - \rho)}{4} (\tilde{m}_r - \gamma (e_h - \nu))^2
\]
\[= (\frac{\epsilon}{4} (\rho (e_l - \nu)^2 + (1 - \rho)(e_h - \nu)^2 - \beta_r)^2
\]
\[-2(\frac{\epsilon}{4} \rho (e_l - \nu) + (1 - \rho)(e_h - \nu)) \tilde{m}_r - \tilde{\beta}_r \gamma + \frac{\epsilon}{4} \tilde{m}_r^2 - \tilde{\beta}_r (\gamma^+)^2 - \beta_u K.
\]
To ensure the existence of a profit-maximizing value of $\gamma$, assume $\frac{\epsilon}{4}(\rho(e_l - \nu)^2 + (1 - \rho)(e_h - \nu)^2) - \beta_r < 0$. Since $Z_1(\gamma, K)$ is a separable function in $\gamma$ and $K$, and concave in $\gamma$ and linear in $K$, $K_1^* = 0$ and $\gamma_1^*$ can be derived from the first-order condition:

$$\gamma_1^* = \frac{\beta_r \gamma_1^* - \frac{\epsilon}{4}(\rho(e_l - \nu)(1 - \rho)(e_h - \nu))m_r}{\beta_r - \frac{\epsilon}{4}(\rho(e_l - \nu)^2 + (1 - \rho)(e_h - \nu)^2)}.$$ 

For this strategy to be consistent with Lemma 1, such that the action $R$ will indeed be optimally adopted for both $e = e_h$ and $e = e_l$, the condition $\gamma^* \leq \gamma_1^* \leq \min\{\gamma^*, \frac{m_r - m_u}{e_l - e_h}\}$ must hold.

2. For the strategy $(R, U)$, the profit is

$$Z_2(\gamma, K) = -\beta_u K - \beta_r (\gamma^+ - \gamma)^2 + \frac{\epsilon}{4}(m_r - \gamma(e_l - \nu)^2) + (1 - \rho)(\frac{1}{4}K^2 + m_u K)$$

$$= -\beta_r - \frac{\epsilon}{4}(\rho(e_l - \nu)^2)\gamma^2 + 2(\beta_r \gamma^+ - \frac{\epsilon}{4}m_r(e_l - \nu))\gamma + (1 - \rho)(\frac{1}{4}K^2 + (m_u - \frac{m_r - m_u}{e_l - e_h})K) - \beta_r (\gamma^+)^2.$$ 

Since $Z_2(\gamma, K)$ is a separable function in $\gamma$ and $K$, and concave in both of them, the profit-maximizing solutions can be solved from the respective first-order conditions:

$$\gamma_2^* = \frac{\beta_r \gamma_2^* - \frac{\epsilon}{4}(\rho(e_l - \nu)m_r)}{\beta_r - \frac{\epsilon}{4}(\rho(e_l - \nu)^2)}, \quad K_2^* = \frac{\epsilon}{2}(m_u - \frac{\beta_u}{1 - \rho}).$$

For this strategy to be consistent with Lemma 1, such that the $R$ action will indeed be adopted when the emission price is low and the $U$ action will be adopted otherwise, the following conditions must hold: $\max\{\gamma^*, \frac{m_r - m_u}{e_l - e_h}\} \leq \gamma_2^* \leq \min\{\gamma^*, \frac{m_r - m_u}{e_l - e_h}\}$, and $\frac{\epsilon}{2}(m_r - \gamma_2^*(e_h - \nu)) \leq K_2^*.$

3. For strategy $(B, U)$, the profit function becomes

$$Z_3(\gamma, K) = -\beta_u K - \beta_r (\gamma^+ - \gamma)^2 + \rho \left( \frac{\epsilon}{4}(m_r - \gamma(e_l - \nu)^2) + ((m_r - \gamma(e_l - \nu)) - m_u K) \right)$$

$$+ (1 - \rho) \left( -\frac{1}{\epsilon}K^2 + m_u K \right)$$

$$= -\beta_r - \frac{\epsilon}{4}(\rho(e_l - \nu)^2)\gamma^2 + 2(\beta_r \gamma^+ - \frac{\epsilon}{4}m_r(e_l - \nu))\gamma + (1 - \rho)m_u - \beta_u + \rho(m_u - (m_r - \gamma(e_l - \nu)))/K - \beta_r (\gamma^+)^2 + \frac{\epsilon}{4}\rho m_r^2.$$ 

To ensure the profit is jointly concave in $K$ and $\gamma$, the associated Hessian matrix has to be negative definite, with

$$H = \begin{bmatrix} 2\frac{\rho(e_l - \nu)^2 - \beta_r}{\rho(e_l - \nu)^2} & \rho(e_l - \nu) \\ \rho(e_l - \nu) & -2\frac{1}{\epsilon} \end{bmatrix},$$

which reduces to $2(\frac{\epsilon}{4}\rho(e_l - \nu)^2 - \beta_r)(-2\frac{1}{\epsilon} - (\rho(e_l - \nu))^2) = (1 - \rho)\beta_r - \frac{\epsilon}{4}\rho(e_l - \nu)^2 > 0$ (note that we already assume $2(\frac{\epsilon}{4}\rho(e_l - \nu)^2 - \beta_r) < 0$). Then from the first-order conditions, the profit-maximizing interior solution can be derived as follows:

$$\gamma_3^* = \frac{\beta_r (1 - \rho) \gamma^+ - \frac{\epsilon}{4}(\rho(e_l - \nu)(m_r - m_u + \beta_u))}{\beta_r (1 - \rho) - \frac{\epsilon}{4}(\rho(e_l - \nu)^2)}, \quad K_3^* = \frac{\epsilon}{2}(m_u - \beta_u) + \frac{\epsilon}{2}\rho \frac{m_r - \beta_u - (m_r - \gamma^+(e_l - \nu))}{\beta_r (1 - \rho) - \frac{\epsilon}{4}(\rho(e_l - \nu)^2)}.$$ 

For this strategy to be consistent with Lemma 1, so that the $B$ action will be adopted when the emission price is low and the $U$ action will be adopted otherwise, the following conditions must hold: $\max\{\gamma^*, \frac{m_r - m_u}{e_l - e_h}\} \leq \gamma_3^* \leq \gamma^+$ and $\max\{0, \frac{1}{2}(m_r - \gamma_3^*(e_h - \nu))\} \leq K_3^* \leq \frac{\epsilon}{2}(m_r - \gamma_3^*(e_h - \nu)).$
4. For strategy \((U, U)\), the profit function is

\[
Z_4(\gamma, K) = -\beta_u K - \beta_r (\gamma^+ - \gamma)^2 - \frac{1}{\epsilon} K^2 + (m_u)K.
\]

The profit-maximizing solution can be derived from the first-order condition equations:

\[
\gamma_4^* = \gamma^+, \quad K_4^* = \frac{\epsilon}{2}(m_u - \beta_u).
\]

For this strategy to be consistent with Lemma 1, such that the action \(U\) will indeed be optimally adopted for both \(e = e_h\) and \(e = e_l\), the condition \(m_r - \gamma^+ e_h \leq m_u - \beta_u\) must hold.

These results are summarized in Proposition 1.

**Proof of Proposition 2**

We start by proving the results for \(\gamma^*\). From Proposition 1, it is straightforward to show that \(\gamma^*\) weakly increases in \(\gamma^+\) and weakly decreases in \(\tilde{m}_r\). Moreover, \(\tilde{m}_r\) is increasing in \(\tilde{p} - c_r\). Therefore, \(\gamma^*\) weakly decreases in \(\tilde{p} - c_r\).

To prove that \(\gamma^*\) is weakly increasing in \(\beta_r\), we first calculate \(\Delta_1 = \gamma^+ - \gamma_1^*\). Specifically,

\[
\Delta_1 = \gamma^+ - \gamma_1^* = \frac{\epsilon}{4} \frac{\beta_r \rho(e_l - \nu)(\tilde{m}_r - \gamma^+(e_l - \nu)) + (1 - \rho)(e_h - \nu)(\tilde{m}_r - \gamma^+(e_h - \nu))}{\beta_r - \frac{\epsilon}{4}(\beta_r - \frac{\epsilon}{4}(1 - \rho)(e_h - \nu)^2)}
\]

which is decreasing in \(\beta_r\). Therefore, \(\gamma_1^*\) is weakly increasing in \(\beta_r\) under the strategy \((R, R)\). The results under other strategies can be proved in a similar way.

To prove the results for \(q_{r,i}^*\), note that when the action \(R\) is adopted by the contingent production strategy after the realization of the emission price, the optimal production quantity has the form of \(q_{r,i}^* = \frac{\epsilon}{4}(\tilde{m}_r - \gamma^*(e - \nu))\), which is increasing in \(\tilde{m}_r\) and hence \(\tilde{p} - c_r\), since \(\gamma^*\) is weakly decreasing in \(\tilde{m}_r\). When the action \(U\) is adopted, \(q_{r,i}^* = 0\). Finally, the action \(B\) is adopted only under the strategy \((B, U)\) and \(e = e_l\). In this case,

\[
q_{r,i}^* = \frac{\epsilon}{2} \frac{\beta_r \rho(e_l - \nu)(\tilde{m}_r - \gamma^+(e_l - \nu)) - (m_u - \beta_u)}{\beta_r(1 - \rho) - \frac{\epsilon}{4}\rho(e_l - \nu)^2}
\]

which is also weakly increasing in \(\tilde{m}_r\).

From Proposition 1, it is straightforward to see that \(K^*\) is weakly increasing in \(m_u\) and weakly decreasing in \(\beta_u\).

**Proof of Proposition 3**

From Proposition 1:

\[
\gamma_1^* - \gamma_2^* = \frac{-\frac{\epsilon}{2}m_r \rho(e_l - \nu)(e_h - e_l) - \beta_r (\tilde{m}_r - \gamma^+(e_h - \nu))}{(\beta_r - \frac{\epsilon}{4}\rho(e_l - \nu)^2 - \frac{\epsilon}{4}(1 - \rho)(e_h - \nu)^2)(\beta_r - \frac{\epsilon}{4}\rho(e_l - \nu)^2)} \leq 0,
\]

\[
\gamma_2^* - \gamma_3^* = \frac{-\frac{\epsilon}{2}(e_l - \nu)K_4^*}{(\beta_r - \frac{\epsilon}{4}\rho(e_l - \nu)^2)} \leq 0, \quad \gamma_3^* \leq \gamma^* = \gamma_4^*.
\]
Therefore, $\gamma^*_1 \leq \gamma^*_2 \leq \gamma^*_3 \leq \gamma^*_4$.

For the production capacity in the unregulated region, direct comparison proves that $K^*_2 \leq K^*_4$ based on Proposition 1. For $K^*_3$, note that

$$K^*_3 = \frac{\epsilon}{2} (m_u - \beta_u) - \frac{\epsilon \rho}{2(1 - \rho)}((m_r - r^*_3(e_l - \nu)) - (m_u - \beta_u))$$

Moreover, as this strategy adopts action $U$ when $e = e_l$, $K^*_3 \leq \frac{\epsilon}{2}(m_r - \gamma^*_3(e_l - \nu))$, which leads to $m_u - \beta_u \leq m_r - \gamma^*_3(e_l - \nu)$. Then $K^*_2 - K^*_3 \leq 0$ when $Z^*_2 \leq Z^*_3$.

**Proof of Proposition 4**

Since $K^{b*}_1$ and $\gamma^*_1$ have similar structures as $K^*_1$ and $\gamma^*_1$ except that $m_u$ is replaced by $m_u - \gamma^- e$, and both $\gamma^*_1$ and $K^*_1$ are weakly increasing in $m_u$, we have $K^{b*}_1 \leq K^*_1$ and $\gamma^{b*}_1 \leq \gamma^*_1$. Combining these with the results in Proposition 3 yields $\gamma^{b*} \leq \gamma^*$ and $K^{b*} \leq K^*$.

On the other hand, the total emission $\Gamma^*_1 = \rho(\gamma^* q^*_e + (\gamma^* + \gamma_u) q^*_u)|_{e = e_l} + (1 - \rho)(\gamma^* q^*_e + (\gamma^* + \gamma_u) q^*_u)|_{e = e_h}$. Under strategy $(R, R)$, the change of $m_u$ to $m_u - \gamma^- e$ introduced by the border tax does not influence the decisions; i.e., $\gamma^{b*}_1 = \gamma^*_1$, $K^{b*}_1 = K^*_1$, $q^{b*}_{e,1} = q^*_{e,1}$ and $q^{b*}_{u,1} = q^*_{u,1}$; therefore, $\Gamma^{b*}_1 = \Gamma^*_1$. Under strategy $(R, U)$, $\gamma^{b*}_2 = \gamma^*_2$ and $K^{b*}_2 \leq K^*_2$. When $e = e_l$, both $q^{b*}_{e,2} = q^*_{e,2}$ and $q^{b*}_{u,2} = q^*_{u,2}$; when $e = e_h$, $q^{b*}_{e,2} = q^*_{e,2}$ but $q^{b*}_{u,2} \leq q^*_{u,2}$. Therefore, $\Gamma^{b*}_2 \leq \Gamma^*_2$. Under strategy $(B, U)$, $\gamma^{b*}_2 \leq \gamma^*_2$ and $K^{b*}_2 \leq K^*_2$; moreover, $q_{e,3} = \frac{m_r - \gamma^*_3(e_l - \nu) - \gamma^*_3(e_l - \nu)}{35}$ always holds. Therefore $\Gamma^{b*}_3 \leq \Gamma^*_3$.

**Proof of Proposition 5**

The proof is similar to the proof of Proposition 4 and is omitted for brevity.

**Proof of Lemma 3**

We solve $\max_{q_e, q_u} \Pi^*(q_e, q_u | e, \gamma, K)$ in Stage 2. The profit function can be rewritten as

$$\Pi^*(q_e, q_u | e, \gamma, K) = \begin{cases} -\frac{1}{2} (q_e + q_u)^2 + q_e(m_r) + q_u(m_u), & \text{for } 0 \leq q_e \leq \frac{\Omega}{\gamma}. \\ -\frac{1}{2} (q_e + q_u)^2 + q_e(m_r - \gamma e) + \Omega e + q_u(m_u), & \text{for } q_e > \frac{\Omega}{\gamma}. \end{cases} \quad (8)$$

Therefore, when $q_e > \frac{\Omega}{\gamma}$, the grandfathered allowance does not influence the optimal profit margin in either region and hence the resulting optimal production decisions remain the same as in the Baseline case. We next solve the following problem

$$\max_{q_e, q_u} \Pi(q_e, q_u | e, \gamma, K) = -\frac{1}{2} (q_e + q_u)^2 + q_e(m_r) + q_u(m_u), \quad \text{subject to } 0 \leq q_u \leq K \quad \text{and } 0 \leq q_e \leq \frac{\Omega}{\gamma}.$$
We form the Lagrangian for maximization

\[ L = -\frac{1}{\epsilon} (q_r + q_u)^2 + q_r (\bar{m}_r + \gamma \nu) + q_u (m_u) + \lambda_1 (q_u) + \lambda_2 (K - q_u) + \mu_1 (q_r) + \mu_2 \left( \frac{\Omega}{\gamma} - q_r \right) \]

The first-order condition equations are:

\[ \frac{\partial L}{\partial q_r} = 0 \Rightarrow 2 \frac{1}{\epsilon} (q_r + q_u) - m_r = \mu_1 - \mu_2 \]

\[ \frac{\partial L}{\partial q_r} = 0 \Rightarrow 2 \frac{1}{\epsilon} (q_r + q_u) - m_u = \lambda_1 - \lambda_2 \]

There are 9 possible cases (excluding the inconsistent ones such as those with \( q_u = 0 \) and \( K - q_u = 0 \)).

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1. \( q_u = 0 \) and \( \mu_1 = \mu_2 = \lambda_2 = 0 \). From the first-order conditions, \( q_r = \frac{\bar{m}_r}{\gamma} \). To ensure \( \lambda_2 \geq 0 \) and \( 0 \leq q_r \leq \frac{\Omega}{\gamma} \) for validity of this candidate solution, the associated constraints are \( m_r \geq m_u \) and \( 0 \leq \frac{\bar{m}_r}{\gamma} \leq \frac{\Omega}{\gamma} \).

All other cases for \( q_r \leq \frac{\Omega}{\gamma} \) can be solved in a similar way.

Following this procedure, we can also solve the following problem for \( q_r \geq \frac{\Omega}{\gamma} \).

\[
\max_{q_r, q_u} \Pi(q_r, q_u | e, \gamma, \nu) = -\frac{1}{\epsilon} (q_r + q_u)^2 + q_r (m_r - \gamma e) + q_u (m_u) + \Omega, \quad \text{subject to} \quad 0 \leq q_u \leq K \quad \text{and} \quad \frac{\Omega}{\gamma} \leq q_r.
\]

Finally, to determine which case of \( q_r \leq \frac{\Omega}{\gamma} \) and \( q_r \geq \frac{\Omega}{\gamma} \) yields the final optimal production action, we compare the resulting profits from each of the two cases. For example, when \( \frac{\bar{m}_r}{\gamma} \leq \frac{\Omega}{\gamma} \) and \( \frac{\nu}{\gamma} m_r \leq \frac{\nu}{\gamma} m_u \), the optimal profit when \( q_r \leq \frac{\Omega}{\gamma} \) is achieved at \( q_r = \frac{\bar{m}_r}{\gamma} \) and \( q_u = 0 \), whereas the optimal profit when \( q_r \geq \frac{\Omega}{\gamma} \) is achieved at \( q_r = \frac{\nu}{\gamma} m_r \) and \( q_u = 0 \). Comparison between the two resulting profits shows that the one in the case \( q_r \leq \frac{\Omega}{\gamma} \) is higher and hence \( q_r = \frac{\nu}{\gamma} m_r \) and \( q_u = 0 \) is the optimal solution. All other cases can be solved in a similar way and the final optimal production decisions are summarized in Lemma 3.

**Proof of Proposition 6**

From Lemma 3, substituting \( e = e_l \) and \( e = e_h \) respectively, we can then identify all the possible non-dominated production strategies. The results are summarized in Proposition 6.

**Details of The Numerical Study**

Regarding the location of offshore production, China seems infeasible because of the relatively high transport costs compared to the typical cement sales price of 90/ton (all monetary values are in €). On the other hand, Egypt is currently the biggest producer of cement in the EMEA.
region and much closer to Europe (MapXL 2012). Lafarge has bought a large Egyptian cement producer in 2007 (Lafarge 2007). Although mainly producing for the Egyptian market, it provides an opportunity to serve the European market in the longer term. Therefore, we assume that Egypt is chosen as the offshore production location, where emission regulation is currently absent.

We first describe the benchmark values of the parameters based on the data in Drake et al. (2010). Two technologies are available for cement production in Europe (the regulated region): the current (more polluting) technology and carbon capture and storage (CCS) (cleaner) technology, with emission intensities 0.075/ton and 0.7/ton respectively (Drake et al. 2010, p. 33). We assume that there exist sufficient alternatives to realize any emission intensity between the current and CCS technology, by altering fuels or other raw materials; i.e., $0.075 = \gamma^- \leq \gamma \leq \gamma^+ = 0.7$. The technology improvement potential of the current technology is measured by $\gamma^+ - \gamma^- = 0.625$. The investment cost for the CCS technology is $14.3$ per unit of capacity (Drake et al. 2010, p. 33). We convert it into cost per period to be consistent with our model, based on a yearly production of $2 \cdot 10^6$ tons (Drake et al. 2010, p. 33), which will be $28.6 \cdot 10^6 (= 14.3 \times 2 \cdot 10^6)$ per year. This investment corresponds to a reduction of emission intensity from $\gamma^+$ to $\gamma$: i.e., $\beta_r(\gamma^+ - \gamma^-)^2 = 28.6 \cdot 10^6$, therefore $\beta_r = \frac{28.6 \cdot 10^6}{0.625} = 73.22 \cdot 10^6$. Based on data, the estimated parameter ($\beta_u$) of the offshore production capacity investment cost ($\beta_u K$) is $\beta_u = 10.1$ (Drake et al. 2010, p. 33). The unit production cost in the regulated region using the current and the CCS production technology are estimated to be $43.6$ and $55.0$, respectively (Drake et al. 2010, p. 33). Therefore $c_r = 43.6$ and $c_r + \nu(\gamma^+ - \gamma^-) = 55$, resulting in $\nu = \frac{55 - 43.6}{0.7 - 0.075} = 18.24$. Production is cheaper in Egypt (due to factors such as lower-cost labor), which we assume to be 80% of the cost in Europe. The cost of transporting the products back to Europe is $0.004/\text{ton}-\text{km}$ (Demailly and Quirion 2006). Therefore, $c_u = 0.8c_r + 0.004\delta$ in Egypt, where $\delta$ represents the distance between the two regions. Estimations from the cement demand data in Drake et al. (2010), Smale et al. (2006, p.39) and Salvo (2010, p.339) yields $Q = 2.5 \cdot 10^6$ and $\epsilon = 2.22 \cdot 10^3$, as consistent with the cement industry.

References


