We analyze product design implications of Extended Producer Responsibility (EPR)-based take-back legislation on durable goods. In particular, we observe that durable product design incentives under EPR may involve an inherent trade-off that has not been explored to date: Durable goods producers can respond to EPR by making their products either more recyclable or more durable, where the former will decrease the unit recycling cost whereas the latter will reduce the volume the producer has to recycle. When these two design attributes do not go hand-in-hand, as is the case for many product categories, product design implications of EPR can be subtle. We find that seemingly similar EPR implementation levers, namely recycling and collection targets, may have opposing effects in driving producers’ design choices. Furthermore, more stringent legislative targets do not always guarantee improved product recyclability and durability. In particular, if the objective of EPR is to induce recyclable product designs, a low recycling target accompanied with a high collection target is preferred. On the other hand, if the objective of EPR is to induce durable product designs, a low collection target accompanied with a high recycling target is preferred.

Key words: Durability, Recyclability, Take-Back Legislation, Extended Producer Responsibility

1. Introduction

Extended Producer Responsibility (EPR) is a policy concept that requires producers to operate or finance the management of their end-of-life products through environmentally friendly processes such as recycling. By assigning product end-of-life management responsibility to producers, EPR legislation is expected to create incentives for producers to design products with environmentally superior attributes (OECD 2001). In particular, because EPR legislation emphasizes recycling (e.g., the WEEE Directive specifies recycling rate targets\(^1\)), it is commonly assumed that it will induce eco-design incentives that are geared towards making products more recyclable (easier and cheaper to recycle). Yet durable goods producers - and most products covered under EPR are durable - have an additional lever at their disposal: investing in durability. In contrast to recyclability that acts on unit cost, durability acts on volume: A more durable product can be priced higher and sold to fewer customers while maintaining profitability. Surprisingly, however, a durable good

\(^1\) A recycling rate target is defined as the percentage, by weight, of the product that needs to be recycled.
producer’s durability choice in response to EPR has received limited attention (see §2 for a detailed discussion). In this paper, we fill this void.

These are timely questions: In recent years, recognizing that most electronic products are durable, durability has emerged as a stated eco-design objective. This is the case both in Europe and in the US. For example, the Sustainable Production and Consumption Unit of the European Commission now considers both recyclability and durability as key eco-design attributes (Misiga 2012). EPR legislation in Rhode Island states that its purpose is encouraging the design of electronic products that are more durable and more recyclable (RIDEM 2013). The environmental rationale on durability is that durability may lead to “source reduction,” the most preferred form of waste reduction (U.S. EPA 2013), by suppressing new production due to two effects: life extension and demand response (a more durable product can be priced higher as described above). However, it is hard to specify a durability target that is akin in ease of enforcement to a recycling rate or a collection rate target\(^2\): This would require regulating how quickly a product’s market value should depreciate or how many years a product should last (both of which have been utilized in the literature as durability measures), neither of which is viable. Therefore, another open question is: How do recycling and collection rate targets imposed by EPR, especially as they become more stringent, indirectly affect durability choice? This is another question this paper seeks to answer.

How recyclability and durability interact matters greatly in this context. On one hand, they may be synergistic design attributes, i.e., they can be enhanced by similar design changes. Eichner and Runkel (2003) argue that both attributes positively correlate with the weight of the product: Cars with thicker aluminum frames are more durable due to their higher rigidity and more recyclable due to a higher content of recyclable material. Similarly, a desktop computer with an aluminum casing is cheaper to recycle and more durable than one with a plastic casing (HP 2009). On the other hand, these two attributes may conflict, as in the following examples: Using screws rather than adhesives increases durability because screws are more chemically stable and withstand heat better, but this increases the recycling cost (i.e., reduces recyclability) as screws require labor for disassembly (Bonnington 2014). Apple replaced PVC by TPE in the sheathings of its new generation lightning cables. These cables are more recyclable yet less durable because TPE is softer (Apple 2015). NiMH batteries are more recyclable than NiCd batteries because they contain more nickel. However, NiMH batteries can be recharged fewer times than the NiCd batteries, i.e., they are much less durable (Langrova 2002).

Photovoltaic Panels (PVPs), which have recently been added to the scope of the WEEE Directive, exhibit a similar durability-recyclability trade-off in multiple design dimensions. For PVPs,

\(^2\)A collection rate target is defined as the proportion of total product volume sold that needs to be collected for recycling.
choosing the main technology for the photovoltaic cells is an important design decision. Currently, the two most prevalent alternatives that are mature enough for commercial use are crystalline silicon (c-Si) and thin-film technologies. In general, the thin-film PVPs are more recyclable than the c-Si PVPs because the thin-film panels contain precious rare metals such as indium and gallium, which can be recovered by recycling and have positive recycling value. However, a c-Si PVP outperforms a thin-film-based one in product durability because it has a lower degradation rate (Jordan and Kurtz 2012). To be more specific, PVPs bring value to consumers by converting sunlight into electric power, so when the efficiency of this functionality depreciates more slowly, as reflected by a lower degradation rate, it means the products are more durable. The trade-off between recyclability and durability exists in determining the build of PVPs as well. A frameless PVP structure facilitates recycling due to easier disassembly, but may compromise the durability when compared to a framed design (Besiou and Wassenhove 2015). One reason is that frameless PVPs are more vulnerable to damage, especially in a dusty or humid environment. Moreover, the glass-glass frameless PVP modules have a higher degradation rate than the glass-polymer framed modules, which indicates lower durability (Jordan and Kurtz 2012). Finally, the design decision on the encapsulant material (which is a layer of adhesive in PVPs) may also involve a durability-recyclability trade-off. Silicone encapsulants are less recyclable than EVA-based encapsulants (NovoPolymers 2015) but are more durable as they are more resistant to discoloration-based deterioration.

In this paper, we develop a model to shed light on a durable good producer’s choice of durability and recyclability under EPR when these two design attributes interact. In particular, we focus on EPR legislation imposing recycling and collection targets, inspired by the WEEE Directive of the European Commission (Europa-Environment 2012), probably the most influential EPR legislation to date, covering 27 countries in Europe for the majority of electrical and electronic equipment categories. We first derive a cornerstone technical result: a joint closed-form characterization of the optimal recyclability and durability choices. To the best of our knowledge, this is the first paper to provide structural results about the durability-recyclability interaction by endogenizing durability choice in a market subject to EPR.

We leverage this characterization to derive a number of managerial insights. When recyclability and durability are synergistic design attributes, a more stringent recycling rate target indeed leads to the most favorable design outcome: a product that is more recyclable and durable. However, a producer’s design choices can be quite counter-intuitive when the two attributes conflict (as in the above examples). Relatively low recycling targets work as intended, i.e., durable goods producers should design for recyclability, but this may come at the expense of durability. However, further increases in recycling targets may drive producers to switch to designing more durable products at the expense of recyclability. This effect is even stronger for producers with higher profit margins.
Furthermore, a similar analysis of the collection targets reveals that seemingly similar EPR implementation levers have very different effects on a durable good producer’s design choices. Collection targets have the complete opposite effect compared to the recycling targets: Relatively low collection targets imply that durable goods producers should design for durability, which helps reduce collection volume obligations by reducing sales volume, but this may come at the expense of lower recyclability. Interestingly, further increases in collection targets may drive producers to switch to designing more recyclable, yet less durable products despite the increased collection volume obligations this strategy may imply. This effect is further strengthened when recycling a unit is profitable and implies a competitive market for access to end-of-life products. In that case, a durable good producer always designs more recyclable but less durable products as collection targets become more stringent. We show that under some conditions, the emphasis on lower durability in response to stringent collection targets translates to increased production and disposal volumes.

We conclude that implicit assumptions on EPR-driven product design changes may not necessarily hold in durable goods markets. In particular, a stricter recycling target does not necessarily translate to a more recyclable product. Interestingly, there are some instances where higher durability may be observed without directly targeting durability. Moreover, depending on the economics of recycling, the collection rate lever can work in different ways. The policy implication is that in durable goods markets, the design outcomes associated with different EPR implementation levers such as collection and recycling targets (and their stringency level choices) should be carefully evaluated and these policy tools should be adapted to the product category.

To demonstrate how the implications of chosen policy levers on product design can be analyzed, we perform a calibrated numerical study for PVPs, a product category that was recently added in the WEEE Directive Recast and that faces clear trade-offs between recyclability and durability as explained above. The set-up of the WEEE Directive is especially interesting for our analysis, as its recent recast (Europa-Environment 2012) calls for collection and recycling targets to become more stringent over time. We focus on the design trade-off in the choice between c-Si and thin-film technologies for the PV cells. Using real market data and expert input from the PVP industry, our analysis allows us to investigate the implications of the WEEE Directive Recast on PVP producers’ technology choice, illustrating the advantages and pitfalls of EPR on design for the environment. This analysis demonstrates the importance of not using uniform legislative targets regarding recycling and collection rates for all product categories, but rather adapting these targets to product and market characteristics and environmental impact priorities.

2. Literature Review
This work draws on and contributes to EPR-related research at the interface of operations management and environmental economics. Environmental economists have long studied the economic
and environmental implications of EPR, particularly to analyze and compare the efficiency of different EPR policy instruments. Among those are (i) financial instruments such as production taxes, advance recycling fees, and refunds (see Turner and Pearce 1994, for a discussion); (ii) information-based instruments, e.g., labeling requirements mandating producers to display the environmental characteristics of their products (Lindqvist 2000); and (iii) administrative instruments, in the form of requirements imposed on producers, such as product take-back mandates (Lee 2002, Toffel 2003). See Callcott and Walls (2002), Eichner and Pething (2001), Fullerton and Wu (1998) and Dinan (1993) for further discussion as to how these different policy instruments compare.

We differ from this stream of research by our operational focus, i.e., we provide a detailed analysis of design choices under product take-back mandates (which are prevalent for a variety of reasons; see Atasu and Van Wassenhove 2010, for a detailed discussion) in the presence of a durability-recyclability trade-off in design. Focusing on operational questions has proven useful in studying various issues regarding environmentally sustainable practices in business (see Guide and Van Wassenhove 2007, Atasu et al. 2008, Ferguson and Souza 2010, for reviews). Studies in this vein include (i) reverse logistics for used or recovered products (Fleischmann et al. 2001, De Brito and Dekker 2004, Savaskan et al. 2004); (ii) inventory management with remanufactured products (Toktay et al. 2000, DeCroix 2006); (iii) consumer returns management (Guide et al. 2006, Ferguson et al. 2006); (vi) joint management of new and remanufactured products in the market (Debo et al. 2005, Tereyagoglu et al. 2015); (v) sustainability of different business models (Agrawal et al. 2012, ¨Ulku et al. 2012); and (vi) sustainable production via responsible sourcing by producers (Kraft et al. 2013, Kraft and Raz 2015) or by suppliers (Agrawal and Lee 2015). In particular, we contribute to the recent operations management literature investigating the implications of environmental legislation in different operational settings. Related research covers topics including (i) the effect of disclosure mandates on producers’ efforts to evaluate their environmental impacts (Kalkanci et al. 2015); (ii) the effect of emission legislation on carbon leakage (Islegen and Reichelstein 2011, Drake 2012); and (iii) the effect of consumer subsidies or environmental taxes on the adoption of green technologies (Cohen et al. 2015, Chamama et al. 2015, Krass et al. 2013). Recent research that shares our focus on EPR legislation includes (i) the effect of incorporating reuse targets in EPR on remanufacturing and the environment (Karakayali et al. 2015, Esenduran et al. 2014); (ii) the effect of collection and recycling cost structures and competition on the efficiency of EPR (Atasu et al. 2009, Toyasaki et al. 2011) and different stakeholders’ perspectives (Atasu et al. 2012); (iii) cost allocation in collective EPR implementations considering network-based operations (Gui et al. 2015) and exogenous cost sharing mechanisms (Atasu and Subramanian 2012, Jacobs and Subramanian 2011); (iv) the effect of secondary market interference practices (Alev et al. 2014); and (v) the effect of EPR on producers’ new product introduction decisions (Plambeck and Wang
2009) and product design choices (Atasu and Subramanian 2012, Subramanian et al. 2009, Atasu and Souza 2012, Raz et al. 2015, Esenduran and Kemahlioglu-Ziya 2015, Esenduran et al. 2015, to name a few). Our work is closely related to the last group of papers that explore the product design implications of EPR. Yet, to the best of our knowledge, none of these study EPR-driven design changes for durable products, or the associated durability-recyclability trade-off.

Our consideration of the design decisions on both recyclability and durability attributes has commonalities with new product development research that involves design with respect to multiple quality dimensions (Lacourbe et al. 2009, Chen 2001, Krishnan and Zhu 2006, Kim and Chhajed 2002, Chambers et al. 2006). We contribute to this stream by integrating a general model of design interactions in a durable goods setting to study the influence of EPR legislation on eco-design attributes. Our work also contributes to research in the durable goods literature (cf. Waldman 2003 for a comprehensive review) that analyzes a producer’s pricing and design decisions (see Coase 1972, Bulow 1982, 1986, Kim 1989, Choi 1994, Waldman 1996, Hendel and Lizzeri 1999, Huang et al. 2001, Agrawal et al. 2012, 2015). To the best of our knowledge, the effect of EPR on a producer’s product durability choice has not been studied to date and our results show that the effect of EPR can be quite nuanced.

3. Model

In this section we build a discrete-time, infinite-horizon model to analyze the implications of EPR on a durable good producer that designs its product for durability and recyclability. Periods are indexed by \( t \geq 0 \) and the timeline of events is as follows. At period \( t = 0 \), a legislator announces the EPR obligations imposed on the producer. Given the EPR obligations, the producer determines its product design, namely the durability and recyclability of the product. In each subsequent period \( t > 0 \), the producer sets its new product sales price \( p'_t \), recycles a portion of end-of-life products arising from past sales, and consumers make purchasing decisions given product durability and price.

Below, we first describe the assumptions regarding our market equilibrium analysis for \( t > 0 \), and characterize the demand. After that, we describe the model assumptions regarding the producer’s product design decisions under EPR, which helps us formulate the producer’s objective.

3.1. Demand Characterization

We consider a discrete-time, infinite-horizon, sequential game between a producer selling a single durable product and consumers, given a product with durability \( \delta \in [0, 1] \) and recyclability \( \rho \geq 0 \). We assume that the product of interest has a two-period lifetime (Desai and Purohit 1998, Huang et al. 2001, Hendel and Lizzeri 1999, Agrawal et al. 2015). A product can be in one of three states during its lifetime: new during the first period of use, used during the second period of use, and
end-of-life after two periods of use. We assume it still provides consumer utility after one period of use, but none after two periods of use.

The market size remains constant in each period $t$. Each consumer uses at most one unit of product at any time. Without loss of generality, we normalize the market size to a unit mass of customers indexed by their type $\theta$. To capture market heterogeneity, we assume $\theta$ is uniformly distributed in $[0, 1]$. A consumer type $\theta$ obtains a gross utility of $\theta$ and $\delta \theta$ from using new and used products for a period, respectively, where product durability is denoted by $\delta$ and represents the relative value of a used product compared to a new one, implying a depreciation or value loss after use that is typical for durable products. At the end of every period, consumers who own a product that still has useful life left may choose to sell the used product on the secondary market at the market-clearing price $p_u^t = p_u^t(p^t)$ and purchase a new one. In analyzing the game between the producer and the consumers, we focus our analysis on stationary equilibria, where the price stays constant in time (Hendel and Lizzeri 1999, Huang et al. 2001, Plambeck and Wang 2009, Agrawal et al. 2015); i.e., $p^t = p$ and $p_u^t = p_u$. This rules out transient effects due to only new products being present in the first period.

We assume that the consumers are forward looking. We also assume that all information regarding the cost structures and preferences is common knowledge, and all players have a common discount factor $\gamma$. Given the two-period lifetime of a product, we need only focus on two-period customer strategies. Moreover, the consumer utility from holding on to the product after using it for one period is equivalent to selling the one-period-old product at that point and buying a used one on the secondary market (cf. Appendix A.1 of Agrawal et al. 2015). Therefore, under stationarity, the per-period net utility from purchasing a new product is $\theta - p + \gamma p_u$, that from purchasing a used product is $\delta \theta - p_u$, and that from remaining inactive is 0. Furthermore, the net utility from these actions is independent of the consumer’s past actions. Consequently, there are only three undominated consumer strategies: buy new products in every period (Bn), buy used products in every period (Bu), and always remain inactive (RI); see also Hendel and Lizzeri (1999a), pp. 1099–1100 for an intuitive explanation of this. The per-period utilities of consumers choosing these undominated strategies are denoted by: $V[Bn, \theta] = \theta - p + \gamma p_u$, $V[Bu, \theta] = \delta \theta - p_u$ and $V[RI, \theta] = 0$, respectively. The differences $V[Bn, \theta] - V[Bu, \theta]$ and $V[Bu, \theta] - V[RI, \theta]$ are increasing functions in $\theta$. Therefore, there exist $\theta_1, \theta_2 \in [0, 1]$ such that consumers of type $\theta \in (\theta_1, 1]$ choose Bn, those of type $\theta \in (\theta_2, \theta_1]$ choose Bu and all others remain inactive. Then the demand for new products, $q(p, \delta)$, can be written as $q = 1 - \theta_1$ and the secondary market clearing price $p_u$ can be obtained by solving $\theta_2 - \theta_1 = 1 - \theta_1$. This analysis gives $\theta_1 = \frac{\gamma + \delta + \gamma \delta}{1 + \gamma + 2 \gamma \delta}$, $\theta_2 = \frac{-1 + 2 \gamma + 2 \gamma \delta}{1 + \gamma + 2 \gamma \delta}$, and $q = 1 - \theta_1 = \frac{1 + \gamma + \delta}{1 + \gamma + 2 \gamma \delta}$. For simplicity, we take $\gamma = 1$ in the remaining of the paper, i.e., $q = 1 - \theta_1 = \frac{1 + \delta}{1 + 3 \delta}$. 

3.2. Design Choices

We model the EPR obligations based on the WEEE Directive implementations in Europe, and consider a collection rate target $\lambda \in [0,1]$ and a recycling rate target $R \in [0,1]$ imposed on the producer (see §5.3 for a discussion on other forms of EPR implementation). We assume that the collection rate target requires the producer to collect a minimum $\lambda$ fraction of all end-of-life products, mimicking the WEEE Directive that specifies a percentage of total products put on the market that producers are required to collect (DBIS 2012). The recycling rate target requires the producer to recycle at least $R$ percentage (by weight) of each unit of product collected. This modeling choice also follows from the recycling rates of the WEEE Directive that are defined in terms of the percentage by weight per appliance, which can vary from 50% – 80% depending on product category (Europa-Environment 2010).

The recycling rate target directly influences the economics of recycling: End-of-life products consist of parts that have different recycling value, and recycling typically concentrates on the parts that have the highest value. For example, cellphone recycling focuses on harvesting the precious metal used in the circuit boards and soldering, such as gold, silver and copper. As the recycling percentage of a cellphone increases, recycling extends to also include the pure handset casings and then the mixed plastics, which have lower recycling value (Mobile Muster 2015). Similarly, for a desktop computer, the CPUs, memory cards, and motherboards are usually the components that recycling focuses on due to their precious metal content and hence higher value. If a higher recycling rate is desired from the item, then the less valuable parts such as the wires, aluminum heat sink or even the plastic casings will be recycled (SMF 2012). Therefore, for recycling every unit of product, the marginal recycling value (denoted as $f(r)$) can be represented as a non-increasing step function in the recycling level (denoted as $r$ and $r \in [0,1]$), with the steps corresponding to different materials/components of the product. $f(r)$ can also be negative for certain values of $r$, which indicates a cost incurred to recycle some parts of the product. This can be the case for products such as CRT monitors: While the yoke and circuit boards have positive recycling value, the leaded glass imposes a cost (MRI 2014). We approximate this relation by a linear function and let $f(r) = \alpha - \beta r$. Here, $\alpha$ represents the marginal recycling value from the most valuable parts (such as the gold from cellphones, or the CPUs from desktop computers). $\beta$ represents the degree to which the marginal recycling value drops as the recycling level increases, for example, $\beta$ for CRT monitors may be large because when the level of recycling rises from only including the yoke (with valuable materials) to also including the leaded glass components (costly to process), the marginal recycling value drops dramatically. In the absence of a given recycling target being specified by EPR legislation, the profit-maximizing producer would optimally only recycle the parts of each product unit that generate positive marginal recycling value. In other words, $r^* = \min\{1, \max\{0, r\}\}$, where
solves \( f(\tau) = 0 \), and the unit recycling value, i.e., the total recycling value obtained from one unit at the chosen recycling rate, equals \( \int_0^{r^*}(\alpha - \beta\phi)d\phi \). If the recycling rate target enforced by EPR is more stringent \((R > r^*)\), the producer is mandated to recycle some parts that do not generate positive marginal recycling value and the unit recycling value obtained \( (\int_0^R(\alpha - \beta\phi)d\phi) \) would be lower and could even be negative.

To lower the unit economic burden associated with mandated recycling, the producer can design for recyclability. For instance, when products are designed for ease-of-disassembly or use more recyclable materials, the unit recycling value will be higher. To model such design improvements for recyclability (denoted by \( \rho \)) in a tractable manner, we assume that the marginal recycling value at any recycling level increases by \( \rho \). In other words, \( f(r, \rho) = \alpha + \rho - \beta r \). As such, for a given recycling level \( r \), recycling a unit with recyclability level \( \rho \) brings a unit recycling value of \( v(r, \rho) = \int_0^R(\alpha + \rho - \beta\phi)d\phi = -\frac{1}{2}\beta r^2 + (\alpha + \rho)r \) (which can be negative). For a given collection target \( \lambda \), this implies an effective unit recycling value of \( \lambda v(r, \rho) \).

Alternatively, a producer subject to EPR legislation can design for durability, which we assume is equivalent to choosing \( \delta \). Increasing durability implies a higher consumer utility from buying new products. Therefore, it enables the producer to price the products higher and sell a smaller quantity, resulting in fewer end-of-life products and hence a smaller volume of products to recycle. This reduces the cost of complying with EPR legislation.

The producer’s design decisions on durability \((\delta)\) and recyclability \((\rho)\) influence the production costs. To capture this, we assume that a product that is non-durable and non-recyclable can be produced at cost \( m > 0 \) per unit, and that the cost of producing a unit of product with durability \( \delta \) and recyclability \( \rho \) is given by \( g(\rho, \delta) = m + \tau \rho^2 + b\delta^2 + d\rho\delta \), where \( \tau, b \geq 0^3 \).

This formulation considers durability and recyclability as different quality dimensions as in Krishnan and Zhu (2006). The coefficients \( \tau \) and \( b \) reflect the rate of cost increase driven by the relevant design attribute, and the associated terms are quadratic to reflect decreasing returns in these design attributes (Purohit 1994, Atasu and Subramanian 2012). The last term in this formulation captures the possible interaction between recyclability and durability choices in product design (paralleling Krishnan and Zhu 2006). When the two attributes are synergistic as in the example of thick aluminum panels in cars, or of metal in desktop computer casings, \( d < 0 \). However, when there is a trade-off between durability and recyclability as in the example of PVPs, \( d > 0 \), reflecting a higher cost to realize improvements in both attributes.

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3 Any collection \((c_c)\) and recycling cost \((c_p)\) that is incurred at a unit level (independent of recycling rate) can be captured by adjusting the unit production cost by \( \lambda (c_c + c_p) \).
4. The Equilibrium Characterization

Following the stationary demand characterization provided in §3.1, we assume that the producer maximizes its per-period profit in steady state. Given the sequential nature of the model, we solve the producer’s profit maximization problem by backward induction: Let \( \Pi(p, r; \rho, \delta) = q(p, \delta) \cdot (p - g(\rho, \delta) + \lambda v(r, \rho)) \) denote the producer’s per-period steady-state profit given durability \( \delta \) and recyclability \( \rho \), and the associated recycling, collection and production costs provided in §3.2 under a mandated collection rate \( \lambda \) (which we assume to be binding due to costly recycling or competition for profitable recycling; both cases are discussed in §5.2). We first solve \( \max_{p, r} \Pi(p, r; \rho, \delta) \) subject to \( r \geq R \) in steady state. Let \( \Pi^*(\rho, \delta) \) denote the solution to this problem. Then, the producer’s design problem can be formulated as \( \max_{\rho, \delta} \Pi^*(\rho, \delta) \).

Substituting previously derived expressions into the profit function, we obtain:

\[
\Pi(p, r; \rho, \delta) = \frac{1 + \frac{\delta - p}{1 + 3\delta}}{1 + \frac{3}{\delta}} \left( p - (m + \tau \rho^2 + b\delta^2 + d\delta \rho) + \lambda(\alpha + \rho)r - \frac{1}{2} 3r^2 \right). \tag{1}
\]

Lemma 1 characterizes the producer’s recycling level and pricing decisions as well as the resulting demand in steady state for given \( (\rho, \delta) \). In our analysis, we assume \( 2\beta\tau > \lambda \) so that \( (A1) \) is jointly concave with respect to \( (\rho, r) \). We also restrict the analysis to parameters \( (m, \tau, b, d, \alpha, \beta) \) where for at least some values of \( (\rho, \delta, r, \lambda) \), the unit profit margin is non-negative and the unit recycling value is lower than the unit production cost. All proofs are provided in the Appendix.

**Lemma 1** Given a product design with \( \rho \) and \( \delta \), the optimal recycling level \( r^*(\rho, \delta) \) and the new product price \( p^*(\rho, \delta) \) can be summarized as follows. When \( \frac{\alpha + \rho}{\beta} \leq R \), \( r^*(\rho, \delta) = R \); when \( R < \frac{\alpha + \rho}{\beta} < 1 \), \( r^*(\delta, \rho) = \frac{\alpha + \rho}{\beta} \); and when \( 1 \leq \frac{\alpha + \rho}{\beta} \), \( r^*(\delta, \rho) = 1 \). The optimal price \( p^*(\rho, \delta) = \frac{1}{2} \left( 1 + \frac{\delta - g(\rho, \delta)}{1 + 3\delta} \right) \). The corresponding new product demand is \( q(p^*(\delta, \rho), \delta) = \frac{1}{2(1+3\delta)} \left( 1 + \delta - g(\rho, \delta) + \lambda(\alpha + \rho)r^*(\rho, \delta) - \frac{1}{2} \beta(r^*(\rho, \delta))^2 \right) \).

Lemma 1 identifies three recycling scenarios that will be observed in equilibrium, which map well to the range of recycling choices in practice. The first case (i.e., \( r^* = R \)) represents scenarios where voluntary recycling is not profitable enough and hence the recycling target is binding at optimality. This outcome represents the case of regulated markets for most electronics, either due to low recycling margins (\( \alpha \) is small) as in the case of fluorescent lamps, or a quick drop in the marginal recycling value when the recycling level increases (\( \beta \) is high) as is typical in CRT recycling. The second case, \( r^* = \frac{\alpha + \rho}{\beta} \), represents voluntary (and partial) recycling beyond compliance requirements, which represents the case of the car industry subject to legislation such as the End of Life Vehicles Directive, where voluntary recycling has reportedly exceeded the compliance requirement levels (Smink 2006). Finally, the last case with full recycling, i.e., \( r^* = 1 \), represents markets...
for valuable commodities such as aluminum cans. Given this characterization of the producer’s optimal recycling and pricing decisions for a given design profile, we next investigate the producer’s design choices.

To build intuition as to the producer’s design choices, we first start with a simple benchmark scenario that ignores the possible design interactions between durability and recyclability, i.e., we assume \( d = 0 \). Proposition 1 presents the producer’s recyclability choice for a given durability level.

**Proposition 1** Let \( d = 0 \). The optimal recyclability choice (and the corresponding optimal recycling level) can be summarized as follows. When \( \alpha \leq R(\beta - \frac{\lambda}{2\tau}) \), \( \rho^* = \frac{\lambda}{\beta - \lambda} \) and \( r^* = R \); when \( R(\beta - \frac{\lambda}{2\tau}) < \alpha < \beta - \frac{\lambda}{2\tau} \), \( \rho^* = \frac{\alpha\lambda}{2\beta\tau - \lambda} \) and \( r^* = \frac{2\alpha\tau}{2\beta\tau - \lambda} \); and when \( \beta - \frac{\lambda}{2\tau} \leq \alpha \), \( \rho^* = \frac{\lambda}{2\tau} \) and \( r^* = 1 \).

Proposition 1 states that in the absence of a durability-recyclability interaction, the recyclability choice of the producer will be primarily driven by the baseline recycling value \( \alpha \), which is a measure of recycling benefits that can be obtained from recycling in the absence of design for recyclability. Two intuitive observations from this proposition are that (i) as \( \alpha \) increases, the recyclability choice of the producer will increase and (ii) the recycling target will be binding only if \( \alpha \) is sufficiently low. Moreover, \( \rho^* \) does not depend on \( \delta \), the durability level of the product: In the absence of the design interaction, the choice of recyclability boils down to minimizing the unit cost subject to the recycling target requirement.

Given this observation, we next characterize the producer’s optimal durability choice. The analysis reveals that the unit profit margin of a non-durable product at its optimal recyclability level is key; we call this quantity \( A^4 \). \( A \) increases when the product has a higher recycling value potential (higher \( \alpha \) or lower \( \beta \)), or a lower production cost (\( m \)). In turn, given \( A \), the producer’s profit for a given \( \delta \) can be written as \( \Pi(\delta) = \frac{1}{4(1+3\beta)}(\delta - \beta^2 + A)^2 \) (see Lemma 2 in the Appendix for a derivation), which leads us to one of the important technical results in this paper.

**Proposition 2** Let \( d = 0 \), \( \overline{A}(b) = \frac{1}{3}(5 - 13b) \), \( \hat{A}(b) = \frac{2}{233}(27 + 8b) + \frac{2}{233}\sqrt{\frac{1}{b}} \frac{729 + 972b + 432b^2 + 64b^3}{186} \) if \( b < \frac{3}{4} \) and 2/3 otherwise, and \( \delta_{\text{int}} = \frac{(3 - 4b) + \sqrt{(3 - 4b)^2 + 36(2 - 3A)}}{186} \). The optimal durability choice of the producer is characterized as follows: When \( 0 \leq b < \frac{5}{13} \), then \( \delta^* = 1 \) if \( A < 1 - b \) and 0 otherwise; when \( \frac{5}{13} \leq b < \frac{5}{11} \), then \( \delta^* = 1 \) if \( A < \overline{A}(b), \delta^* = \delta_{\text{int}} \) if \( \overline{A}(b) \leq A < \hat{A}(b) \), and \( \delta^* = 0 \) otherwise; and when \( \frac{5}{11} \leq b \), then \( \delta^* = \delta_{\text{int}} \) if \( A < \hat{A}(b) \), and 0 otherwise.

Proposition 2 characterizes the optimal durability choice of the producer in the absence of the durability-recyclability interaction in design, which is illustrated in Figure 1: The optimal durability choice of the producer simply depends on the marginal cost of designing for durability (\( b \)).

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4 Following Proposition 1, \( A \) is given by \( 1 - m + \lambda(Ra - \frac{2\beta \lambda - A}{2\tau})^2 \), \( 1 - m + \frac{\alpha^2 \lambda}{2\beta \tau - \lambda} \), \( 1 - m + \lambda(\alpha - \frac{2\beta \lambda - A}{2\tau}) \) for the three cases represented in equilibrium, respectively.
and the unit profit margin for a non-durable product (evaluated at the optimal recyclability level). In essence, a large unit profit margin from a non-durable product (i.e., high $A$) implies that the producer would like to sell as many of those products as possible. Hence, the larger the $A$, the lower the optimal durability choice of the producer. Note that a number of factors determine the magnitude of $A$: In particular, a higher recycling target $R$ (weakly) reduces $A$ - that is, more stringent recycling targets will lead the producer to design equally or more durable products. In contrast, a large $\alpha$, or a low $\beta$ and $m$ imply a larger $A$, meaning that products that are inherently easier to recycle and cheaper to produce will be subject to design for product obsolescence (low durability), irrespective of the directional improvement achieved by more stringent recycling targets. To the best of our knowledge, this is the first closed-form characterization of a producer’s durability choice, jointly considering the implications of EPR and markets for durable goods (i.e., the interaction between new product sales and secondary markets), which advances our understanding of the interactions between EPR and durable goods markets. It also lays the foundation for analyzing the design interaction we focus on, which is investigated next.

We now turn to the case of a non-zero interaction term $d \neq 0$. While omitted here for brevity, we first prove a result paralleling Proposition 1 to characterize the optimal recyclability choice in this case (see Lemma 3 in the Appendix for details). Proposition 3 then characterizes the optimal durability choice.

**Proposition 3** The producer’s profit function is of the form $\Pi_j(\delta) = \frac{F_j^2}{4(1+4\delta)} (\delta - b_j\delta^2 + A_j)^2 \forall d$, defined by a set of $F_j$, $b_j$ and $A_j$ (detailed in the Appendix). The optimal durability choice is unique and of the form $\delta^* = \arg \max_j \{ \Pi_j(\delta^*_j) \}$.

Proposition 3 essentially states that the optimal durability characterization in the presence of a non-zero interaction term $d \neq 0$ follows from the characterization in Proposition 2 with $d =$
0. The only difference is that depending on the value $d$, there exist sets of mutually exclusive and collectively exhaustive cases with corresponding expressions of $F_j$, $b_j$ and $A_j$ such that the producer’s profit $\Pi(\delta)$ is fully characterized by a high-order polynomial. Yet the characterization of the optimal $\delta$ within each case $j$ (denoted as $\delta^*_j$) is identical to that in Proposition 2, and the globally optimal unique $\delta^*$ is easily obtained by a profit comparison across those cases. The significance of this result is that it provides us with a closed-form solution that allows us to analyze the implications of EPR on the design of durable products, which we investigate in detail in the next section.

5. The Effect of EPR on the Design of Durable Products

We are now poised to answer the core research questions posed in this paper by investigating how the recycling and collection targets affect the producer’s design choices. We start with the simpler case of a synergistic design interaction.

Proposition 4 When $d \leq 0$, the optimal durability and recyclability choices are (weakly) increasing in the recycling target $R$ and the collection target $\lambda$.

When there is a synergistic design interaction between durability and recyclability choices (e.g., as in desktop computers), an increase in the recycling and collection targets ($R$ and $\lambda$) leads to a simultaneous increase in recyclability and durability: More stringent recycling or collection targets naturally generate a positive impetus for increasing $\rho^*$ and $\delta^*$ because when higher collection and recycling levels are mandated, the former increases the marginal recycling value, whereas the latter helps reduce the sales volume and hence the volume of recycling.

The same intuition however, does not hold when the two attributes are conflicting in design (as in the examples provided in the introduction), i.e., when $d > 0$. Accordingly, in what follows, we focus our attention to $d > 0$. We further focus on the case where the recycling target is binding (i.e., $r^* = R$) in equilibrium, which allows us to focus on legislation-induced design choices. To derive the key insights while keeping the analysis concise, we assume $b = \tau$, i.e., the costs of improving the two attributes are comparable. We first analyze the effect of the recycling target $R$, and then analyze the same for the collection target $\lambda$.

5.1. Recycling Targets

Proposition 5 When $d > 0$, $\exists R_{dl}, R_d$, where $0 \leq R_{dl} \leq R_d \leq 1$ such that the optimal durability $\delta^*$ decreases if $R \in [R_{dl}, R_d]$ and (weakly) increases otherwise.

Proposition 5 provides an important result regarding the directional impact of the recycling target $R$ on the producer’s choice of product durability. In particular, this result reveals that the
durability choice is non-monotonic in the recycling target when \( d > 0 \), and that there exists a range of the recycling target in which an increase in \( R \) implies reduced durability. This result can be explained by analyzing the effect of \( R \) on the recyclability choice and its interaction with the durability level, which is explored in the next proposition and illustrated in Figure 2.

**Proposition 6** When \( d > 0 \), (i) \( \exists R_r, R_{rr} \) where \( 0 \leq R_r \leq R_{rr} \leq 1 \) and \( d \geq 0 \), such that the optimal recyclability is decreasing if \( R \in [R_r, R_{rr}] \) and (weakly) increasing otherwise; (ii) \( R_{rr} = 1 \) when \( d > \overline{d} \); and (iii) \( R_d \leq R_r \), with \( R_d = R_r \) when \( m \leq \overline{m} \).

The first part of Proposition 6 says that the effect of \( R \) on the producer’s recyclability choice is also non-monotonic. That is, there exists a range of \( R \) for which increasing \( R \) may reduce the producer’s recyclability choice. Furthermore, the second part of the proposition states that when the design trade-off is critical (i.e., \( d > \overline{d} \)), \( \rho^* \) decreases over the entire range \([R_{rr}, 1]\). The last part of the proposition states that when the marginal production cost of a (non-recyclable, non-durable) product is sufficiently small, the range where the recyclability of a product goes down perfectly coincides with the range where the product durability increases. In sum, this proposition clarifies the design dependency we explore under EPR as follows:

When there is a conflict between the two design options, design improvements will need to concentrate more heavily on the attribute that more effectively eases the economic burden of recycling obligations, but the effectiveness of each design option heavily depends on the chosen recycling targets. Propositions 5 and 6 explain the resolution to the design trade-off as follows (illustrated in Figure 2 for the parameter range in which the design trade-off is most influential):

At low \( R \) levels (i.e., \( R \) remains below \( R_{dl} \)), an increase in \( R \) implies increased durability only (\( \rho^* = 0 \)). This happens because at low values of \( R \), the marginal value of making the product more recyclable is lower than the same at high values of \( R \) (\( \partial^2 v(R, \rho) / \partial \rho \partial R > 0 \)). Therefore, when \( R \leq R_{dl} \), the producer relies on increasing durability alone. Beyond (\( R > R_{dl} \)), however, the marginal value
of recyclability is more substantial, and the producer starts investing in recyclability. Given the
design interaction, this makes durability expensive to maintain. Thus, the producer reduces product
durability. This trend continues only up to a certain point \((R_r)\), beyond which the cost of making
the product more recyclable implies a narrow profit margin, and the producer instead invests in
increasing durability so as to increase the product price and hence the unit margin. An additional
benefit of increasing durability is reducing the sales volume, and hence the total cost associated
with recycling. Because of the design interaction in cost, the producer starts decreasing the product
recyclability once \(R\) goes beyond this threshold. The key conclusion from this analysis is that very
stringent recycling targets do not necessarily imply the highest levels of recyclability; rather they
will drive the producer towards more durable but less recyclable designs.

In sum, the non-monotonicity of \(\rho^*\) and \(\delta^*\) with respect to the recycling target implies that
legislation that aims to promote easier recycling cannot assume higher recycling targets will drive
producers towards more recyclable designs. Rather, in the presence of a design trade-off between
durability and recyclability, producers may opt for durability improvement at the expense of dimin-
ished recyclability. Inducing durable goods producers to maximize the recyclability of their products
may imply imposing rather conservative recycling targets. This however, may come at the expense
of reducing product durability, i.e., design for obsolescence.

5.2. Collection Targets

We next investigate the implications of a binding collection target on the producer’s design choices.
In doing so, we distinguish between two scenarios: costly and profitable recycling. We first focus
our analysis on \(v(R,0) \leq 0\), i.e., recycling is costly. We then consider the implications of profitable
recycling for \(v(R,0) > 0\), under which the producer needs to compete with third parties to gain
access to end-of-life products with recoverable value. As before, we focus our analysis on cases
where the recycling target is binding.

5.2.1. Costly Recycling

**Proposition 7** When \(d > 0\), \(\exists \lambda_d \geq 0\) such that the optimal durability is decreasing if \(\lambda \geq \lambda_d\), and
(weakly) increasing otherwise.

Proposition 7 states that the producer’s durability choice is non-monotonic in the collection rate
target as well. While the non-monotonicity is similar to the situation with the recycling rate target,
interestingly the collection target’s effect on the design choices of interest is reversed. In particular,
very stringent collection targets can backfire and drive the producer to design products with lower
durability. As before, this effect is driven by the conflict between recyclability and durability, which
can be further clarified with the help of Proposition 8 below.
Proposition 8 When \( d > 0, \exists \lambda_{rl}, \lambda_r \) such that \( 0 \leq \lambda_{rl} \leq \lambda_r \leq 1 \) and the optimal recyclability choice is decreasing in \( \lambda \in [\lambda_{rl}, \lambda_r] \), and (weakly) increasing otherwise. Furthermore, \( \lambda_r \leq \lambda_d \).

Similar to the earlier discussion on the recycling rate targets, the presence of the design trade-off may render simultaneous increase in recyclability and durability too costly. As such, the producer needs to rely more on one lever than the other. Propositions 7 and 8 collectively resolve the design trade-off under the collection rate target as follows: At a low collection target \( \lambda \), the producer utilizes the durability lever first as it has a direct impact on the sales volume and hence the collection volume (i.e., a more durable product sells at a higher price and leads to a lower sales volume), and reduces the product recyclability. When the collection rate target goes beyond a certain threshold (\( \lambda_r \)) though, the marginal benefit of increased durability starts shrinking and the producer starts designing for recyclability as well. When the collection rate further increases beyond a certain threshold (\( \lambda_d \)), the benefit from recyclability dominates the same from durability, as recyclability increases the recycling value for each unit of the higher collected volume. Therefore, the producer chooses more recyclable but less durable product designs.

5.2.2. Profitable Recycling and Competition for Valuable Waste

When the inherent recyclability in a product is sufficiently high (e.g., \( \alpha \) is large enough) or a binding recycling rate target maintains a net profit from recycling in a regulated market (i.e., \( v(R, 0) > 0 \)), we need to consider an important phenomenon: competition for cores (i.e., end-of-life products) by third parties who are attracted to the collection and recycling market (see Esenduran et al. 2015 for a detailed discussion). For example, the removal of toxic materials from batteries led to a flourishing battery recycling industry involving a large number of third-party recyclers (BU 2015). In the remainder of this section, we analyze recyclability and durability choices under such third-party competition.

Since EPR requires only the producer to collect \( \lambda \) portion of end-of-life products for recycling (Esenduran et al. 2015), we first need to analyze the implications of competition with independent third parties, based on which we can study the implications of \( \lambda \) on the design decisions of durability and recyclability when waste has value. To model such competition, we consider the existence of \( n \) independent for-profit third parties in the collection and recycling market along with the producer. The third parties and the producer pay buyback prices to consumers for returning end-of-life products to their recycling facilities. We assume that the producer, facing the product end-of-life obligation, has built enough capacity to accept and recycle any returns that arrive. In contrast, the capacities of the individual third parties are limited. Yet without loss of generality, we assume that in equilibrium there is sufficient aggregate third-party capacity to cover the whole collection and recycling volume. (We also study more restrictive values of total third-party capacity in the
Appendix, and we show that all the major insights remain robust.) We also assume that the unit recycling value extractable by the third parties from each returned unit, denoted by \( w \), will be lower than that for the producer. This could be due to such factors as not having the best adapted technology or not being familiar with the design specifications.

To capture the effect of the collection target on the competition and the buyback prices, we construct a model similar to the one in Arnold (2001). To adapt the model to our setting, we consider that when consumers are faced with this capacitated market, they perform a random search among the producer and all the third parties to decide where to return the core. In every search attempt, the consumer incurs a cost \( s \) (which can be regarded as a search or delivery cost). When the consumer encounters a third party that is currently out of capacity, another search (along with the associated cost) is needed, until the core is accepted for a buyback payment. In this case, with third parties who set their buyback prices for profit maximization, we derive the buyback price for the producer to secure the collection rate of \( \lambda \) to be \( p_\lambda = w - s((2 - \lambda)^2 - 1) \) (see details in the Appendix).

With a binding collection target \( \lambda \) for the producer and the corresponding buyback price \( p_\lambda \), we first calculate the consumer demand of new products to be \( \frac{(1 + \delta) - p + (p_\lambda - s)}{1 + 3\delta} \). Comparing this to the demand in §3.1, the change in demand is due to consumers’ anticipation that each product will generate an expected net extra value \( p_\lambda - s \) at the end-of-life when returned for recycling. Then the producer maximizes the following profit by determining the recyclability and durability in the design stage, and then setting the market price and the actual recycling level:

\[
\Pi(p, r; \rho, \delta) = -(1 + \delta) - p + (p_\lambda - s) \left(p - (m + \tau \rho^2 + d \rho \delta + b \delta^2) + \lambda(-\frac{1}{2} \beta r^2 + (\alpha + \rho) r - p_\lambda)\right).
\]

We analyze this problem following a similar procedure as discussed in previous sections. The next proposition sheds light on the design incentives created by the collection target.

**Proposition 9** When \( v(R, 0) > 0 \) and \( d > 0 \), \( \rho^* \) is (weakly) increasing and \( \delta^* \) is (weakly) decreasing in \( \lambda \).

Proposition 9 suggests that improving recyclability in a competitive market in fact may have an additional benefit because of the following: As before, when making the decisions on recyclability and durability to deal with the EPR obligation, the producer is essentially comparing the marginal benefits of the two attributes, while controlling the associated cost. First, to meet a higher collection target, the producer is also retaining a larger percentage of the marginal return from improving recyclability, hence it is profitable to invest more on recyclability. Meanwhile, a higher \( \rho^* \) creates a pressure to compromise durability to achieve cost-effectiveness in the presence of the design
trade-off. Second, expanding demand has an additional benefit now because every new product can be sold at a higher price to reflect the buyback value of the product at end-of-life, but part of this benefit is “free” to the producer since a portion of the buyback payments to consumers are made by third parties. Consequently, the producer redirects more investment from durability to recyclability.

In sum, the analysis in this section suggests that the direct impact of very stringent collection targets may be a combination of design for recycling and product obsolescence (i.e., reduced durability), and more so if the inherent recycling value in the product of interest is already high.

5.3. Welfare and Environmental Implications

The analysis above suggests that due to the inherent trade-off between product durability and recyclability, the design implications of EPR for durable products may not be straightforward. In particular, our results show that stringent recycling targets may drive the producer to design more durable yet less recyclable products, and that stringent collection targets may drive the producer to design for recyclability at the expense of reduced durability.

In turn, these contrasting effects of recycling and collection targets on the two design for environment options suggest that EPR may not necessarily increase welfare in a durable goods setting. To shed light onto this issue, consider an additive welfare measure that consists of three traditional economic measures: Producer Profits, Consumer Surplus and Environmental Externalities (see Atasu et al. 2009, Jacobs and Subramanian 2011, Atasu and Souza 2012, Krass et al. 2013, for similar approaches). In our context, it is straightforward to show that both producer profits and consumer surplus measures decrease in the stringency of the recycling and collection targets. Hence, imposing stricter $R$ and $\lambda$ will only improve welfare if the environmental externalities are reduced by such increases. In the existing literature (e.g., Atasu et al. 2009), which often overlooks product durability, this is often the case. When product durability is considered, on the other hand, a reduction in environmental externalities through increased targets is not guaranteed. For instance, Propositions 6 and 8 suggest that even though the fraction of products that end up in landfills may go down with higher $R$ and $\lambda$, more hazardous substances may find their ways to landfill streams due to reduced product recyclability. Similarly, Propositions 5, 7 and 9 show that an emphasis on design for obsolescence may imply increased production and higher landfilling. To shed further light onto this observation, consider the following result:

**Proposition 10** \(\exists \alpha_R, m_R, \overline{m}_R\) such that \(q(p^*(\rho^*, \delta^*), \delta^*)\) is increasing in \(R\) if \(\alpha_R < \alpha\) and \(m_R < m < \overline{m}_R\), and decreasing otherwise. In addition, \(\exists \alpha_\lambda, m_\lambda, \overline{m}_\lambda\), such that \(q(p^*(\rho^*, \delta^*), \delta^*)\) is increasing in \(\lambda\) if \(\alpha_\lambda < \alpha\) and \(m_\lambda < m < \overline{m}_\lambda\).
Proposition 10 states that increasing collection or recycling targets can increase the overall production and the associated waste generation. In particular, products that have moderate production costs can face this dilemma. For such products, if the inherent recycling value is high, increased recycling or collection targets can lead to increased production because of an emphasis on increasing recyclability at the expense of durability. Figure 3 shows such an example with respect to $R$. In sum, the take-away from this figure is that the environmental impact and welfare implications of EPR are not clear cut in the durable goods context.

In closing this section, we also note that our discussion so far has focused on instances of legislation where a producer would directly reap the benefits of what it has sown with respect to recyclability improvements in its products. However, it is also important to note that there are other forms of EPR implementations under which a producer’s capability of fully realizing its return on recyclability can be limited. In particular, collective EPR implementations that use weight- or volume-based cost allocations that do not take into account product recyclability, or advance recycling or disposal fees based EPR implementations (see Atasu and Van Wassenhove 2011 and Plambeck and Wang 2009 for examples) can significantly hinder a producer’s ability to capture its recyclability investments. The implications of our analysis for such circumstances are as follows: A limitation in the return on recyclability investments can be modeled as a lower return on investments in recyclability in the form of a multiplier $\chi < 1$ in front of $\rho$ in the unit recycling value function $v(R, \rho)$. This will imply a lower $\rho^*$ in equilibrium, and that an increase in the stringency of collection and recycling targets will drive the producer more towards adjusting product durability.
6. The Case of Photovoltaic Panels under the WEEE Directive

Our work provides a framework for analyzing the producer response to EPR legislation under the economics of a particular industry and product. To illustrate the insights that can be generated by such an analysis, we choose the PVP industry. PVPs are particularly relevant for our framework for four main reasons: First, PVPs are the most recently added product category under the WEEE Directive Recast that imposes ambitious collection and recycling targets: PVP producers have to meet a 65% collection target by 2019, and an 80% recycling target by 2018. Second, there are two dominant technologies, c-Si and thin-film, which differ in their recyclability and durability. In particular, the c-Si technology produces more durable and less recyclable PVPs than the thin-film technology (see details below). Third, PVPs already have active secondary markets; hundreds of used PVP transactions (with thousands of panels sold) can be found in online consumer-to-consumer trade channels on a daily basis (Ebay 2015) and there already exist third parties selling used PVPs (SecondSol 2015). Finally, PVP producers face tight profit margins, well below 10% before interest and taxes (BNEF 2013, Ng 2014). As such, the product end-of-life obligation imposed by EPR will significantly affect PVP producers’ profit margins. In turn, although there may be many factors influencing a PVP producer’s technology choice, the implications of EPR cannot be ignored. In particular, PVP producers will need to carefully weigh future recycling cost liabilities associated with potentially higher end-of-life volumes (through less durable but more recyclable panels produced under the thin-film technology) and potentially higher unit recycling costs (through less recyclable but more durable panels produced under the c-Si technology) in making this choice.

To shed some light on this trade-off, we conduct a calibrated numerical study using data from industry reports and practitioner interviews (Coker 2015, Haroon 2015). We set the current status where legislation is absent as the baseline case, and compare it to the scenarios where legislation with binding collection and recycling targets is enforced. As discussed earlier, the durability-recyclability trade-off exists in multiple design dimensions for PVPs. An important one is to choose the technology for the PV cells. Currently, the major commercialized technology categories are c-Si and thin-film. The c-Si technology was the first in the market, and it still dominates with more than 90% market share in 2013 (Fraunhofer 2014). On the other hand, the thin-film technology requires less material in manufacturing and exhibits higher flexibility in application, and hence thin-film investments are expected to grow (GBI 2011, TSS 2015). The thin-film technology consists of three primary technologies, including Copper Indium Gallium Selenide (CIGS), Cadmium Telluride (CdTe) and Amorphous Silicon (a-Si). We focus on CIGS, as it is a leading technology in the thin-film category in terms of both efficiency and market share. We use the subscripts $j = 1, 2$ for the c-Si and CIGS technologies, respectively.
We conduct the calibration study in four steps. First, we look into the economics of recycling. We consider an estimated current collection rate $\lambda_0 = 20\%$ in the baseline case (BIO 2011). Under a binding collection target, the collection rate will be $\lambda (> 20\%)$ as required by legislation. Recycling will both incur processing costs (i.e., costs of treatment for recycling) and generate recoverable material value. The processing costs, denoted as $c_j$, can vary significantly between different recycling procedures and facilities. These costs are also expected to change in the coming years due to the fast evolving dynamics of the PVP recycling industry. Therefore, we use a processing cost of $c = \$200/\text{ton}$ as a benchmark (Choi and Fthenakis 2014, Fthenakis and Moskowitz 1999, BIO 2011) and then show how the result changes with different processing cost values. We derive recoverable material values from the PVP composition and raw material price data (see Appendix for details of the data and the calculation procedure). We convert all the unit values and costs into dollars per kilowatt ($/\text{kW}$), which is the measure commonly used in the industry. Considering both the recoverable material value and the processing costs, we calculate the recycling value parameters $\alpha_j + \rho_j$ and $\beta_j$. Based on these values for each technology, in the baseline case without legislation (where an additional subscript 0 is used), we calculate the optimal recycling levels $r^*_j$ and the unit recycling values $v_{j0} = -\frac{1}{2} \beta_j (r^*_j)^2 + (\alpha_j + \rho_j) r^*_j$ (see the Appendix for details). Notably, $v_{10} = 8.3$ and $v_{20} = 10$, suggesting that PVPs with the CIGS technology have a higher unit recycling value than those with the c-Si technology, because of their higher rare metal content that can be recovered through recycling (e.g., indium and gallium). When a binding recycling target is in effect, the unit recycling values become $v_j = -\frac{1}{2} \beta_j R^2 + (\alpha_j + \rho_j) R$.

Second, we estimate the durabilities of PVPs with the c-Si and the CIGS technologies. As discussed earlier, the degradation rate reflects how the efficiency of converting sunlight into electric power – the most important functionality of PVPs to generate consumer valuation – depreciates with time. In particular, when used panels are traded in consumer secondary markets (e.g., on eBay or other established marketplaces online), the remaining efficiency after degradation largely determines the resale value, which is consistent with our model. Therefore, the degradation rate is a good proxy for product durability in this context. Studies show that the degradation rates for c-Si and CIGS technologies are approximately $0.5%/\text{year}$ and $0.96%/\text{year}$, respectively (Jordan and Kurtz 2012, Coker 2015, Haroon 2015). Moreover, PVPs commonly have an expected lifespan of 25 years, which makes a 12.5-year period consistent with our model assumption of a two-period product useful life. Therefore, we estimate the durabilities as $\delta_1 = (1 - 0.5\%)^{12.5} = 0.94$ and $\delta_2 = (1 - 0.96\%)^{12.5} = 0.89$; i.e., the c-Si technology is more durable than the CIGS technology. This comparison once again highlights the durability-recyclability trade-off in PVPs: While the CIGS technology is more recyclable, c-Si is more durable.
Third, we study the market for PVPs. Recall that in our model, we assume consumer types $\theta$ that reflects the consumers’ product valuations) to be uniformly distributed over $[0, 1]$. In reality, however, the distribution of consumer types can have a more general support on $[0, X]$ with $X > 0$. We infer the values of $X_j$ by using the current PVP market prices $p_j$ and production costs $g_j$ obtained from market studies and reports, based on a generalization of our equilibrium model with $p_j = \frac{1}{2}(X_j(1+\delta_j) - g_j + \lambda_0v_j)$. This analysis allows us to calibrate our study with $X_1 = 1.26 \times 10^3$ and $X_2 = 1.43 \times 10^3$ (see Appendix for details). We extend our model to account for general $X_j$ values. In this case, the producer profits in equilibrium can be calculated as $\Pi_j = \frac{(X_j(1+\delta_j) - g_j + \lambda_0v_j)^2}{4X_j(1+3\delta_j)}$ in the baseline case and $\Pi_j = \frac{(X_j(1+\delta_j) - g_j + \lambda v_j)^2}{4X_j(1+3\delta_j)}$ when legislation with a binding recycling target $R$ and collection target $\lambda$ is enforced.

Finally, we analyze the influence of EPR on the producers’ product technology choices through its impact on profitability: As discussed earlier, maintaining positive profit margins is a challenge for PVP producers, where margins are tight (BNEF 2013, Ng 2014). As such, the influence of EPR on a PVP producer’s profit can be expected to be substantive. To this end, we calculate the relative change in profit due to EPR for a given product technology choice as $\Delta \Pi_j = \frac{\Pi_j - \Pi_{j,0}}{\Pi_{j,0}}$, where the technology $j$ with the larger $\Delta \Pi_j$ is more vulnerable to legislation because the end-of-life obligation causes a more significant erosion to its profit margin. While various factors may influence a producer’s technology choice, $\Delta \Pi_j$ is useful for illustrating the directional impacts of EPR on the possible technology choice of a PVP producer. Focusing on this measure, Figure 4 suggests that the c-Si technology (that is more durable but less recyclable) will face a lower profit margin erosion from a high recycling target under EPR, and the more recyclable but less durable CIGS technology’s profit margin erosion will be lower only if the collection target imposed by EPR is very high.

The figure also suggests that the effect of EPR on these technology choices will depend on market dynamics with respect to processing cost efficiency improvements for recycling (represented by the shifting dashed lines in the figure): The more efficient the PVP recycling process, the lower the EPR-driven profit margin erosion with the CIGS technology. This is an important observation for our purposes because only a low volume of the panels have reached end-of-life to date due to their long lifespans, implying that the PVP recycling market is not mature enough to operate at a sufficiently high volume that achieves economies of scale (Besiou and Wassenhove 2015). At the current processing cost of $200/ton, our results suggest that with a recycling target above 70%, which is within the range proposed by the WEEE Directive Recast, the profit margin erosion of the more durable c-Si technology is lower than the CIGS technology (as shown by the solid line in the figure). As such, the proposed recycling targets encourage producers to choose a technology with higher durability and lower recyclability. As volumes increase and the processing cost goes down
Figure 4  Predicted PVP technology choice under different collection and recycling targets and processing cost efficiency levels

(e.g., below $190/ton), however, the region within which the profit margin erosion of the CIGS technology is lower expands (as shown by the dashed lines). In other words, provided that a certain processing cost efficiency is achieved through scale economies over time, the CIGS technology can be preferred for a wider range of legislative targets. Only then will EPR have achieved its objective of making PVPs more recyclable, especially if supported by high collection targets.

7. Conclusions

In this paper, we analyze a durable good producer’s design choices regarding product durability and recyclability under EPR. In particular, we posit that design incentives induced by EPR are not restricted to design for recyclability. Rather, EPR can also provide incentives to alter product durability. This could be a direct incentive: Designing durable products can reduce production, and in turn the volume of products a producer is responsible for recycling under EPR. This could also be an indirect incentive: If the producer chooses to increase product recyclability, the design trade-off may result in reduced durability.

Our analysis regarding the effect of recycling targets, a key component of EPR implementations such as the WEEE Directive, on producers’ EPR-driven design choices, suggests the following: When recyclability and durability are synergistic design attributes, i.e., they are reinforcing, stringent recycling targets work as intended: They lead a producer to design easier to recycle and more durable products. Essentially, easier to recycle product designs help increase unit recycling value at the end-of-life, and durable product designs (directly) reduce the total volume a producer has to recycle. In contrast, when recyclability and durability are conflicting design attributes, i.e.,
improving either dimension makes it harder to improve the other as in the example of PVPs, the effect of recycling targets can be ambiguous. Our analysis regarding this design trade-off suggests that low but sufficiently stringent recycling targets will drive product design for recyclability, yet this will be at the expense of product durability. However, the opposite may occur under high recycling targets, and a producer will improve durability at the expense of recyclability. This suggests that contrary to intuition, recycling target stringency - a lever that appears to deal purely with recycling processes, does not necessarily imply producer incentives to design for recyclability. More importantly, very strict recycling targets may even compromise recyclable product designs, and more so if the base production costs are already low.

Another interesting finding from our analysis is that the effect of increased collection targets on EPR-driven product design choices differs substantially from that of recycling targets. To be more specific, when a durability-recyclability trade-off exists in product design, a low but sufficiently stringent collection target induces an increase in durability while compromising recyclability. In contrast, a high collection target induces an increase in recyclability while compromising durability. In particular, when recycling is valuable (be it driven by commodity price dynamics for recycled materials or the inherent product characteristics), higher collection targets always imply more recyclable yet less durable product designs.

From the policy maker perspective, our results suggest that the environmental implications of EPR may be far from intuitive in the case of durable products: Although recycling and collection targets appear to focus on environmental concerns associated with the end-of-life phase of the product life-cycle, they may also influence the production and use phases in the product life-cycle through their effect on design for durability. In view of this, we show that seemingly similar legislative targets may work in opposing directions in driving producers’ design choices (see Plambeck and Wang 2009 for a similar discussion in the context of new product introduction frequency). More importantly, more stringent legislative targets do not always guarantee improved environmental performance (see Esenduran et al. 2015 and Krass et al. 2013 for similar discussion). In particular, our results suggest that if the policy maker’s objective is to induce recyclability (i.e., increase landfill diversion or achieve higher quality of recycling), a low recycling target accompanied with a high collection target may be ideal. Likewise, if the policy maker’s objective is to reduce consumption through durability via EPR, a high recycling target accompanied by a low collection target may be ideal.

We conclude that the economic and environmental impact of increasing legislative stringency is more subtle than appears at first glance. As illustrated by our calibrated data analysis for PVPs, our model provides a framework for analyzing the producer response to different legislative targets under the economics of the particular industry and product. Such an analysis can support the setting of legislative targets that are adapted to different product categories.
References


Coker, A. 2015. Personal communications, Hannah Solar LLC, Atlanta, GA.


Haroon, Sol. 2015. Personal communications, Pursuit Engineering, Atlanta, GA.


Appendix

Proof of Lemma 1

Under the recycling target $R$ and the collection target $\lambda$ imposed by legislation, the producer solves the following maximization problem:

$$\max_{p, r} = \Pi(p, r; \rho, \delta) = (1 - \theta_1) \left( p - g(\rho, \delta) + \lambda \left( -\frac{1}{2} \beta r^2 + (\alpha + \rho)r \right) \right),$$

subject to: $R \leq r \leq 1$, $1 - \theta_1 \geq 0$, $\theta_1 - \theta_2 \geq 0$, $\theta_2 \geq 0$, $p_u \geq 0$.

The market clearing condition $1 - \theta_1 = \theta_1 - \theta_2$ means $\theta_1 - \theta_2 \geq 0$ is redundant as we keep $1 - \theta_1 \geq 0$. Furthermore, consumer $\theta_2$ (by definition) is indifferent between choosing Bn or Bu, i.e., $\delta \theta_2 - p_u = 0$, yielding $p_u \geq 0$ is redundant as we retain $\theta_2 \geq 0$. Therefore, the full set of constraints to analyze the problem is $1 - r \geq 0$, $r - R \geq 0$, $1 - \theta_1 \geq 0$ and $\theta_2 \geq 0$. Substituting the expressions of $\theta_1$ and $\theta_2$ from the main text, the last two constraints can be written as $\frac{1 + \delta - \rho}{1 + 3 \delta} \geq 0$ and $\frac{2 \rho - 1 + \delta}{1 + 3 \delta} \geq 0$, which are equivalent to $1 + \delta - p \geq 0$ and $2p - 1 + \delta \geq 0$ since $1 + 3 \delta \geq 0$. We associate Lagrange multipliers with all the constraints to form the Lagrangian for maximization.

$$L = -\frac{1}{1 + 3 \delta} (p - (1 + \delta)) \left( p - g(\rho, \delta) + \lambda \left( -\frac{1}{2} \beta r^2 + (\alpha + \rho)r \right) \right) + \lambda_1 (1 - r) + \lambda_2 (r) + \lambda_3 (1 + \delta - p) + \lambda_4 (2p + \delta - 1).$$

The Karush-Kuhn-Tucker (KKT) conditions yield the following:

$$0 = \frac{\partial L}{\partial p} = \left( -\frac{1}{1 + 3 \delta} \right) \left( 2p - (1 + \delta + g(\delta, \rho) - \lambda((\alpha + \rho)r) - \frac{1}{2} \beta r^2) \right) - \lambda_3 + 2 \lambda_4$$

$$0 = \frac{\partial L}{\partial r} = \left( -\frac{1}{1 + 3 \delta} \right) ((\alpha + \rho) - \beta r) - \lambda_1 + \lambda_2.$$

There are 9 possible cases (excluding inconsistent cases such as the one with both $1 - r = 0$ and $r - R = 0$).

<table>
<thead>
<tr>
<th>Case</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - r$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td></td>
</tr>
<tr>
<td>$r - R$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 + \delta - p$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2p - (1 - \delta)$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We analyze each of them below, and we suppress the arguments of $g(\rho, \delta)$ for brevity.

(1) In this case, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0$. Substituting them into the first-order conditions, we derive one candidate for the optimal solution to be $r = \frac{2 + \rho}{\beta}$ and $p = \frac{1}{2} (1 + \delta + g - \lambda \frac{(\alpha + \rho)^2}{2 \beta})$. Moreover, the resulting Hessian is negative definite, confirming this pair of values to be a local maximizer.

To ensure the validity of this candidate solution, we need $R \leq r = \frac{2 + \rho}{\beta} \leq 1$ and $\frac{1}{2} (1 - \delta) \leq p = \frac{1}{2} (1 + \delta + g - \lambda \frac{(\alpha + \rho)^2}{2 \beta}) \leq 1 + \delta$. Simplifications reduce them to $0 \leq \alpha + \rho \leq \beta$ and $-(1 + \delta - g) \leq \lambda \frac{(\alpha + \rho)^2}{2 \beta} \leq (g + 2 \delta)$. Note that the latter inequality always holds because $\lambda \frac{(\alpha + \rho)^2}{2 \beta}$ is the effective unit recycling value at $r = \frac{2 + \rho}{\beta}$, which has to be lower than the unit production cost, otherwise it leads to the unreasonable case where the producer can generate a steady profit stream by simply producing
and then recycling the products. Therefore, $\lambda \frac{(a+\rho)^2}{2g} \leq g \leq g + 2\delta$. Moreover, $1 + \delta - g + \lambda \frac{(a+\rho)^2}{2g}$ is the unit profit for the producer (accounting for both the production cost and the recycling value), which has to be non-negative to keep the producer in the market, and hence $-(1 + \delta - g) \leq \lambda \frac{(a+\rho)^2}{2g}$.

(2) In this case, $r = 1$ and $p = 1 + \delta$, which lead to zero production and zero profit. This uninteresting case is discarded.

(3) In this case, $r = 1$ and $p = \frac{1-\delta}{2}$, with $\lambda_1 = \frac{(a+\rho)-\beta}{1+3\delta}$ and $\lambda_4 = \frac{-2\delta - g + \lambda((a+\rho) - \frac{1}{2}\beta)}{2(1+3\delta)}$. However, $\lambda_1 < 0$ because $\lambda(-\frac{1}{2}\beta + (a+\rho)) < g < g + 2\delta$, with the LHS being the effective unit recycling value at $r = 1$ that is lower than the unit production cost. Therefore, this candidate solution is invalid and hence discarded.

(4) In this case, $r = R$ and $p = \frac{1-\delta}{4}$, with $\lambda_2 = \frac{(a+\rho)-\beta R}{1+3\delta}$ and $\lambda_4 = \frac{-2\delta - g + \lambda((a+\rho)R - \frac{1}{2}\beta R^2)}{2(1+3\delta)}$. However, similar to Case (3), $\lambda_1 < 0$ and hence this candidate solution is discarded.

(5) In this case, $r = R$ and $p = 1 + \delta$, which lead to zero production and zero profit. This uninteresting case is discarded.

(6) In this case, $r = 1$ and $p = \frac{1}{2}(1 + \delta + g - \lambda((a+\rho) - \frac{1}{2}\beta))$. To ensure the validity of this candidate solution, we need $\lambda_1 = \frac{\alpha + \rho - \beta}{1+3\delta} \geq 0$ and $\frac{1}{2}(1 - \delta) \leq p = \frac{1}{2}(1 + \delta + g - \lambda((a+\rho) - \frac{1}{2}\beta)) \leq 1 + \delta$, with the latter inequality being satisfied following an argument similar to that in Case (1).

(7) In this case, $r = R$ and $p = \frac{1}{2}(1 + \delta + g - \lambda((a+\rho)R - \frac{1}{2}\beta R^2))$. To ensure the validity of this candidate solution, we need $\lambda_2 = \frac{\beta R - (a+\rho)}{1+3\delta} \geq 0$ and $\frac{1}{2}(1 - \delta) \leq p = \frac{1}{2}(1 + \delta + g - \lambda((a+\rho)R - \frac{1}{2}\beta R^2)) \leq 1 + \delta$, with the latter inequality begin satisfied following an argument similar to that in Case (1).

(8) In this case, $p = 1 + \delta$, which lead to zero production and zero profit and hence this uninteresting case is discarded.

(9) In this case, $r = \frac{\alpha + \rho}{\beta}$ and $p = \frac{1-\delta}{2}$. However, similar to Case (3), $\lambda_4 = \frac{-2\delta - g + \lambda((a+\rho)^2}{2(1+3\delta)} < 0$ and hence this candidate solution is discarded.

Finally, the optimal new product price and recycling level are summarized below.

<table>
<thead>
<tr>
<th>Case</th>
<th>Condition</th>
<th>Optimal Price</th>
<th>Optimal Recycling</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$\frac{\alpha + \rho}{\beta} \leq R$</td>
<td>$\frac{1}{2}(1 + \delta + g - \lambda((a + \rho) - \frac{1}{2}\beta))$</td>
<td>$R$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$R \leq \frac{\alpha + \rho}{\beta} \leq 1$</td>
<td>$\frac{1}{2}(1 + \delta + g - \lambda((a + \rho)R - \frac{1}{2}\beta R^2))$</td>
<td>$\frac{\alpha + \rho}{\beta}$</td>
</tr>
<tr>
<td>(iii)</td>
<td>$1 \leq \frac{\alpha + \rho}{\beta}$</td>
<td>$\frac{1}{2}(1 + \delta + g - \lambda((a + \rho) - \frac{1}{2}\beta))$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

Equivalently, the optimal price is $p^*(\rho, \delta) = \frac{1}{2}(1 + \delta + g(\rho, \delta) - \lambda(-\frac{1}{2}\beta (r^*(\rho, \delta))^2 + (a + \rho)r^*(\rho, \delta)))$.

**Proof of Proposition 1**

By backward induction, we study $\rho^*$ and $\delta^*$ in three cases, corresponding to the three cases of $p^*$ and $r^*$ in Lemma 1. In solving this problem, we start with a general value of $d$, and then focus on the case of $d = 0$. 
Case(i) When \( \frac{\alpha + \rho}{\beta} \leq R \). The producer is faced with the following optimization problem:

\[
\max_{\rho, \delta} \Pi(p^\ast, r^\ast; \rho, \delta) = \frac{1}{4(1 + 3\delta)} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda((\alpha + \rho)R - \frac{1}{2}\beta R^2) \right)^2,
\]

subject to: \( 0 \leq \delta \leq 1, \quad \rho \geq 0, \quad \alpha + \rho \leq R\beta \).

We form the Lagrangian of the problem:

\[
L_1 = \frac{1}{4(1 + 3\delta)} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda((\alpha + \rho)R - \frac{1}{2}\beta R^2) \right)^2 + \mu_1(1 - \delta) + \mu_2(\delta) + \mu_3(R\beta - \alpha - \rho) + \mu_4(\rho).
\]

Applying the first-order conditions, we have

\[
0 = \frac{\partial L_1}{\partial \rho} = \frac{1}{2(1 + 3\delta)} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda((\alpha + \rho)R - \frac{1}{2}\beta R^2) \right) (-2\tau\rho - d\delta + R\lambda) - \mu_3 + \mu_4,
\]

\[
0 = \frac{\partial L_1}{\partial \delta} = \frac{1}{4(1 + 3\delta)^2} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda((\alpha + \rho)R - \frac{1}{2}\beta R^2) \right) \times \left( -3(1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda((\alpha + \rho)R - \frac{1}{2}\beta R^2)) + 2(1 + 3\delta)(1 - d\rho - 2b\delta) \right) - \mu_1 + \mu_2.
\]

Note that \( (1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda((\alpha + \rho)R - \frac{1}{2}\beta R^2)) \) is the unit profit and hence should be non-negative. There are 3 candidate solutions for \( \rho^\ast \).

1. \( \rho = \beta R - \alpha \). To ensure \( \rho = \beta R - \alpha \geq 0 \) and \( \mu_3 \geq 0 \) for the validity of the candidate solution, the associated constraint is \( \beta R - \frac{R\lambda - d\delta}{2\tau} \leq \alpha \leq \beta R \).
2. \( \rho = 0 \). To ensure \( \mu_4 \geq 0 \), the associated constraint is \( R\lambda \leq d\delta \).
3. \( \rho = \frac{R\lambda - d\delta}{2\tau} \). The constraints to ensure \( \mu_1, \mu_2 \geq 0 \) are \( d\delta \leq R\lambda \) and \( \alpha \leq R\beta - \frac{R\lambda - d\delta}{2\tau} \).

Case(ii) When \( \frac{\alpha + \rho}{\beta} > 1 \). The producer is faced with the following optimization problem:

\[
\max_{\rho, \delta} \Pi(p^\ast, r^\ast; \rho, \delta) = \frac{1}{4(1 + 3\delta)} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda \frac{(\alpha + \rho)^2}{2\beta} \right)^2,
\]

subject to: \( 0 \leq \delta \leq 1, \quad \rho \geq 0, \quad \beta R \leq \alpha \leq \beta \).

We form the Lagrangian of the problem:

\[
L_2 = \frac{1}{4(1 + 3\delta)} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda \frac{(\alpha + \rho)^2}{2\beta} \right)^2 + \mu_1(1 - \delta) + \mu_2(\delta) + \mu_3(\beta - \alpha - \rho) + \mu_4(\rho) + \mu_5(\alpha + \rho - R\beta).
\]

Applying the first-order conditions, we have

\[
0 = \frac{\partial L_2}{\partial \rho} = \frac{1}{2(1 + 3\delta)} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda \frac{(\alpha + \rho)^2}{2\beta} \right) (-2\tau\rho - d\delta + \lambda \frac{\alpha + \rho}{\beta}) - \mu_3 + \mu_4 + \mu_5,
\]

\[
0 = \frac{\partial L_2}{\partial \delta} = \frac{1}{4(1 + 3\delta)^2} \left( 1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda \frac{(\alpha + \rho)^2}{2\beta} \right) \times \left( -3(1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda \frac{(\alpha + \rho)^2}{2\beta}) + 2(1 + 3\delta)(1 - d\rho - 2b\delta) \right) - \mu_1 + \mu_2.
\]

Note that \( (1 + \delta - m - \tau \rho^2 - d\rho\delta - b\delta^2 + \lambda \frac{(\alpha + \rho)^2}{2\beta}) \) is the unit profit and hence should be non-negative. There are 4 candidate solutions for \( \rho^\ast \).

1. \( \rho = 0 \). The associated constraints to ensure the validity of the candidate solution are \( R\beta - \rho \leq 0 \leq \beta - \alpha \) and \( \mu_1 \geq 0 \), which reduce to \( \beta R \leq \alpha \leq \beta - \frac{\lambda}{\beta} \leq d\delta \).
(2) $\rho = \beta R - \alpha$. The associated constraints are $\rho = \beta R - \alpha \geq 0$ and $\mu_3 \geq 0$, which reduce to $\alpha \leq \beta R$ and $2\alpha - R(2\beta - \lambda) \leq d\delta$.

(3) $\rho = \beta - \alpha$. The associated constraints are $\rho = \beta - \alpha \geq 0$ and $\mu_2 \geq 0$, which reduce to $\alpha \leq \beta$ and $d\delta \leq 2\alpha - (2\beta - \lambda)$.

(4) $\rho = \frac{\alpha \lambda - \beta d\delta}{2\beta - \tau}$. The associated constraints are $\max[0, R\beta - \alpha] \leq \rho \leq \beta - \alpha$, which reduce to

- $\alpha \leq \beta R$ and $2\alpha - (2\beta - \lambda) \leq d\delta \leq 2\alpha - R(2\beta - \lambda)$; or
- $\beta R \leq \alpha \leq \beta$ and $2\alpha - (2\beta - \lambda) \leq d\delta \leq \frac{\alpha \lambda}{\beta}.

Case(iii) When $1 \leq \frac{2\tau}{\beta}$. The producer is faced with the following optimization problem:

$$\max_{\rho, \delta} \Pi(\delta, \rho, \rho^*, \tau^*) = \frac{1}{4(1 + 3\delta)} \left(1 + \delta - m - \tau \rho^2 - d\rho \delta - b\delta^2 + \lambda((\alpha + \rho) - \frac{1}{2}\beta)\right)^2,$$

subject to $0 \leq \delta \leq 1$, $\rho \geq 0$, $\alpha + \rho \geq \beta$.

We form the Lagrangian of the problem:

$$L_3 = \frac{1}{4(1 + 3\delta)} \left(1 + \delta - m - \tau \rho^2 - d\rho \delta - b\delta^2 + \lambda((\alpha + \rho) - \frac{1}{2}\beta)\right)^2 + \mu_1(1 - \delta) + \mu_2(\delta) + \mu_3(\alpha + \rho - \beta) + \mu_4(\rho).$$

Applying the first-order conditions, we have

$$0 = \frac{\partial L_3}{\partial \rho} = \frac{1}{2(1 + 3\delta)} \left(1 + \delta - m - \tau \rho^2 - d\rho \delta - b\delta^2 + \lambda((\alpha + \rho) - \frac{1}{2}\beta)\right)(-2\tau \rho - d\delta + \lambda) + \mu_3 + \mu_4,$n

$$0 = \frac{\partial L_3}{\partial \delta} = \frac{1}{4(1 + 3\delta)^2} \left(1 + \delta - m - \tau \rho^2 - d\rho \delta - b\delta^2 + \lambda((\alpha + \rho) - \frac{1}{2}\beta)\right)$$

$$\left(-3(1 + \delta - m - \tau \rho^2 - d\rho \delta - b\delta^2 + \lambda((\alpha + \rho) - \frac{1}{2}\beta)) + 2(1 + 3\delta)(1 - d\rho - 2b\delta)\right) - \mu_1 + \mu_2.$$

Note that $(1 + \delta - m - \tau \rho^2 - d\rho \delta - b\delta^2 + \lambda((\alpha + \rho) - \frac{1}{2}\beta))$ is the unit profit and hence should be non-negative. There are 3 candidate solutions for $\rho^*$.

1. $\rho = 0$. The associated constraints to ensure the validity of the candidate solution are $\beta \leq \alpha$ and $d\delta \geq \lambda$.

2. $\rho = \beta - \alpha$. The associated constraints are $\alpha \leq \min[\beta, \beta - \frac{\lambda - d\delta}{2\tau}]$.

3. $\rho = \frac{\lambda - d\delta}{2\tau}$. The associated constraints are

- $\alpha \leq \beta$ and $\beta - \frac{\lambda - d\delta}{2\tau} \leq \alpha$; or
- $\beta \leq \alpha$ and $d\delta \leq \lambda$.

All the candidate solutions can be summarized below.

(i) \( \frac{\alpha}{\beta} \leq R \).

<table>
<thead>
<tr>
<th>$r^*$</th>
<th>$d\delta \leq 2\alpha - (2\beta - \lambda)$</th>
<th>$2\alpha - (2\beta - \lambda) \leq d\delta \leq 2\alpha - R(2\beta - \lambda)$</th>
<th>$2\alpha - R(2\beta - \lambda) \leq d\delta \leq R\lambda$</th>
<th>$R\lambda \leq d\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^* = \frac{\alpha - \beta}{\beta}$</td>
<td>$\rho = \beta - \alpha$</td>
<td>$\rho = \frac{\lambda - d\delta}{2\tau}$</td>
<td>$\rho = \beta - \alpha$</td>
<td>$\rho = \beta - \alpha$</td>
</tr>
</tbody>
</table>

(ii) \( R \leq \frac{\alpha}{\beta} \leq 1 \).

<table>
<thead>
<tr>
<th>$r^*$</th>
<th>$d\delta \leq 2\alpha - (2\beta - \lambda)$</th>
<th>$2\alpha - (2\beta - \lambda) \leq d\delta \leq \lambda \frac{\alpha}{\beta}$</th>
<th>$\lambda \frac{\alpha}{\beta} \leq d\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^* = \frac{\alpha - \beta}{\beta}$</td>
<td>$\rho = \beta - \alpha$</td>
<td>$\rho = \frac{\lambda - d\delta}{2\tau}$</td>
<td>$\rho = 0$</td>
</tr>
<tr>
<td>$r^* = 1$</td>
<td>$\rho = \beta - \alpha$</td>
<td>$\rho = 0$</td>
<td>$\rho = 0$</td>
</tr>
</tbody>
</table>
Next, we need to eliminate the dominated candidate solutions. To this end, we compare the profits resulting from the candidate solutions listed in the same column because they are the valid candidate solutions under the same condition (excluding the first column which simply shows the corresponding recycling level \( r^* \)). The optimal solution derived from the comparisons, after an equivalent transformation, is presented below, which we will refer to as the “General Solution” hereafter.

(i) \( \frac{\alpha}{\beta} \leq R \).

\[
\begin{array}{|c|c|}
\hline
\text{d} \delta & \text{r}^* = 1 \\
\hline
2 \alpha \tau - (2 \beta \tau - \lambda) \leq d \delta & \rho^* = \frac{\lambda - d \delta}{2 \tau} \\
\hline
2 \alpha \tau - (2 \beta \tau - \lambda) \leq d \delta & \rho^* = \frac{\alpha \lambda - d \delta}{2 \beta - \lambda} \\
\hline
2 \alpha \tau - R(2 \beta \tau - \lambda) \leq d \delta & \rho^* = \frac{\frac{R \lambda - d \delta}{2 \tau}}{2} \\
\hline
\text{d} \delta \leq R \lambda & \rho^* = 0 \\
\hline
\end{array}
\]

(ii) \( R \leq \frac{\alpha}{\beta} \leq 1. \)

\[
\begin{array}{|c|c|}
\hline
\text{d} \delta & \text{r}^* = 1 \\
\hline
2 \alpha \tau - (2 \beta \tau - \lambda) \leq d \delta & \rho^* = \frac{\lambda - d \delta}{2 \tau} \\
\hline
2 \alpha \tau - (2 \beta \tau - \lambda) \leq d \delta & \rho^* = \frac{\alpha \lambda - d \delta}{2 \beta - \lambda} \\
\hline
\lambda \frac{\alpha}{\beta} \leq d \delta & \rho^* = 0 \\
\hline
\end{array}
\]

(iii) \( 1 \leq \frac{\alpha}{\beta} \).

\[
\begin{array}{|c|c|}
\hline
\text{d} \delta & \text{r}^* = 1 \\
\hline
\rho^* = \frac{\lambda - d \delta}{2 \tau} & \rho^* = \frac{\lambda - d \delta}{2 \beta - \lambda} \\
\hline
\end{array}
\]

In the special case where there is no durability-recyclability interaction, i.e., \( d = 0 \), the General Solution reduces to the following one, presented with the corresponding recycling level and producer profit.

(Ri) When \( \alpha \leq R(\beta - \frac{\lambda}{2 \tau}) \) and \( \beta \leq \frac{\lambda}{2} \), \( r^* = R \) and

\[
\Pi = \frac{(\delta - b \delta^2 + (1 - m + \lambda R(2 \beta - \frac{\lambda}{2 \tau} - \frac{\lambda}{2 \tau}))^2}{4(1 + 3 \delta)}
\]

(Rii) When \( R(\beta - \frac{\lambda}{2 \tau}) \leq \alpha \leq \beta - \frac{\lambda}{2 \tau} \) and \( \beta \leq \frac{\lambda}{2} \), \( r^* = R \) and

\[
\Pi = \frac{\left(\delta - b \delta^2 + (1 - m + \lambda \frac{\beta}{2} - \frac{\lambda}{2 \tau})^2\right)}{4(1 + 3 \delta)}
\]

(Riii) When \( \beta - \frac{\lambda}{2 \tau} \leq \alpha \), \( r^* = R \) and

\[
\Pi = \frac{\left(\delta - b \delta^2 + (1 - m + \lambda \frac{\beta}{2} - \frac{\lambda}{2 \tau} + \frac{\lambda}{2})^2\right)}{4(1 + 3 \delta)}
\]

These results are summarized in Proposition 1.

**Proof of Proposition 2**

To prove Proposition 2, we start by stating Lemma 2.

**Lemma 2** For a given durability \( \delta \), the producer’s profit at the optimal recyclability choice can be written as

\[
\Pi(\delta) = \frac{(\delta - b \delta^2 + A)^2}{4(1 + 3 \delta)}. \tag{A1}
\]

Specifically,

\[
A = \begin{cases} 
1 - m + \lambda (R \alpha - \frac{2 \beta \tau - \lambda}{4 \tau} R^2) & \text{when } \alpha \leq R(\beta - \frac{\lambda}{2 \tau}) \\
1 - m + \lambda \frac{\beta^2}{2 \beta - 1} & \text{when } R(\beta - \frac{\lambda}{2 \tau}) \leq \alpha \leq \beta - \frac{\lambda}{2 \tau} \\
1 - m + \lambda (\alpha - \frac{\beta}{2} + \frac{\lambda}{4 \tau}) & \text{when } \beta - \frac{\lambda}{2 \tau} \leq \alpha
\end{cases}
\]

**Proof:** The lemma follows from Proposition 1 by rearranging the terms of the profit function.
Based on Lemma 2, the profit function has the form $\Pi(\delta) = \frac{(\delta - b\delta^2 + A\delta)^2}{4(1 + 3\delta)^2}$, where $A$ is independent of $\delta$. We solve the first-order condition equation where

$$0 = \frac{\partial \Pi}{\partial \delta} = -\frac{(A + \delta - b\delta^2)(-2 + 3A + \delta(-3 + b(4 + 9\delta)))}{4(1 + 3\delta)^2},$$

which yields the candidate solutions:

$$\delta_{a,b} = \frac{1}{2b} \sqrt{1 + 4\delta^5}, \quad \delta_{c,d} = \frac{(3 - 4b) + \sqrt{(3 - 4b)^2 + 3b(2 - 3A)}}{18b}.$$

Note that $\delta_{a,b} \in \mathbb{R}$ with $\delta_{a} \geq 0$ and $\delta_{a} \leq 0$. Moreover, since $\delta_{b}$ can be shown to be a minimizer, both $\delta_{b}$ and $\delta_{a}$ are eliminated as optimal solution. When $\delta_{c,d} \notin \mathbb{R}$, then the profit maximizing solution $\delta^*$ can only take the value of 0 or 1, at the boundaries. The optimal solution can be selected by comparing the profits at $\delta = 0$ and $\delta = 1$. When $\delta_{c,d} \in \mathbb{R}$, by pairwise comparisons, we can determine the relative relation between the $\delta_{i}$‘s to be $\delta_{c} \leq \delta_{a} \leq \delta_{b}$; hence the optimal $\delta^*$ can only take the value of 0, 1 or $\delta_{a}$. The optimal solution can be selected by comparing the profits at these three points and the solution is characterized below, with $\bar{A}(b) = \frac{1}{3}(5 - 13b)$, $\bar{A}(b) = \frac{2}{27}(27 + 8b) + \frac{2}{27} \sqrt{729 + 972b + 432b^2 + 64b^3}$ and $\delta_{a,b} = \delta_{d} = \frac{(3 - 4b) + \sqrt{(3 - 4b)^2 + 3b(2 - 3A)}}{18b}$.

These results are summarized in Proposition 2.

**Proof of Proposition 3**

To prove Proposition 3, we start by stating Lemma 3.

**Lemma 3** When $d \neq 0$, the optimal recyclability choice (and the corresponding optimal recycling level) can be summarized below, with $\delta^1 = \frac{1}{2}(2\alpha \tau - (2\beta \tau - \lambda))$, $\delta^2 = \frac{1}{2}(2\alpha \tau - R(2\beta \tau - \lambda))$, $\delta^3 = \frac{1}{2}R\lambda$, $\delta^4 = \frac{1}{2}(2\beta \lambda)$ and $\delta^5 = \frac{1}{2}(\lambda)$.

When $d < 0$, and

(i) when $0 < \tau \leq R$.

$$\begin{array}{ll}
\text{(Ni1)} & \delta \leq \delta^2 \\
\rho^* = \frac{d - \delta^2}{2\tau} \\
r^* = R
\end{array} \quad \begin{array}{ll}
\text{(Ni2)} & \delta^2 \leq \delta \leq \delta^4 \\
\rho^* = \frac{d - \delta^2}{2\tau} \\
r^* = \frac{2\lambda - \delta}{2\tau - \lambda}
\end{array} \quad \begin{array}{ll}
\text{(Ni3)} & \delta^4 \leq \delta \\
\rho^* = \frac{d - \delta^2}{2\tau - \delta} \\
r^* = 1
\end{array}$$

$\Pi(\delta) = \Pi_1(\delta)$

(ii) when $R \leq \frac{\tau}{2} \leq 1$.

$$\begin{array}{ll}
\text{(Ni1)} & \delta \leq \delta^4 \\
\rho^* = \frac{d - \delta^4}{2\tau} \\
r^* = \frac{2\lambda - \delta}{2\tau - \delta}
\end{array} \quad \begin{array}{ll}
\text{(Ni2)} & \delta^4 \leq \delta \\
\rho^* = \frac{d - \delta^4}{2\tau - \delta} \\
r^* = 1
\end{array}$$

$\Pi(\delta) = \Pi_2(\delta)$

(iii) when $1 \leq \frac{\tau}{2}$.

$$\begin{array}{ll}
\text{(Ni1)} & \forall \delta \\
\rho^* = \frac{d - \delta}{2\tau} \\
r^* = \frac{1}{2}
\end{array}$$

$\Pi(\delta) = \Pi_3(\delta)$.
When $d > 0$, and

(i) When $\frac{a}{R} \leq R$.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{(Pi1) when } & \text{(Pi2) when } & \text{(Pi3) when } & \text{(Pi4) when } \\
\delta \leq \delta^* & \delta^* \leq \delta \leq \delta^* & \delta^* \leq \delta \leq \delta^* & \delta \leq \delta^* \\
\rho^* = \frac{d - d'A}{2r} & \rho^* = \frac{\lambda - dA}{2r - 2dA} & \rho^* = \frac{b \lambda - dA}{2r - 2dA} & \rho^* = 0 \\
r^* = 1 & r^* = \frac{2r - \lambda}{2r - 2dA} & r^* = \frac{2r - \lambda - da}{2r - 2dA} & r^* = R \\
\Pi(\delta) = \Pi_3(\delta) & \Pi(\delta) = \Pi_2(\delta) & \Pi(\delta) = \Pi_1(\delta) & \Pi(\delta) = \Pi_4(\delta) \\
\hline
\end{array}
\]

(ii) When $R \leq \frac{a}{R} \leq 1$.

\[
\begin{array}{|c|c|c|}
\hline
\text{(Pi1) when } & \text{(Pi2) when } & \text{(Pi3) when } \\
\delta \leq \delta^* & \delta^* \leq \delta \leq \delta^* & \delta \leq \delta^* \\
\rho^* = \frac{a}{2r} & \rho^* = \frac{\lambda - dA}{2r - 2dA} & \rho^* = 0 \\
r^* = 1 & r^* = \frac{2r - \lambda}{2r - 2dA} & r^* = \frac{2r - \lambda - da}{2r - 2dA} \\
\Pi(\delta) = \Pi_3(\delta) & \Pi(\delta) = \Pi_2(\delta) & \Pi(\delta) = \Pi_1(\delta) \\
\hline
\end{array}
\]

(iii) When $1 \leq \frac{a}{R}$.

\[
\begin{array}{|c|c|}
\hline
\text{(Pi1) when } & \text{(Pi2) when } \\
\delta \leq \delta^* & \delta \leq \delta^* \\
\rho^* = \frac{a}{2r} & \rho^* = 0 \\
r^* = 1 & r^* = 1 \\
\Pi(\delta) = \Pi_3(\delta) & \Pi(\delta) = \Pi_0(\delta) \\
\hline
\end{array}
\]

Proof: These results can be derived by adapting the General Solution to the cases with $d < 0$ and $d > 0$. The lemma fully characterizes $\rho^*$ for $d \neq 0$, paralleling Proposition 1 for $d = 0$.

Based on Lemma 3, the profit function can be rewritten as

\[
\Pi_j(\delta) = \frac{(\delta - b_j \delta^2 + A_j^2(F_j)^2}{4(1 + 3\delta)}, \tag{A2}
\]

for $j \in \{1, 2, 3, 4, 5, 6\}$, where $A_j$'s, $b_j$'s and $F_j$ are presented below:

\[
\begin{array}{|c|c|c|c|c|}
\hline
j & A_j & b_j & F_j \\
\hline
1 & \frac{R^2 \lambda^2 + 4(1 - m) + 4(Ra - R)^2 \lambda r}{-2dA + 4\tau} & \frac{4r - \lambda - \frac{R^2 \lambda}{2}}{2r - 2dA} & \frac{2r - dA}{2r} \\
2 & \frac{\alpha^2 + \lambda^2 + 1 + \lambda + 2\lambda r}{-2A + 2\alpha r} & \frac{2\beta - 2\alpha - \lambda - dA}{2\beta - 2\alpha - dA} & \frac{2\beta - 2\alpha - dA}{2\beta - 2\alpha} \\
3 & \frac{\alpha^2 + \lambda^2 + 1 + \lambda + 2\lambda r}{-2A + 2\alpha r} & \frac{2\beta - 2\alpha - \lambda - dA}{2\beta - 2\alpha - dA} & \frac{2\beta - 2\alpha - dA}{2\beta - 2\alpha} \\
4 & 1 - m + (\alpha - \frac{R^2 \lambda}{2}) \lambda & b & 1 \\
5 & 1 - m + \frac{\alpha^2 \lambda}{2} & b & 1 \\
6 & 1 - m + (\alpha - \frac{R^2 \lambda}{2}) \lambda & b & 1 \\
\hline
\end{array}
\]

Since we can unify the profit function into Equation (A2), which has a similar structure as Equation (A1), the solution in Proposition 2 applies. That means for $\Pi(\delta) = \Pi_j(\delta)$, the optimal $\delta$ in this case (denoted by $\delta_j^*$) takes the value of $\arg\max_3 \Pi(\delta)$ as specified in Proposition 2 with $A = A_j$ and $b = b_j$. Then the optimal solution can be identified to be the $\delta_j^*$ that yields the highest profit across the different $j$ cases. To be specific,

\[
\delta^* = \arg\max_j \{\Pi_j(\delta_j^*)\}, \quad \text{where } \begin{cases} 
\delta_j^* = \{1, 2, 3, 4\} & \text{when } \frac{a}{R} \leq R \\
\delta_j^* = \{2, 3, 5\} & \text{when } R \leq \frac{a}{R} \leq 1 \\
\delta_j^* = \{3, 6\} & \text{when } 1 \leq \frac{a}{R} 
\end{cases}
\]

Proof of Proposition 4

Based on the closed-form solutions of $\delta^*$ and $\rho^*$ from Proposition 1–3 and Lemma 3, we can prove that $\frac{\partial \delta^*}{\partial R} \geq 0$, $\frac{\partial \delta^*}{\partial \lambda} \geq 0$, $\frac{\partial \rho^*}{\partial R} \geq 0$ and $\frac{\partial \rho^*}{\partial \lambda} \geq 0$ when $d \leq 0$. Details are omitted for brevity.
Proof of Proposition 5
This result is derived from the closed-form solution of $\delta^*$ for $d > 0$ from Proposition 3 and its proof, based on which we can calculate $\frac{\partial \delta^*}{\partial R}$. Note that all the thresholds $R_{dl}$ and $R_d$ are in closed-form. They are omitted for brevity.

Proof of Proposition 6
This result is derived from the closed-form solution of $\rho^*$ for $d > 0$ from Lemma 3 and its proof, based on which we can calculate $\frac{\partial \rho^*}{\partial R}$. Specifically, $\bar{d} = \sqrt{\frac{6 \lambda}{d}}$. The thresholds $R_r$ and $R_{rr}$ are also found in closed-form but omitted for brevity.

Proof of Proposition 7
This result is derived from the closed-form solution of $\delta^*$ for $d > 0$ from Proposition 3 and its proof, based on which we can calculate $\frac{\partial \delta^*}{\partial \lambda}$. Note that the threshold $\lambda_d$ is found in closed-form but omitted for brevity.

Proof of Proposition 8
This result is derived from the closed-form solution of $\rho^*$ for $d > 0$ from Lemma 3 and its proof, based on which we can calculate $\frac{\partial \rho^*}{\partial \lambda}$. The thresholds $\lambda_{rl}$ and $\lambda_r$ are found in closed-form but omitted for brevity.

Profitable Recycling and Competition for Cores
In order to elicit returns, the producer and the third parties offer respective buyback options to consumers for the cores. Such buyback options can take various forms including trade-in discounts, coupons, checks or cash. Denote the unit buyback price offered by the producer as $p_0$ and the one offered by the $i$-th third parties as $p_i$ (regardless of the product condition). We assume all the $n$ third parties are homogeneous, so as to obtain insights while keeping the analysis tractable.

While the producer is assumed to have a large enough capacity to accept and recycle all the returns that arrive, a third party only has a limited capacity that can accept and recycle a fraction $k$ ($0 \leq k \leq 1$) of all the end-of-life products. When the aggregate third-party capacity covers the whole collection and recycling volume, $k = 1/n$.

We construct a model similar to the one in Arnold (2001) and solve the problem by first looking at the consumers’ return strategy. When all the consumers are risk neutral, there exists a symmetric mixed strategy for consumers represented by $\pi = \{\pi_0, \pi_1, \cdots, \pi_n\}$ such that in every attempt to return the core, a consumer chooses the producer and the $i$-th third party with probabilities $\pi_0$ and $\pi_i$ ($i = 1, \cdots, n$), respectively. Given this strategy, in every period, a portion $\pi_i$ of the total returns arrive at the $i$-th third party. Each of these recyclers accept returns up to capacity. Then they start recycling and stop accepting returns during a time period $Y$, which is assumed to be exponentially
Proof of Proposition 9

When $v(R, 0) > 0$ and $d > 0$, we can solve for the closed-form $\rho^*$ and $\delta^*$ that maximizes the producer profit in this case, following a similar procedure as above. Then we can prove that $\frac{\partial \rho^*}{\partial \lambda} \leq 0$ and $\frac{\partial \delta^*}{\partial \lambda} \geq 0$ for $0 \leq \lambda \leq 1$.

Proof of Proposition 10

Note that $g(\rho^*(\rho^*, \delta^*), \delta^*) = \frac{1 + \delta - (m + R\rho^2 + d\delta + d^2) - (1 + \delta)R^2 + (\alpha + \rho)R \lambda}{4(1 + \delta)}$, with $\rho^*$ and $\delta^*$ as specified earlier. Based on the resulting expression of the new production volume, we can calculate $\frac{\partial g(\rho^*(\rho^*, \delta^*), \delta^*)}{\partial R}$ and $\frac{\partial g(\rho^*(\rho^*, \delta^*), \delta^*)}{\partial \lambda}$. All the thresholds in the proposition are in closed-form but omitted for brevity.
Details of Calibrated Numerical Study

We first derive the values of \( \alpha_j + \rho_j \) and \( \beta_j \) for both the c-Si \((j = 1)\) and CIGS \((j = 2)\) technologies by using the recycling data. The processing cost is estimated to be \( c = $200/\text{ton} \) based on recent data (Choi and Fthenakis 2014, Fthenakis and Moskowitz 1999, BIO 2011), and it is comparable for both of the technologies. To convert it into \( c_1 \) and \( c_2 \) in terms of \$/kW, we use the average weight of 102 kg/kW and 200 kg/kW for the c-Si and CIGS technologies, respectively (Okopol et al 2007).

Next we collect data on the material components of PVPs, as well as the recovery rates and market prices of all the composition elements. Note that the market prices are corresponding to a 100% purity level, which may not always be achieved by the current recycling technologies. Therefore, the actual values of the recovered materials may be lower but these values are good approximations.

The table shown in Figure A5 summarizes the data (Cucchiella et al. 2015, BIO 2011), note that it only reflects the recoverable material values but has not yet captured the processing cost.

<table>
<thead>
<tr>
<th>Materials composition</th>
<th>Al</th>
<th>Glass</th>
<th>Cd</th>
<th>Cu</th>
<th>Ga</th>
<th>In</th>
<th>Mo</th>
<th>Plastic</th>
<th>Se</th>
<th>Si</th>
<th>Sn</th>
<th>Te</th>
<th>Zn</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-Si [c-Si, p-Si, a-Si]</td>
<td>17.5</td>
<td>65.8</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(% of weight)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CdTe</td>
<td>96.8</td>
<td>0.08</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(% of weight)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIGS</td>
<td>96.9</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.12</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(% of weight)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recycling rates of the material (%)</td>
<td>100</td>
<td>97</td>
<td>98</td>
<td>78</td>
<td>99</td>
<td>75</td>
<td>99</td>
<td></td>
<td>80</td>
<td>85</td>
<td>99</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>Market prices of the material ($/kg)</td>
<td>1.00</td>
<td>0.08</td>
<td>0.95</td>
<td>3.69</td>
<td>153.08</td>
<td>417.69</td>
<td>14.62</td>
<td>0.07</td>
<td>32.31</td>
<td>1.17</td>
<td>12.69</td>
<td>59.23</td>
<td>1.12</td>
</tr>
</tbody>
</table>

**Figure A5** The material composition of different PVPs.

We can now calculate the recycling levels \( (r_{m1} \text{ and } r_{m2}) \) that only account for the recoverable material values in the following way: For every composition element, multiply its percentage of the total weight by its recovery rate, and then sum over all the elements of the panel. We can also derive the unit recycling material values \( v_{mj} \) in the following way: For each element, multiply its percentage of the total weight by the average weight of the panel, then multiply the result by the market price, and sum over all the elements of the panel. Next, let \( \alpha_{mj} + \rho_j \) represent the highest marginal recycling value of the product that only accounts for the recoverable material value, given the recyclability of the technology. Then based on \( r_{mj} = \min \{1, \max \left\{ \frac{\alpha_{mj} + \rho_j}{\beta_j}, 0 \right\} \} \) and \( v_{mj} = -\frac{1}{2} \beta_j r_{mj}^2 + (\alpha_{mj} + \rho_j) r_{mj} \), we can calculate \( \alpha_{mj} + \rho_j \) and \( \beta_j \) for both the technologies, by \( \beta_j = \frac{2v_{mj}}{r_{mj}} \) and \( \alpha_{mj} + \rho_j = \frac{2v_{mj}}{r_{mj}} \) (since \( 0 < \frac{\alpha_{mj} + \rho_j}{\beta_j} < 1 \)). Now we incorporate the processing cost in the estimation. Note that the processing cost applies to all the recycled components of the panel. Therefore, the unit recycling value \( v_{j0} \) after accounting for the processing cost (we use the additional subscript “0” for the values in the baseline case without legislation) should be the unit recycling material value net of the processing cost; i.e., \( v_{j0} = v_{mj} - c_j \). We assume the processing
cost applies in a uniform manner to the recycled components and hence $\beta_j$ remains unchanged. Substituting the relevant data into the equations, we arrive at the following results: $\beta_1 = 0.08$ and $\beta_2 = 0.112$; $v_{10} = 8.3$ and $v_{20} = 10$. Then we can also solve for $r_{j0}$ and $\alpha_j + \rho_{j0}$ based on $r_{j0} = \min\{1, \max\{0, -\frac{\alpha_j + \rho_{j0}}{\beta_j}\}\}$ and $v_j = -\frac{1}{2} \beta_j r_j^2 + (\alpha_j + \rho_j) r_j$. The results are: $r_{10} = 0.455$ and $r_{20} = 0.424$; $\alpha_1 + \rho_1 = 0.0366$ and $\alpha_2 + \rho_2 = 0.0476$.

Next, we show how the $X_j$ values are recovered from the market data. First, we collect data on the current prices of PVPs and the production costs: $p_1 = $1.75 $\times 10^3$/kW, $p_2 = $2.1 $\times 10^3$/kW, $g_1 = $1.05 $\times 10^3$/kW and $g_2 = $1.5 $\times 10^3$/kW (ISET 2010, Reddy 2012). In the baseline case where legislation is absent, when consumer types are distributed in the generalized range of $[0, X_j]$, we re-solve our model in a similar way as before and derive $p_j = \frac{1}{2}(X_j(1 + \delta_j) + g_j - \lambda_0 v_{j0})$. Therefore, $X_j = \frac{2p_j - g_j + \lambda_0 v_{j0}}{2 + \delta_j}$, and calculations give $X_1 = 1.26 \times 10^3$ and $X_2 = 1.43 \times 10^3$.

Next, based on the generalization of our model (with $X_j$), the producer profit in the baseline case is $\Pi_{j0} = \frac{(X(1 + \delta_j) - g_j + \lambda_0 v_{j0})^2}{4X_j(1 + \delta_j)}$, whereas the profit in the legislated case will be $\Pi_j = \frac{(X(1 + \delta_j) - g_j + \lambda_0 v_{j0})^2}{4X_j(1 + \delta_j)}$ with $r_j = \max\{R, r_{j0}\}$ and $v_j = -\frac{1}{2} \beta_j (r_j)^2 + (\alpha_j + \rho_j) r_j$ where $R$ and $\lambda$ are the legislative recycling and collection targets, respectively. Then $\Delta \Pi_j = \frac{\Pi_j - \Pi_{j0}}{\Pi_{j0}}$. Figure 4 shows how the regions defined by $\min\{\Delta \Pi_1, \Delta \Pi_2\}$ shift as we vary the processing cost from the benchmark value $200$/ton to $185$/ton (as shown by the dashed lines moving downwards).

References


Knut Sander, Institut fur Okologie und Politik. Study on The Development of a Take Back and Recovery System for Photovoltaic Products.