Submitted to *Management Science*
manuscript

Design Incentives under Collective Extended Producer Responsibility: A Network Perspective

Luyi Gui
Paul Merage School of Business, University of California Irvine, Irvine, CA 92697, luyig@uci.edu

Atalay Atasu
Scheller College of Business, Georgia Institute of Technology, Atlanta, GA 30308, atalay.atasu@scheller.gatech.edu

Özlem Ergun
Department of Mechanical and Industrial Engineering, Northeastern University, Boston, MA 02115, o.ergun@neu.edu

L. Beril Toktay
Scheller College of Business, Georgia Institute of Technology, Atlanta, GA 30308, beril.toktay@scheller.gatech.edu

A key goal of Extended Producer Responsibility (EPR) legislation is to provide incentives for producers to design their products for recyclability. EPR is typically implemented in a collective system, where a network of recycling resources are coordinated to fulfill the EPR obligations of a set of producers, and the resulting system cost is allocated among these producers. Collective EPR is prevalent because of its cost efficiency advantages. However, it is considered to provide inferior design incentives compared to an individual implementation (where producers fulfill their EPR obligations individually). In this paper, we revisit this assertion and investigate its fundamental underpinnings in a network setting. To this end, we develop a new biform game framework that captures producers' independent design choices (non-cooperative stage) and recognizes the need to maintain the voluntary participation of producers for the collective system to be stable (cooperative stage). This biform game subsumes the network-based operations of a collective system and captures the interdependence between producers' product design and participation decisions. We characterize the manner in which design improvement may compromise stability and vice versa. Yet we establish that a stable collective EPR implementation can match and even surpass an individual implementation with respect to product design outcomes. Our analysis uncovers network properties that can be exploited to develop cost allocations that achieve such superior design outcomes.

Key words: Extended Producer Responsibility, Design for Environment, Biform Games, Recycling, Network Operations

1. Introduction

The growing volume of electronic waste (e-waste) has become one of the biggest environmental concerns in recent years. It is expected to reach 93.5 million tons in 2016 globally, increasing at an annual growth rate of 17.6 percent in the past five years (Environmental Leader 2014). To respond to the e-waste problem, policy instruments that aim to create incentives for more environmentallyfriendly product design have been developed. In particular, Extended Producer Responsibility (EPR) is an instrument that has been widely adopted due to its potential to effect design improvements (Lifset and Lindhqvist 2008). By mandating producers to be financially responsible for product end-of-life costs, EPR is expected to motivate producers to improve the recyclability of their products in order to reduce these costs. Providing such design incentives is explicitly stated as a major goal in EPR-based legislation, e.g., in the Waste Electrical and Electronic Equipment (WEEE) Directive in the EU (WEEE 2012) and several EPR bills in the US (e.g., Washington State Legislature 2006, Maine Revised Statutes 2007).

While the design principle behind EPR is simple, implementing EPR-based e-waste legislation is a complex process influenced by multiple economic and operational factors. Most prominently, due to the cost burden EPR imposes on electronics producers, EPR implementations mainly focus on operational cost efficiency: State programs are committed to providing collection and recycling services at low cost subject to legislative standards (e.g., WMMFA 2012, European Recycling Platform 2012, Department for Business Innovation & Skills 2013). This cost efficiency focus has contributed to the prevalence of *collective* implementations of EPR, in which a large-scale collection and recycling system handles a mixture of e-waste manufactured by different producers. Such collective systems enjoy cost benefits deriving from economies of scale and from synergies obtained by sharing capacitated resources (i.e., system-wide cost reduction by more efficient product routing).

However, collective implementations have been criticized for muting the design incentives intended by EPR. Criticism has most often been directed at the weight-based proportional cost allocations that are frequently adopted. A widely-used example is cost allocation by return share, in which a producer's share of the total cost equals the percentage of its products in the total return volume by weight. Because such cost allocations do not penalize or reward producers according to their product recyclability, one producer's bad design and in turn, high recycling cost at end-of-life is absorbed jointly by all participating producers in a collective system (IPR Works 2012). As such, some producers have indicated that they lost design incentives under collective implementations (Greenpeace 2007).

These concerns have resulted in strong advocacy for the principle known as Individual Producer Responsibility (IPR), i.e., each producer should only bear the cost of *its own products* in an EPR implementation (Lifset and Lindhqvist 2008, IPR Works 2012). In particular, it is often argued that an *individual* implementation (in which every producer sets up a system to recycle its own products) provides the best design incentives for recyclability (Atasu and Subramanian 2012, Plambeck and Wang 2009). An alternative view is that "elements of IPR can be realized in practice in collectively organized compliance systems" by allocation mechanisms based on the "actual costs associated with managing individual producers' products" (Sander et al. 2007, IPR Working Group

2012, Dempsey et al. 2010), and there have been heuristic attempts at qualitatively calibrating the proportional allocations currently used based on the "IPR principle" (Dempsey et al. 2010, Mayers et al. 2013, Department for Business Innovation & Skills 2013).

The two central questions in this ongoing debate are: Can a collective EPR implementation match the design-for-recyclability potential of an individual EPR implementation? What is the role of cost allocation mechanisms in the relative performance of these approaches? These are the questions we resolve in this paper. The answers are far from obvious (and closely linked) for the following reasons: In a collective system, producers make design decisions independently, but then their products are processed collectively and the resulting total recycling cost is allocated among them. The design decisions by the full set of producers, the operations of the collective system and the allocation mechanism all combine to determine each producer's total recycling cost. Because participation in a collective system is *voluntary*, if its allocated cost is higher than what a producer can achieve independently or with a subgroup of producers, that producer may "defect" from the collective system, i.e. the collective system is not stable. (This is a real phenomenon that has resulted in the fragmentation of a number of collective implementations to date (Atasu and Van Wassenhove 2012, Gui et al. 2015).) Thus, there is an interdependence between product design decisions and participation decisions of producers, and this interdependence is moderated by the operational infrastructure and the cost allocation mechanism in effect. This makes it challenging to establish whether and under what operating conditions each type of system dominates. We provide a comprehensive analysis of these questions in this paper.

To capture the interdependence between design and participation decisions, we develop a new biform game formulation. The first-stage is a non-cooperative game that models producers' independent product design choices. The second stage is a cooperative game that models producers' decisions to participate in the collective system or not given the cost allocation mechanism in effect. This cooperative game is defined based on a capacitated recycling network (RN) model of the collective system that optimally routes returns to the set of processing facilities in order to achieve the highest cost efficiency. We use this cooperative game to evaluate whether the cost allocated to any producer or producer sub-coalition is no higher than its stand-alone cost, i.e., the minimum cost that can be achieved if it operates independently. This property, called *group incentive compatibility* in game theory, implies stability of the collective system against sub-coalition formation.

We focus on two fundamental cost allocation mechanisms from the literature (dual based and marginal cost based), and uncover the existence of a fundamental design effectiveness - stability tradeoff in collective implementations: The dual based allocation ensures stability of the collective implementation, but the design outcome may be inferior compared to that in an individual implementation. The marginal cost based allocation yields a superior design outcome in a collective implementation relative to an individual implementation, but this mechanism is not guaranteed to be group incentive compatible. We establish the fundamental underpinnings of this trade-off. In particular, we find that whether this tradeoff is in effect depends on the capacity configuration of different processing facilities in the collective system, and how the unit recycling costs at these facilities change as a function of product design improvements.

We further analyze the root cause of the design improvement-stability tradeoff by considering a more general class of cost allocation mechanisms that encompasses the major mechanisms studied in the literature and used in practice. We prove that superior design and group incentive compatibility can be simultaneously achieved if and only if the second-stage cooperative game satisfies a convexity condition. Intuitively, this is because convexity implies higher synergy in larger coalitions, hence group incentive compatibility imposes less stringent conditions on the cost allocation. In that case, we show how to tailor the components of this class of allocation mechanisms to meet the design and stability criteria. These findings provide game theoretical insights into why there is a conflict between design incentives and coalitional stability, and under what conditions this conflict can be overcome by choosing the cost allocation mechanism appropriately.

The contribution of our results is three-fold. First, and at the most fundamental level, we uncover a surprising result: There are many networks for which there is no mechanism that ensures the collective implementation achieves better design outcomes than the individual implementation, and yet maintains stability. In such networks, adopting a cost allocation mechanism that ensures strong design incentives comes at a price: It causes instability of the collective system (with respect to producer coalitions), which in turn, undermines the cost efficiency advantage of collective operations. Second, when the network exhibits certain infrastructural characteristics, which we refer to as "design reinforcing conditions," we prove the existence of (and identify) cost allocation mechanisms that enable collective implementations to achieve better design outcomes than individual implementations and maintain stability. By characterizing the design-reinforcing conditions, we identify infrastructural factors that need to be taken into account when evaluating the design impact of collective implementations. In particular, managing the design-stability tradeoff requires looking into how available processing technologies interact with product design improvement in reducing recycling costs, and the capacity mix of these technologies. Finally, we provide a new policy insight: There are situations where collective EPR will primarily act as an enabler of cost efficiency, but not of design improvement, in which case policy makers should resort to direct design incentives such as requiring or subsidizing the preferred design.

2. Literature Review

A stream of research in the environmental economics literature analyzes the product design impact of environmental policy instruments including product take back (e.g., Walls 2006, Fullerton and Wu 1998). These studies take an economics perspective through general equilibrium models, and do not address the impact of implementation-level phenomena (e.g. product routing on a network, cost allocation) on design choices (Atasu and Van Wassenhove 2012). The industrial ecology literature qualitatively discusses the lack of design incentives in collective EPR implementations (e.g., Lifset and Lindhqvist 2008, Kalimo et al. 2012, Dempsey et al. 2010, Mayers et al. 2013). These papers position IPR as the main solution for this problem, but do not establish the effectiveness of the proposed approaches under the prevailing operational characteristics of EPR implementations. This paper contributes to both streams by modeling fundamental features of collective recycling operations such as product routing based on shared resources and cost allocation, and by analyzing their design implications.

One stream in the operations management literature highlights the importance of understanding the impact of operational features or implementation choices on policy outcomes (e.g., Islegen et al. 2015, Drake et al. 2015, Raz and Ovchinnikov 2015, Granot et al. 2014). Another stream of research employs analytical models to evaluate the economic and environmental tradeoffs associated with green product design decisions (e.g., Chen 2001, Agrawal and Ülkü 2012). At the intersection are papers that study product design implications of EPR policy, in the contexts of product market competition (Atasu and Subramanian 2012), processing technology choice (Zuidwijk and Krikke 2008), compliance scheme choice (Esenduran and Kemahlıoğlu-Ziya 2015), new product introduction (Plambeck and Wang 2009), supply chain coordination (Subramanian et al. 2009), and secondary market concerns (Alev et al. 2014). We contribute to this research stream by capturing the effects of capacitated resource sharing on EPR outcomes and considering the cost allocation mechanism as an implementation choice with design implications.

Network models of recycling operations have been adopted in the reverse logistic network design literature outside of a policy context, focusing on studying self-forming recycling networks when recycling is profitable (e.g., Fleischmann et al. 2001, Nagurney and Toyasaki 2005). In this paper, we study regulated recycling networks in a setting where products are costly to process and these networks are formed only to comply with regulation. We analyze how the design incentives induced by EPR-based regulation are influenced by network effects. Gui et al. (2015) adopt a similar network approach to devise group incentive compatible cost allocation mechanisms for collective EPR systems. This paper differs by focusing on design incentive provision as a major policy objective, and analyzing the tradeoff between design incentives and group incentive compatibility in the choice of cost allocation mechanisms.

Recycling coalitions are studied in Tian et al. (2014), who analyze the formation dynamics of recycling coalitions with product market competition considerations. Their work considers a given cost allocation rule and evaluates the stability of different coalition structures, while ours searches for a cost allocation that ensures both stability in the cooperative stage and superior design incentives in the non-cooperative stage. Our work contributes to the operations management literature based on biform games, which generally analyze traditional supply chain management issues without the product recovery component (e.g., Granot and Sošić 2003, Chod and Rudi 2006, Anupindi et al. 2001, Plambeck and Taylor 2005). To the best of our knowledge, this paper is the first to adopt a biform game to explicitly investigate the interaction between the non-cooperative (e.g., independent product design decision-making) and cooperative (e.g., collective recycling operations) dimensions of EPR implementations.

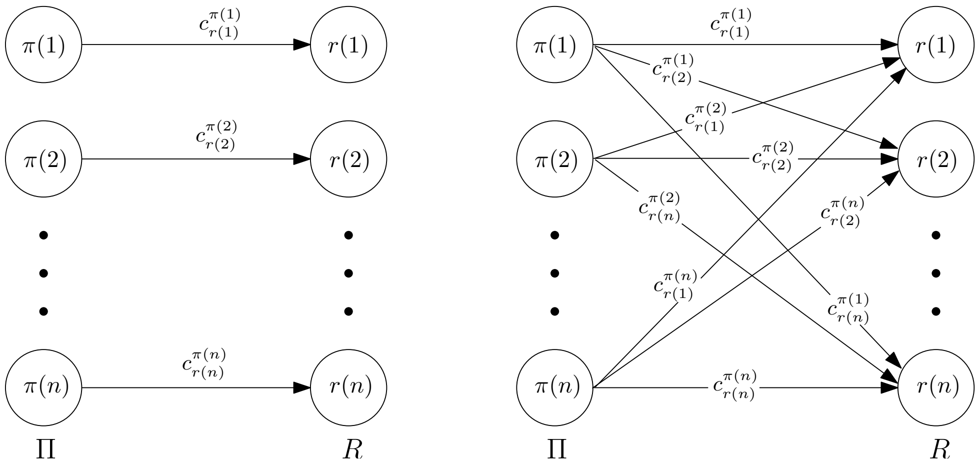
From a methodological perspective, this paper studies the tradeoff between multiple properties desired from a cost allocation mechanism (i.e., promoting design incentives and group incentive compatibility). In the literature, tradeoffs in cost allocation are studied as a theoretical topic in economics and computer science, mostly in the context of auction design (e.g., Roughgarden and Sundararajan 2006, Immorlica et al. 2008, Sundararajan 2009). Our work contributes by studying this problem in the context of public policy design. Moreover, by adopting a biform game model, we explicitly capture the tradeoff between ensuring a superior equilibrium outcome in the non-competitive stage and ensuring stability in the cooperative stage.

Our work also contributes to the game theory literature by introducing a new application of a classic notion of the marginal contribution. In the literature, marginal contribution is a fundamental concept studied in cost allocation problems (e.g., Moulin 1999, Shapley 1952). We show that this notion gives rise to an allocation that provides superior design incentives under collective EPR relative to the individual EPR benchmark. In addition, a set of papers in this literature shows that convexity of the cooperative game ensures the group incentive compatibility of allocations based on marginal contribution (e.g., Shapley 1971, Greenberg 1985, Branzei et al. 2003). Our study contributes by extending this result to the biform game setting.

3. Model Description

In this section, we describe the biform game model used to study producers' design choices and participation in a collective EPR implementation. This biform game is defined based on a network flow characterization of the recycling operations in a collective EPR implementation. In the following, we first introduce this recycling network model (§3.1). We then present a model of product design decisions, based on which we introduce the biform game that captures producers' participation decisions in the collective system (§3.2). We conclude with a roadmap of the analysis to be presented in §4-7. Table 1 summarizes the notation used in this paper.

Figure 1 A network representation of an individual system and the transportation network model of a collective recycling network (RN).



(a) An individual system (b) A collective system

3.1. The Recycling Network Model

In practice, recycling operations typically consist of collection, consolidation, transportation, and processing (dismantling, shredding and component/material separation) stages. In this paper, we focus on the processing stage because it is the most relevant one for affecting design incentives, i.e., product recyclability directly determines processing costs. We consider two types of recycling systems, corresponding to collective versus individual EPR implementations, and compare the resulting design outcomes. Our analysis is based on the simplest network characterization that can capture the fundamental difference between the two systems and provide structural insight. In particular, we consider a set of $N = \{1, 2, ..., n\}$ producers, and assume each producer i makes one product $\pi(i)$ with a return volume of $d^{\pi(i)}$ to be recycled to fulfill EPR obligations. In an individual system, each producer recycles its own products independently using the capacity it has access to, as illustrated in Figure 1(a). We denote the level of capacity that producer i has access to with $k_{r(i)}$ and assume it is no smaller than $d^{\pi(i)}$. In practice, this can either be contracted capacity at third-party service providers or capacity at the producer's own recycling facility.

In contrast, a collective system handles the end-of-life products of all participating producers by using all the available capacities in order to achieve cost efficiency. To model this so as to allow a direct comparison with the individual system, we combine the set of products $\Pi = \{\pi(i) : \forall i \in N\}$ as well as the set of processors $R = \{r(i) : \forall i \in N\}$, and construct a transportation network as shown in Figure 1(b), which we call the recycling network (RN). In this RN, a returned product may be alloted (i.e. routed) to any of the processors. In practice, this allotment is typically determined by a "control tower" entity that operates the collective system, which we discuss in detail in §3.2.2.

In the rest of this paper, we sometimes drop (i) and refer to a generic product and processor as π and r, respectively. We model the unit cost to process a product π at a processor r, denoted as c_r^π and hereafter called the unit recycling cost, as the edge cost between the two corresponding nodes in the RN. In practice, better product recyclability and higher processor cost efficiency typically lead to a lower cost. For example, it costs less to process LCD TVs than CRT TVs due to the leaded glass contained in CRT TVs, which is expensive to recycle subject to environmental standards. Recycling leaded glass is particularly costly at manual processors that are not equipped with the required separation technology, and who thus have to outsource the process to downstream brokers and bear a higher cost. To capture this, we introduce a recyclability measure of each product π and an efficiency measure of each processor r, denoted by λ^π and τ_r respectively. Intuitively, λ^π can be interpreted as the unit cost to process π at a standard processor with benchmark efficiency, and τ_r represents the cost ratio between processor r and the standard processor. Accordingly, we model the unit recycling cost c_r^π as a multiplicative function of λ^π and τ_r, plus a base cost $\bar{c}^\pi$, which represents the base recycling cost required to process π at any processor, for example, the minimum labor cost required to dismantle a CRT TV. In this function, λ^π is the decision variable of the producer that makes product π; a lower λ^π value indicates a more recyclable product.

$$c_r^\pi(\lambda^\pi) = \lambda^\pi \cdot \tau_r + \bar{c}^\pi \quad \forall \pi \in \Pi \quad \forall r \in R. \tag{1}$$

We note that this multiplicative cost model implicitly assumes that the unit recycling cost reduction in response to a design improvement is higher at a less efficient processor (i.e., a processor with a larger τ_r). Examples of design attributes that lead to this situation are ease of manual dismantling and using less toxic materials (such as in the case of LCD versus CRT TVs, where the cost saving due to a design with no leaded glass is higher for manual processors). We discuss the complementary case (i.e., product design improvements generate higher cost savings at a more efficient processor) as an extension of the main analysis in §7.1. We show that the insights derived based on the multiplicative model continue to hold. We also note that the cost function in (1) models the case where the efficiency parameter τ_r at processor r applies to all products. In §7.2, we extend the analysis to cases in which processor efficiency may be differentiated by product.

3.2. Product Design Decisions

We focus on the regulatory impact of EPR on producers' design choices, and assume that each producer i chooses a recyclability level $\lambda^{\pi(i)}$ to minimize its own recycling cost in an individual system or its allocated cost in a collective system, plus the design investment cost required in product development. We assume that a more recyclable product (i.e., one with a smaller $\lambda^{\pi(i)}$) requires a higher investment as well as a larger marginal investment for further improvement. Accordingly, the investment cost is modeled as a convex decreasing function $Q^i(\lambda^{\pi(i)})$ for each producer i with the following continuity conditions.

Table 1 Table of notation used in this paper

RN notation	i and N	A producer and the set of producers
	$\pi(i)$ and $r(i)$	Product of producer i and processor where producer i has access to capacity
	π and r	A generic product and a generic processor
	Π and R	Set of products and processors
	d^{π} and k_r	Return volume of product π and capacity at processor r
	τ_r	Efficiency level of processor r
Design-related notation	λ^{π}	Recyclability level of product π (*the design variable*)
	Q^i	Design investment function of producer i (as an function of $\lambda^{\pi(i)}$)
	$\bar{c}^{\pi}$	Base unit cost to recycle π at any processor
	c_r^{π}	Unit recycling cost function for product π at processor r
	Λ	A generic design profile of all products
Biform game notation	(C)	Centralized allotment problem in the collective system
	f^*	Socially optimal allotment
	$Z(f^*)$	Minimum total recycling cost in the collective system
	x	Cost allocation mechanism
	x^i	Cost allocated to producer i under mechanism x such that $\sum_{i\in N} x^i = Z(f^*)$
Design outcome notation	Λ_{ind}	Optimal design profile induced by an individual system
	Λ_x^{ne}	Equilibrium design profile under allocation mechanism x in a collective system

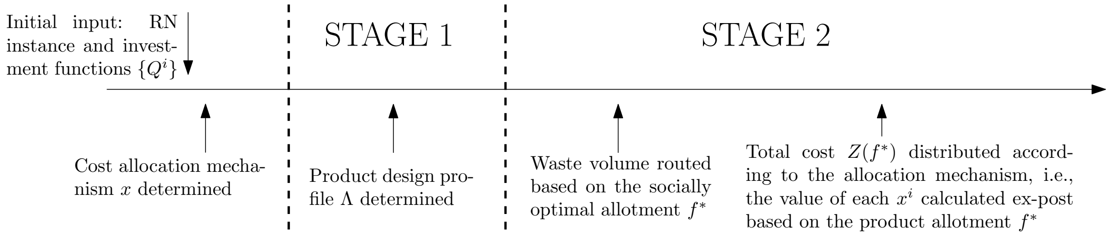
Assumption 1. *We assume that (i) Q^i is continuously differentiable with respect to $\lambda^{\pi(i)}$; (ii) the first order derivative $Q^{i\prime}$ is strictly monotonic, i.e., Q^i is strictly convex; and (iii) $Q^{i\prime}$ satisfies $\lim_{\lambda^{\pi(i)}\to 0} Q^{i\prime}(\lambda^{\pi(i)}) = -\infty$ and $\lim_{\lambda^{\pi(i)}\to\infty} Q^{i\prime}\left(\lambda^{\pi(i)}\right) = 0$.*

Clearly, a function Q^i that satisfies the above continuity conditions has an invertible derivative $Q^{i\prime}$ on $(0,\infty)$ and we denote the corresponding inverse function by $(Q^{i\prime})^{-1}$.

3.2.1. Product Design under Individual Implementation We first calculate the design outcome under an individual implementation, where each producer i recycles its product using processor $r(i)$, and incurs a total *stand-alone* recycling cost of $c_{r(i)}^{\pi(i)} \cdot d^{\pi(i)}$. Hence, producer i's optimal design choice is the value of $\lambda^{\pi(i)}$ that minimizes its total cost $c_{r(i)}^{\pi(i)} \cdot d^{\pi(i)} + Q^i(\lambda^{\pi(i)}) = (\lambda^{\pi(i)} \cdot \tau_{r(i)} + \bar{c}^{\pi(i)}) \cdot d^{\pi(i)} + Q^i(\lambda^{\pi(i)})$. This optimal recyclability level, which we denote by $\lambda_{ind}^{\pi(i)}$, can be calculated as $(Q^{i\prime})^{-1}\left(-d^{\pi(i)} \cdot \tau_{r(i)}\right)$ by taking the first-order derivative of producer i's total cost function. We denote the collection of $\lambda_{ind}^{\pi(i)}$ of all producers by $\Lambda_{ind} \doteq \{\lambda_{ind}^{\pi(i)} : \forall i \in N\}$, and call it the optimal design profile induced by an individual system.

3.2.2. Product Design under Collective Implementation: A Biform Game Model Under a collective implementation, producers choose the recyclability levels of their products to minimize their *cost allocation* in the collective system plus their design investment cost. In the rest of this sub-section, we introduce a biform game that models this design decision.

Figure 2 The sequence of events in the two-stage biform game.



In this game, we first assume that the design choices under a collective implementation are made based on a given RN instance (i.e., an RN with known parameters except the recyclability levels of the products). This includes a given set of products and processors, their capacity levels and their cost efficiency parameters, and the return volume of each product[1]. We also assume that the cost allocation mechanism to be used in the collective system is pre-determined. For example, most EPR legislation adopts allocation by return share, where each producer is allocated a portion of the total cost incurred that equals the percentage of its products in the total volume by weight. In our model, the allocation mechanism is assumed to be known to all producers before design choices are made. Note that we distinguish between the allocation mechanism (i.e., the functional forms of producers' costs, determined ex-ante), and the allocated cost which is calculated ex-post based on the design profile chosen and the associated product allotment. The biform game proceeds in the following two stages:

- In the first stage, each producer i makes the product design choice $\lambda^{\pi(i)}$ independently. We model this as a non-cooperative game and study its equilibrium outcome.
- The second stage occurs when the products with the chosen design are returned and recycled in a collective system. Then the total recycling cost is allocated to the participating producers based on the pre-determined allocation mechanism and the product allotment. A key issue in this stage is whether the cost allocation provides participation incentives to the producers, i.e., whether it ensures the stability of the collective implementation. The cooperative game is constructed to address this issue.

Figure 2 summarizes the sequence of events in this two-stage biform game. Next, we present the detailed formulation of the cooperative game in the second stage, and then analyze the equilibrium design choices in the first stage.

Stage Two: The Cooperative Product Allotment (PA) Game A typical collective EPR implementation is operated based on the centralized allotment of waste volumes to the available

[1] For tractability, we assume that the return volume is independent of the product improvement considered (this is particularly relevant when recyclability attributes have negligible impact on sales, e.g., the number of screws used), and leave the more general case for future research. The known return volume assumption is relaxed in §7.3.

capacities. The allotment is often determined by a system operator (i.e., a "control tower" entity), aiming to minimize the total recycling cost (see p. 6 in (Gui et al. 2015) for a detailed description of the control tower approach). We capture this by a transportation problem on the RN shown in Figure 1(b), called the *centralized allotment problem* (C).

$$(C): \quad \min \quad Z(f) = \sum_{\pi\in\Pi}\sum_{r\in R} c_r^\pi \cdot f_{[\pi,r]} \tag{2}$$

$$s.t. \quad \sum_{r\in R} f_{[\pi,r]} = d^\pi \quad \forall \pi\in\Pi \tag{3}$$

$$\sum_{\pi\in\Pi} f_{[\pi,r]} \le k_r \quad \forall r\in R \tag{4}$$

$$f_{[\pi,r]} \ge 0 \quad \forall \pi\in\Pi, \forall r\in R. \tag{5}$$

In the above program, $f_{[\pi,r]}$ is the flow variable that represents the amount of product π alloted to processor r. Constraints (3) and (4) make sure that all return volume is processed, and the capacity at each processor is not exceeded, respectively. Let f^* denote the optimal solution to (C), which we call the *socially optimal allotment.* The minimum total recycling cost $Z(f^*)$ is then distributed among producers based on the pre-determined cost allocation mechanism. Note that $Z(f^*)$ depends on the design profile Λ of the products returned. Hence, we model the cost allocation mechanism as a set of functions of Λ, denoted by $x(\Lambda) \doteq \{x^i(\Lambda) : \forall i \in N\}$, such that $\sum_{i\in N} x^i(\Lambda) = Z(f^*)$ holds. Each function x^i represents the cost allocated to producer i, whose functional form is pre-determined but whose value is calculated ex-post based on the socially optimal allotment given the design profile. In the rest of this paper, we will suppress the functional notation and refer to the allocation mechanism as $x \doteq \{x^i\}$ when its dependence on Λ does not need to be highlighted.

Note that a cost allocation undermines producers' incentive to participate in a collective implementation if it charges them too high a cost compared to their operating independently either individually or with others in a sub-coalition. This can lead to producers' defection from the collective system individually or as sub-groups. In practice, such instability problems are observed in many collective implementations (Gui et al. 2015). The PA game described in this section provides a framework to model this dynamic. This game models the stand-alone cost of each coalition $S \subset N$ (including those consisting of a single producer) by a *characteristic function* $v(S)$. This stand-alone cost can be calculated as the minimum total cost of the centralized allotment problem restricted to the coalition S, in which the product set Π and the processor set R are replaced by those associated with the sub-coalition S, i.e., $\Pi^S \doteq \{\pi(i) : \forall i \in S\}$ and $R^S \doteq \{r(i) : \forall i \in S\}$. When S equals the set of all producers N, we call this coalition the *grand coalition*, and $v(N)$ equals the minimum total recycling cost $Z(f^*)$ of (C). In the PA game, each sub-coalition S aims to minimize its own cost. Therefore, any cost allocation to S that is at most equal to $v(S)$ will motivate its participation in the grand coalition. In game theory, this property is called group incentive compatibility.

Definition 1. Given an RN instance, a cost allocation mechanism $x(\Lambda) \doteq \{x^i(\Lambda)\}$ is defined to be *group incentive compatible* if the condition $\sum_{i \in S} x^i(\Lambda) \leq v(S)$ is satisfied for any sub-coalition $\forall S \subsetneq N$ given any product design profile Λ.

Note that given any product design profile, the set of group incentive compatible mechanisms that allocate the entire cost constitute the *core* of the PA game, which is a fundamental concept widely studied in cooperative game theory. Compared to the existing literature that characterizes core allocations, a unique feature of our work is that it focuses on how a group incentive compatible mechanism affects the equilibrium design choices in the first stage, which we model as follows.

Stage One: The Non-Cooperative Product Design Game We define an equilibrium design profile as the set of design choices under which no producer can lower its total cost (i.e., the sum of its cost allocation in the collective system plus the design investment) by unilaterally switching to another design for its own product. Mathematically, we denote this equilibrium as $\Lambda_x^{ne} \doteq \{(\lambda_x^{ne})^{\pi(i)} : \forall i \in N\}$ given a cost allocation mechanism x. To formally define the concept, we denote the set of design decisions of all producers except i under the equilibrium as $(\Lambda_x^{ne})^{-i} \doteq \{(\lambda_x^{ne})^{\pi(j)} : \forall j \in N \setminus \{i\}\}$.

Definition 2. Given an RN instance and a cost allocation mechanism x, an *equilibrium design profile* Λ_x^{ne} is defined such that the recyclability level $(\lambda_x^{ne})^{\pi(i)}$ minimizes producer i's total cost given that others adopt $(\Lambda_x^{ne})^{-i}$, i.e., $x^i\left(\Lambda_x^{ne}\right) + Q^i\left((\lambda_x^{ne})^{\pi(i)}\right) \leq x^i\left(\lambda^{\pi(i)} \cup (\Lambda_x^{ne})^{-i}\right) + Q^i\left(\lambda^{\pi(i)}\right)\ \forall \lambda^{\pi(i)}$.

In the rest of this paper, we aim to identify cost allocation mechanisms that are group incentive compatible, and under which the equilibrium design profile is (weakly) superior to the optimal design profile induced under the individual system. The main challenge with this analysis derives from the interdependence between the two stages of the biform game: The design profile affects the product allotment of the collective system and thus the cost allocation, which itself determines producers' design incentives. Such interactions can be intractable if the biform game is defined based on a general network. Here, we exploit the bipartite structure of the RN model we developed to explicitly characterize the interaction between the design choices and the cost allocation.

Using the above methodology, our analysis starts with a widely-studied class of group incentive compatible allocations, called the dual-based cost allocation, and explore if/when it can induce superior design choices (§4). After that, we propose a cost allocation that ensures design choices superior to those in an individual system, and explore when it guarantees group incentive compatibility (§5). These two studies reveal a tradeoff between design incentives and group incentive compatibility. We then develop a general cost allocation mechanism whose structure encompasses the main cost allocation approaches discussed in the literature or adopted in practice. Based on this mechanism, we formally characterize how the design-stability tradeoff is related to the features of the RN (§6). Finally, we discuss extensions of the results obtained from the main analysis (§7).

4. Design Incentives under Dual-based Cost Allocation

The product allotment game in the second stage of our biform game model is a linear programming (LP) game defined based on a network optimization problem. According to the cooperative game theory literature, in LP games, cost allocation based on duality is group incentive compatible (Owen 1975). Hence, in this section, we develop a dual-based cost allocation mechanism and analyze its design outcomes. Let $\beta^{*\pi}$ and α_r^* denote the optimal dual solutions associated with the demand and capacity constraints in the centralized allotment problem (C) (constraints (3) and (4), respectively). Intuitively, $\beta^{*\pi}$ can be interpreted as the additional cost to process another unit of product π under the socially optimal allotment f^*. Similarly, α_r^* represents the reduction in total recycling cost under f^* when an additional unit of capacity at processor r is added to the RN, i.e., the shadow price of the capacity at r. Then we define the dual-based cost allocation to each producer i as

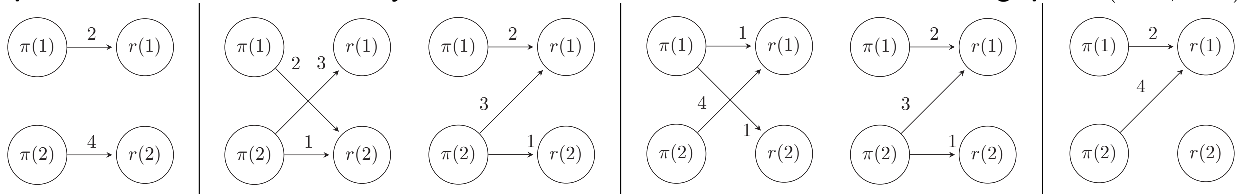
$$x_d^i = \beta^{*\pi(i)} \cdot d^{\pi(i)} - |\alpha_{r(i)}^*| \cdot k_{r(i)} \,. \tag{6}$$

This allocation charges i the dual cost $\beta^{*\pi(i)}$ for every unit of its product $\pi(i)$, and also distributes a capacity reward to i proportional to the shadow price $|\alpha_{r(i)}^*|$ of the capacity at $r(i)$; this reward reflects the value of including the capacity at $r(i)$ in the collective system. The allocation x_d recognizes the product recycling cost and capacity access differentials among producers. A natural question is whether such differentiation helps promote design incentives. We show that the answer to this question critically depends on the capacity configuration in the RN. To formally present the result, we define $\Lambda_1 \le \Lambda_2$ if $\lambda_1^{\pi(i)} \le \lambda_2^{\pi(i)}$ $\forall i \in N$, i.e., no producer adopts a worse design under Λ_1 than under Λ_2. We call Λ_1 superior to Λ_2 and Λ_2 inferior to Λ_1. In the rest of this paper, we will use "superior" and "inferior" in the weak sense unless stated otherwise.

Proposition 1. *If the RN instance satisfies $d^{\pi(i)} \ge \sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}$ $\forall i \in N$, then under the dual-based cost allocation mechanism x_d, any equilibrium design profile Λ_d^{ne} is superior to the optimal design profile induced by an individual system, i.e., $\Lambda_d^{ne} \le \Lambda_{ind}$.*

Proposition 1 provides an insight that is contrary to common intuition: a collective EPR implementation can achieve superior design improvements than an individual implementation when a particular network condition is satisfied. The condition in Proposition 1 means that the total capacity at processors that are more cost efficient than processor $r(i)$'s is less than or equal to the volume of product $\pi(i)$. This characterizes an RN structure with limited efficient capacity in the following sense: In the collective system, no producer can have its entire return volume recycled using more efficient capacity than in the individual system. Proposition 1 indicates that this network feature drives the superior design incentives under the collective system with dual-based cost allocation

Figure 3 Product allotment in the two-producer RN instances considered in Example 1. Panel (a) shows the allotment in the individual system, which is the same for all three instances. Panel (b)-(f) show the socially optimal allotment in the collective system under each instance and as a function of the design profile $(\lambda^{\pi(1)}, \lambda^{\pi(2)})$**.**



(a) $k_{r(1)} = 3$, 5 or 7 and any Λ (b) $k_{r(1)} = 3$ and $\lambda^{\pi(1)} \leq \lambda^{\pi(2)}$ (c) $k_{r(1)} = 3$ and $\lambda^{\pi(1)} > \lambda^{\pi(2)}$ (d) $k_{r(1)} = 5$ and $\lambda^{\pi(1)} \leq \lambda^{\pi(2)}$ (e) $k_{r(1)} = 5$ and $\lambda^{\pi(1)} > \lambda^{\pi(2)}$ (f) $k_{r(1)} = 7$ and any Λ

relative to that in the individual system. Intuitively, this impact is derived from the way this network feature affects the socially optimal allotment f^* and thus the associated dual solutions $\beta^{*\pi}$ and α^*_r, based on which the dual-based cost allocation is calculated.

To understand Proposition 1 in more depth, below we analyze RNs with two producers in detail. Without loss of generality, we assume that the processor $r(1)$ is more cost efficient, i.e., $\tau_{r(1)} < \tau_{r(2)}$. In this setting, the network condition identified in Proposition 1 reduces to $d^{\pi(2)} \geq k_{r(1)}$. In fact, for a two-producer RN, we can show that when this condition does not hold, both producers have inferior design incentives under the dual-based cost allocation (see Proposition 7 in Appendix A.1). The following example illustrates the underlying intuition.

Example 1. We consider three instances of an RN with two producers, where $r(1)$ has 3, 5 and 7 units of capacity, respectively. In all instances, we assume there are 2 units of product $\pi(1)$ and 4 units of product $\pi(2)$. Hence, the first instance represents the case when the condition $d^{\pi(2)} \geq k_{r(1)}$ is met. The remaining two instances illustrate the other case, and satisfy the condition $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$ and $d^{\pi(1)} + d^{\pi(2)} < k_{r(1)}$, respectively. We also assume there are 5 units of capacity at $r(2)$, and that the base recycling cost is zero for both products (i.e., $\bar{c}^{\pi(1)} = \bar{c}^{\pi(2)} = 0$ so that the unit recycling cost $c^{\pi}_r(\lambda^\pi) = \lambda^\pi \cdot \tau_r$).

The socially optimal allotment f^* in each of these RN instances is depicted in Figure 3(b)-(f). Take panel (b) as an example: When product $\pi(1)$ is more recyclable than $\pi(2)$, the optimal allotment routes $\pi(2)$ to the efficient capacity at $r(1)$ to the extent possible. Hence, in the first RN instance, $\pi(2)$ fully uses the 3 units of capacity at $r(1)$ and the remaining 1 unit of its volume is allotted to $r(2)$; the entire volume of $\pi(1)$ is sent to $r(2)$. Based on f^*, we calculate the optimal dual solution $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$. For example, in the case of panel (b), the dual cost equals $\beta^{*\pi(1)} = c^{\pi(1)}_{r(2)}$ and $\beta^{*\pi(2)} = c^{\pi(2)}_{r(2)}$, and the marginal value of the capacity equals $\alpha^*_{r(1)} = c^{\pi(2)}_{r(2)} - c^{\pi(2)}_{r(1)}$ and $\alpha^*_{r(2)} = 0$. Table 2 summarizes the producers' dual-based cost allocation in this example (see formulas (11), (14), (16), (19) and (22) in Appendix A.1 for the derivation).

Table 2 Recycling cost comparison between the individual system and the collective system with the dual-based cost allocation x_d based on the RN instances in Example 1. Each bolded expression is the rate of change in the producer's total recycling cost as a function of its design choice.

		Stand-alone cost in an individual system	Dual-based cost allocation x_d in a collective system	
			When $\lambda^{\pi(1)} \leq \lambda^{\pi(2)}$	When $\lambda^{\pi(1)} > \lambda^{\pi(2)}$
When $k_{r(1)} = 3$ ($d^{\pi(2)} \geq k_{r(1)}$)	Producer 1	$\boldsymbol{2\tau_{r(1)}} \cdot \lambda^{\pi(1)}$	$\boldsymbol{2\tau_{r(2)}} \cdot \lambda^{\pi(1)}$ $-3\left(\tau_{r(2)} - \tau_{r(1)}\right) \cdot \lambda^{\pi(2)}$	$\boldsymbol{2\tau_{r(1)}} \cdot \lambda^{\pi(1)}$ $-\left(\tau_{r(2)} - \tau_{r(1)}\right) \cdot \lambda^{\pi(2)}$
	Producer 2	$\boldsymbol{4\tau_{r(2)}} \cdot \lambda^{\pi(2)}$	$\boldsymbol{4\tau_{r(2)}} \cdot \lambda^{\pi(2)}$	
When $k_{r(1)} = 5$ ($d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$)	Producer 1	$\boldsymbol{2\tau_{r(1)}} \cdot \lambda^{\pi(1)}$	$\left(\boldsymbol{5\tau_{r(1)} - 3\tau_{r(2)}}\right) \cdot \lambda^{\pi(1)}$	$\boldsymbol{2\tau_{r(1)}} \cdot \lambda^{\pi(1)}$ $-3\left(\tau_{r(2)} - \tau_{r(1)}\right) \cdot \lambda^{\pi(2)}$
	Producer 2	$\boldsymbol{4\tau_{r(2)}} \cdot \lambda^{\pi(2)}$	$\boldsymbol{4\tau_{r(1)}} \cdot \lambda^{\pi(2)}$ $+4\left(\tau_{r(2)} - \tau_{r(1)}\right) \cdot \lambda^{\pi(1)}$	$\boldsymbol{4\tau_{r(2)}} \cdot \lambda^{\pi(2)}$
When $k_{r(1)} = 7$ ($d^{\pi(1)} + d^{\pi(2)} < k_{r(1)}$)	Producer 1	$\boldsymbol{2\tau_{r(1)}} \cdot \lambda^{\pi(1)}$	$\boldsymbol{2\tau_{r(1)}} \cdot \lambda^{\pi(1)}$	
	Producer 2	$\boldsymbol{4\tau_{r(2)}} \cdot \lambda^{\pi(2)}$	$\boldsymbol{4\tau_{r(1)}} \cdot \lambda^{\pi(2)}$	

We perform the equilibrium design analysis (see the proof of Proposition 7 in Appendix A.1), and conclude that in this example, the dual-based cost allocation leads to superior design incentives in the first RN instance (with $k_{r(1)} = 3$, presented in Figure 3 panel b), and inferior design incentives in the other two. The key to this result is how the rate at which the dual-based cost allocation x_d changes in design compares to the rate at which producers' stand-alone costs change in design. In Table 2, these rates are highlighted in bold. It can be observed that it is only in the first RN instance where the rate of change in design is higher under x_d (recall that we assume $\tau_{r(2)} > \tau_{r(1)}$), i.e., $\frac{\partial x_d^i}{\partial \lambda^{\pi(i)}} \geq \frac{\partial\left(c_{r(i)}^{\pi(i)} \cdot d^{\pi(i)}\right)}{\partial \lambda^{\pi(i)}} = d^{\pi(i)} \cdot \tau_{r(i)}$ $i = 1, 2$. To see why this occurs, we first calculate based on formula (6) that $\frac{\partial x_d^i}{\partial \lambda^{\pi(i)}} = \frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} - \frac{\partial |\alpha_{r(i)}^*|}{\partial \lambda^{\pi(i)}} \cdot k_{r(i)}$. This points us to the following two network effects associated with $\beta^{*\pi}$ and α_r^* respectively.

Network effect 1: In the first RN instance, a unit decrease in $\lambda^{\pi(i)}$ reduces the dual cost $\beta^{*\pi(i)}$ no less (and strictly more for producer 2) than it does for the unit recycling cost that producer i incurs operating alone, i.e., $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \geq \frac{\partial c_{r(i)}^{\pi(i)}}{\partial \lambda^{\pi(i)}}$ $i = 1, 2$. This happens because in this RN instance, an additional unit of $\pi(i)$ will be recycled at the same or less cost efficient processor than $r(i)$ under the socially optimal allotment. It can be further observed from Figure 3 that this is not the case in the other two instances. Our analysis proves that it is the network condition $d^{\pi(2)} \geq k_{r(1)}$ that plays the deciding role.

Network effect 2: We further observe that in the first and the third RN instances, the shadow price $|\alpha_{r(i)}^*|$ is independent of producer i's design choice, i.e., $\frac{\partial \alpha_{\pi(i)}^*}{\partial \lambda^{\pi(i)}} = 0$, because under the socially optimal allotment, an additional unit of capacity at $r(i)$ will not be used to recycle $\pi(i)$. Hence, the capacity reward part of the dual-based allocation mechanism does not affect the design outcome. However, in the second RN instance, it may be the case that $\frac{\partial \alpha^*_{\pi(1)}}{\partial \lambda^{\pi(1)}} < 0$, indicating an unfavorable situation where producer 1 receives a smaller capacity reward due to its own design improvement and thus its design incentives are weakened. This happens because an additional unit of capacity at $r(1)$ will be used to recycle producer 1's own product $\pi(1)$ (see Figure 3(d)). Thus $r(1)$'s marginal value is calculated as the unit cost saving when $\pi(1)$ is rerouted from $r(2)$ to $r(1)$, which decreases as $\pi(1)$ becomes more recyclable. We can show that which of the above two cases occurs crucially depends on the network condition $d^{\pi(2)} \geq k_{r(1)}$.

To summarize, in this example, the key effect of the network condition $d^{\pi(2)} \geq k_{r(1)}$ is that under the socially optimal allotment, (i) an additional unit of each product is recycled at a processor of the same or lower efficiency compared to that in the individual system, and (ii) an additional unit of capacity at $r(i)$ is not used to process i's own product. These allotment features are reflected in the dual solution, and lead to superior design incentives under the dual-based cost allocation. We also note that in this two-producer RN, the total recycling cost is strictly lower in the collective system than in an individual system when $\pi(2)$ can be recycled at the more efficient processor $r(1)$. This is how synergy is derived from capacity sharing. Hence, limited capacity at $r(1)$ relative to the volume of $\pi(2)$ constrains such synergy potential of a collective system with two producers. Thus we refer to $d^{\pi(2)} \geq k_{r(1)}$ as a "low-synergy" condition on the capacity configuration of the RN.

The proof of Proposition 1 indicates that the above intuition can be generalized to RNs with any number of producers (see Appendix A.1.2). The key observation is that the network condition $d^{\pi(i)} \geq \sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}$ $\forall i \in N$ naturally extends the "low-synergy" capacity configuration characterized in Example 1. That is, the volume of any product $\pi(i)$ can saturate all capacity that is more efficient than $r(i)$. Via similar network effects as characterized in Example 1, this condition ensures superior design incentives, and we call RNs that satisfy such a condition *design reinforcing*.

Finally, we note that the dual-based cost allocation has been proposed in the literature to guarantee group incentive compatibility. In this section, we have shown that under certain conditions, this cost allocation is also effective at design improvement. In the next section, we take the complementary angle, i.e., we develop an allocation mechanism particularly targeted at enhancing design incentives, and then analyze its group incentive compatibility.

5. A Marginal Contribution Based Cost Allocation

In Section 4, we observe that the dual-based cost allocation x_d provides superior design incentives compared to the individual system when the cost allocated to each producer exhibits a higher rate of change in design than that producer's stand-alone cost (i.e., its cost in the individual system) does. This provides us with a direction in constructing an allocation mechanism that guarantees superior design incentives in any RN. That is, for each producer i, we consider the sub-coalition of producers with less efficient capacities than $r(i)$'s, i.e., $\{j : \tau_{r(j)} > \tau_{r(i)}\}$, and propose to charge producer i the additional recycling cost incurred when i joins this sub-coalition, denoted as

$$x_m^i = v(i \cup \{j : \tau_{r(j)} > \tau_{r(i)}\}) - v(\{j : \tau_{r(j)} > \tau_{r(i)}\}), \quad \forall i \in N. \tag{7}$$

The allocation x_m leads to a superior design outcome in RNs with two producers. To explain, we consider a two-producer RN where $r(1)$ is more cost efficient. Then we can calculate that $x_m^1 = v(\{1,2\}) - v(2)$ and $x_m^2 = v(2)$. That is, producer 2 is allocated its stand-alone recycling cost, and the remainder of the total cost in the collective system is allocated to producer 1. It is obvious that producer 2 will experience the same level of design incentives under x_m as in an individual system. As for producer 1, we observe that the rate at which x_m^1 changes with respect to its design choice $\lambda^{\pi(1)}$ is the same as that of $v(N)$ (since $v(2)$ is independent of $\lambda^{\pi(1)}$). This rate of change is at least as large as that of producer 1's stand-alone recycling cost, because in the collective system, producer 1's product $\pi(1)$ may be processed at the less efficient processor $r(2)$, where an improved design of $\pi(1)$ leads to higher cost savings. This results in superior design incentives for producer 1 under the allocation x_m.

This discussion confirms our intuition about the design advantage of x_m: After participating in the sub-coalition $\{j : \tau_{r(j)} > \tau_{r(i)}\}$, producer i may have its product processed at a less efficient processor compared to $r(i)$. Thus producer i may save more in recycling cost from its design effort, which is reflected in its cost allocation under the mechanism x_m. We formalize this insight in the next proposition for the general case.

PROPOSITION 2. *Given any RN instance, the equilibrium design profile Λ_m^{ne} under x_m is superior to the optimal design profile induced by an individual system, i.e., $\Lambda_m^{ne} \leq \Lambda_{ind}$.*

Note that the mechanism x_m is an application of the widely-studied *marginal contribution* concept in cooperative game theory. By definition, one's marginal contribution to a coalition in a cost game is the additional cost incurred after one joins this coalition. Based on this concept, a number of allocation mechanisms have been discussed in the literature. The most prominent example is the Shapley value, which is defined as the average of one's marginal contributions to all sub-coalitions (Shapley 1952). The novelty of the allocation x_m proposed in this paper lies in the choice of the sub-coalition with respect to which the marginal contribution of each producer i is measured in order to achieve superior design incentives, i.e., we choose the sub-coalition of producers with access to less efficient capacity, where, by our modeling assumption, design improvements lead to higher cost savings than at $r(i)$. In the rest of the paper, we refer to x_m as the marginal contribution based cost allocation.

Group Incentive Compatibility of the Marginal Contribution Based Allocation. We now turn to the stability of the mechanism x_m. We first show that x_m guarantees that the cost allocated to each individual producer is no higher than its stand-alone recycling cost (i.e., $x^i \leq v(i)$ $\forall i$), a property referred to as *individual rationality* in game theory. The proof shares the same spirit with a classic result in the literature that in a cost minimization setting, the Shapley value is individually rational in any sub-additive game, i.e., a game in which $v(S \cup T) \leq v(S) + v(T)$ is satisfied for any two mutually exclusive sub-coalitions S and T (Shapley 1953). In the problem considered in this paper, we can show that the PA game is sub-additive as the synergy in merging two independent RNs leads to a cost reduction. Hence, the following proposition holds.

PROPOSITION 3. *The cost allocation x_m is individually rational in any RN instance.*

However, simple examples indicate that the allocation x_m may not be group incentive compatible. That is, it may charge higher costs to producer sub-coalitions than their stand-alone costs. In fact, this is a well-known problem for the Shapley value as well, and has been extensively discussed in the cooperative game theory literature (e.g., Shapley 1971, Maschler et al. 1971, Greenberg 1985, Branzei et al. 2003). A major result in this stream of research is that the Shapley value is group incentive compatible if the cooperative game is *convex*. Our analysis indicates that a parallel result can be developed for the cost allocation x_m, which requires a weaker condition. Intuitively, a game is defined to be convex if a larger coalition enjoys more benefits from the participation of the same player. In the cost minimization setting, this condition can be written as follows: For any sub-coalition $T \subseteq N$,

$$v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S) \quad \forall S \subset T \quad \forall i \notin T\,. \tag{8}$$

We propose a *weak convexity* condition that only requires (8) to hold for sub-coalitions T with less cost efficient capacities than i's own.

DEFINITION 3. A PA game (v, N) is defined to be *weakly convex* if $\forall i \in N$ and $\forall S \subset T \subseteq \{j : \tau_{r(j)} > \tau_{r(i)}\}$, the inequality $v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S)$ holds.

Our next result shows that weak convexity guarantees group incentive compatibility of the cost allocation x_m.

LEMMA 1. *Given any RN instance, the allocation mechanism x_m is group incentive compatible if the PA game is weakly convex under any product design profile Λ.*

Lemma 1 is intuitive: Each x_m^i equals producer i's marginal contribution to the sub-coalition with less efficient capacities than $r(i)$'s, hence it is natural that a convexity condition defined based on these sub-coalitions ensures group incentive compatibility of x_m. Combining Proposition 2 and

Lemma 1, we conclude that given an RN instance for which the PA game is weakly convex, the allocation x_m ensures both superior design incentives and group incentive compatibility. A natural question that follows is what network feature of the RN contributes to the weak convexity of the PA game. It turns out that the design-reinforcing property discussed in §4 plays the key role.

PROPOSITION 4. *If the RN instance satisfies $d^{\pi(i)} \geq \sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}$ $\forall i > 2$, then the PA game is weakly convex under any product design profile Λ, and the allocation mechanism x_m is group incentive compatible.*

To intuitively understand the connection between the design-reinforcing property of the RN and the weak convexity of the PA game, note that according to Definition 3, weak convexity indicates that cost efficient capacity can be better utilized in larger coalitions and lead to higher cost reduction. This occurs when the efficient capacity is limited so that its value monotonically increases as the size of the coalition increases. Proposition 4 indicates that the limited availability of efficient capacity characterized by the low-synergy capacity configuration is sufficient to ensure weak convexity.

Table 3 The tradeoff between design incentives and coalitional stability under the allocations x_d and x_m.

	Induce superior design incentives	Group incentive compatibility
x_d	In design-reinforcing RNs (*Proposition 1*)	Always (*known result in literature Owen 1975*)
x_m	Always (*Proposition 2*)	In design-reinforcing RNs (*Proposition 4*)

Summarizing the results obtained so far (Table 3), we find that the same RN property, i.e., the design reinforcing property, ensures superior design incentives under the dual-based allocation x_d and group incentive compatibility of the marginal contribution based allocation x_m. Moreover, counter-examples show that when the condition does not hold, either the design outcome or the group incentive compatibility can be undermined. These results motivate us to question whether there exists a fundamental tradeoff between design incentives and group incentive compatibility that cannot be resolved by cost allocation unless the network is "design reinforcing". We study this issue in the next subsection and show this is indeed the case.

6. The Design-Stability Tradeoff under General Cost Allocation Mechanisms

In this subsection, we analyze the existence of a general cost allocation under which a collective implementation can achieve both superior design incentives (compared to an individual system) and coalitional stability. To this end, we first observe that many prevalent cost allocation models considered in practice and theory share a common functional form when applied to the setting considered in this paper. That is, they are all piecewise linear functions of producers' design choices. Examples include the dual-based allocation x_d and the allocations based on the concept of marginal contribution (e.g., x_m and the Shapley value). Proportional cost allocation mechanisms that have been widely adopted in practice (such as the one based on return share) are also piecewise linear in the design variables. Please refer to Appendix A.3.1 for a detailed discussion.

Hence, in this subsection, we define and consider a general class of allocation mechanisms, which we call *linear design-based* cost allocations, denoted by x_l. Specifically, for each producer i, let $\{h_1^i, h_2^i, ..., h_U^i\}$ be a finite set of numbers independent of i's design variable $\lambda^{\pi(i)}$. They divide the space $(0,\infty)$ into $U+1$ intervals that we denote as I_1, I_2,... I_{U+1}. Then the linear design-based cost allocation to each producer i is calculated as

$$x_l^i = \begin{cases} a_1^i \cdot \lambda^{\pi(i)} + b_1^i & \text{if } \lambda^{\pi(i)} \in I_1 \doteq (0, h_1^i] \\ a_u^i \cdot \lambda^{\pi(i)} + b_u^i & \text{if } \lambda^{\pi(i)} \in I_u \doteq (h_{u-1}^i, h_u^i] \quad \forall u = 2,3,...,T \\ a_U^i \cdot \lambda^{\pi(i)} + b_U^i & \text{if } \lambda^{\pi(i)} \in I_{U+1} \doteq (h_U^i, \infty) \end{cases} \tag{9}$$

In the above formula, the coefficients in each linear piece, a_u^i and b_u^i, are independent of $\lambda^{\pi(i)}$. We can interpret the part $a_u^i \cdot \lambda^{\pi(i)}$ as the recycling cost burden due to producer i's own design choice, where a_u^i measures the rate of change of the allocation with respect to design improvements. We call a_u^i the design sensitivity parameter. The part b_u^i represents a side payment to i independent of i's design choice, e.g., one that reflects the cost reduction in processing others' products due to i's capacity contribution.

We search within this general class of linear design-based allocation mechanisms for one that (i) guarantees superior design incentives relative to an individual system, and (ii) is group incentive compatible. That is, we study how to tailor the design sensitivity parameter a_u^i and the side payment parameter b_u^i to meet these two goals. Our previous analysis of the mechanisms x_d and x_m indicates that to do so can be challenging. Another concrete example of linear design-based allocations are proportional allocations such as return share. Return share performs even worse: It leads to inferior design incentives for at least one producer in any RN (see Appendix A.3 for details) and is not group incentive compatible except in entirely homogeneous RNs (Gui et al. 2015). The next theorem shows that it is infeasible to achieve both superior design and group incentive compatibility under a general linear design-based allocation except in an RN with the design-reinforcing property.

Theorem 1. *Given any RN instance, the following three statements are equivalent.*

1. *The RN instance satisfies* $d^{\pi(i)} \geq \sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}$ $\forall i > 2$.
2. *The PA game is weakly convex under any product design profile* Λ.
3. *There exists a linear design-based allocation mechanism* x_l *such that (i)* x_l *is group incentive compatible, and (ii) any equilibrium design profile under* x_l *is superior to the optimal design profile induced by an individual system, i.e.,* $\Lambda_l^{ne} \leq \Lambda_{ind}$.

Theorem 1 shows that the design reinforcing network structure is a necessary and sufficient condition for the existence of a group incentive compatible cost allocation that also provides superior design incentives compared to that in an individual system. This clearly indicates potential incompatibility between improved design incentives and a coalitionally stable collective system. Whether this incompatibility can be resolved is determined by the capacity configuration of the RN infrastructure.

Synopsis: The proof of Theorem 1 enables us to better understand the cause of the design-stability tradeoff. Under the general linear design-based allocation model x_l, there are two levers, the design sensitivity parameter a_u^i and the side payment parameter b_u^i, that can be used to simultaneously achieve superior design incentives and group incentive compatibility. To do so, we ideally need a large a_u^i value to ensure a high rate of change of producer i's cost allocation with respect to its own design choice, giving i a strong design incentive. At the same time, i's participation in the collective system can only be guaranteed if compensated by a large side payment (i.e., adopting a small b_u^i parameter) that sufficiently reduces its net cost allocation. However, this may be infeasible when all producers' design incentives and all sub-coalitions' participation incentives are considered jointly. The challenge is that a unit increase in a_u^i also increases i's cost allocation by $\lambda^{\pi(i)}$, which may not be offset by increasing the side payment under any design profile, since the side payment parameter b_u^i is independent of $\lambda^{\pi(i)}$. In that case, ensuring group incentive compatibility may require a sufficiently small a_u^i, which undermines design incentives. We show whether this problem occurs depends on the weak convexity of the PA game (Corollary 1 in Appendix A.3). The insight from the proof of this result is that the convexity condition implies larger coalitions have stronger synergy such that group incentive compatibility imposes less stringent conditions on the two levers (a_u^i and b_u^i), which can then be tailored separately to meet the design and the stability criteria, respectively.

The design-stability tradeoff characterized by Theorem 1 constitutes a major finding of this paper. In particular, combining this result with our previous observations in Table 3, we show that, unless individual rationality is sufficient to maintain stability (in which case the cost allocation mechanism x_m introduced in §5 can be implemented to ensure superior design outcomes), collective implementations will often induce inferior design incentives compared to individual systems. In these cases, collective implementations primarily act as enablers of cost efficiency at the expense of design improvement. Yet there are also recycling infrastructures where design improvement and collective system stability do go hand in hand, if the cost allocation is designed appropriately. Thus, the point of view that cost allocation adjustments can restore design incentives in collective systems (Sander et al. 2007, IPR Working Group 2012, Dempsey et al. 2010) does have validity, but is not uniformly true.

7. Extensions

In this section, we enrich the main model and provide three extensions of the main analysis. We demonstrate that the tradeoff between design incentives and coalitional stability persists and continues to be driven by refined variations of the design-reinforcing property of the RN.

7.1. Extension in Product-Process Technology Interaction

Our main analysis focuses on the case where design improvements lead to higher savings in unit recycling cost at less efficient processors. This extension considers the opposite case, that is, it is at the *more* efficient processors where the cost savings in response to design improvements are higher. For example, using higher value materials would bring more value at more automated processors using advanced shredding and material separation technologies.

The key finding in this extension is that in this case, a design-reinforcing RN is one where the capacity configuration generates high synergy from capacity sharing. The technical challenge in analyzing this case is that, due to the change in the product-process interaction, the multiplicative function no longer naturally models the unit recycling cost. Hence, our analysis in this subsection is based on the following assumption of the unit recycling cost c_π^r that captures the type of product-process interaction considered here.

Assumption 2. *Let c_π^r be a convex increasing function of the design variable λ^π. We assume that for each product π, within a sufficiently large interval (a,b) of the λ^π variable, $c_\pi^{r(i)} < c_\pi^{r(j)}$ implies $\frac{\partial c_\pi^{r(i)}}{\partial \lambda^\pi} > \frac{\partial c_\pi^{r(j)}}{\partial \lambda^\pi}$, i.e., the unit cost to recycle product π at the more cost efficient processor $\pi(i)$ changes at a faster rate with respect to the design variable λ^π.*

Based on this model, we derive a set of results that parallel those obtained in the main analysis.

Proposition 5. *Given any RN instance where the unit recycling cost is modeled as described in Assumption 2, the following statements hold.*

1. *Under the dual-based cost allocation x_d, which is group incentive compatible, if the RN instance satisfies $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \leq k_{r(i)}\ \forall i \in N$, then any equilibrium design profile (if it exists) is superior to the optimal design profile induced by an individual system, i.e., $\Lambda_d^{ne} \leq \Lambda_{ind}$.*

2. *Consider a cost allocation mechanism* $\bar{x}_m$ *where* $\bar{x}_m^i = v(i \cup \{j : \tau_{r(j)} < \tau_{r(i)}\}) - v(\{j : \tau_{r(j)} < \tau_{r(i)}\})$ $\forall i \in N$. *Then any equilibrium design profile under* $\bar{x}_m$ *is superior to the optimal design profile induced by an individual system, i.e.,* $\Lambda_{\bar{m}}^{ne} \leq \Lambda_{ind}$. *Moreover,* $\bar{x}_m$ *is guaranteed to be individually rational.*

The first result in Proposition 5 parallels Proposition 1 in §4. It indicates that when design improvements lead to higher cost savings at more efficient processors, a design-reinforcing RN structure requires ample efficient capacity such that high synergy can be derived under capacity sharing. To see this, note that the condition $\sum_{j:\tau_{r(j)} > \tau_{r(i)}} d^{\pi(j)} \leq k_{r(i)}$ indicates that any processor $r(i)$ has at least as much capacity as the return volume of all producers who have access to less efficient capacity than $r(i)$ in the individual system. In other words, $r(i)$ can cover all products from which a cost reduction can be generated if being re-routed to $r(i)$ in the collective system. This is the opposite of the low-synergy condition identified in Proposition 1. Yet at a high level, Proposition 5 and Proposition 1 share the same spirit, that is, an RN is design-reinforcing if the majority of the capacity is at processors where design improvements lead to high cost savings.

The second result parallels Propositions 2 and 3 in §5. It shows that cost allocation based on marginal contribution can lead to superior design incentives and ensure individual rationality in the setting of this extension. Note that $\bar{x}_m$ is defined differently from the marginal contribution based allocation x_m proposed in §5: Each $\bar{x}_m^i$ equals producer i's marginal contribution to the sub-coalition with *more* efficient capacity rather than to the sub-coalition with less efficient capacity under x_m. This is due to the change in the product-process interaction, i.e., design improvements lead to higher cost savings at more efficient processors in this extension rather than at less efficient ones in the main analysis. Combining the above observations, we conclude that the infrastructure features of the RN and the type of product-process technology interaction jointly determine the design outcome and stability in a collective implementation.

7.2. Extension to Product-differentiated Processor Efficiency

In practice, the recycling efficiency at a given processor may be differentiated by product. This can happen when the recycling technology required for different products varies, and processors specialize in processing certain product types. For example, some processors have advanced CRT TV/monitor recycling technology, especially in leaded glass recycling that accounts for a significant portion of the end-of-life treatment cost of TVs.

To capture this heterogeneity, we consider a more general version of the multiplicative model of the unit recycling cost considered in the main analysis. That is, in this subsection, we allow the efficiency measure of a processor r to depend on the product π and denote it as τ_r^π. Thus unit recycling cost c_r^π equals

$$c_r^\pi = \lambda^\pi \cdot \tau_r^\pi + \bar{c}^\pi \quad \forall \pi \in \Pi \quad \forall r \in R. \tag{10}$$

Let $J^i \doteq \{j : \tau^{\pi(i)}_{r(j)} < \tau^{\pi(i)}_{r(i)}\}$ denote the set of producers whose capacity is more efficient than i's own when processing product $\pi(i)$.

Proposition 6. *Given any RN instance where the unit recycling cost is modeled as in (10), if there exists a linear design-based allocation x_l that is group incentive compatible and ensures a superior equilibrium design profile compared to an individual system, i.e., $\Lambda^{ne}_l \leq \Lambda_{ind}$, then the RN instance satisfies $d^{\pi(i)} \geq \sum_{j \in J^i} k_{r(j)}$ $\forall i \in N$ for which $|J^i| \geq 2$.*

Note that the set J^i, which is product dependent here, reduces to $\{j : j > i\}$ when the recycling efficiency at a given processor is not differentiated by product, as in the main analysis. Hence, the condition identified above generalizes the low-synergy one proposed in Theorem 1 to the case with product-dependent heterogeneity in processing efficiency. We conclude that the design-reinforcing characterization derived in the main analysis can be extended to this more general setting.

7.3. Extension to Return Volume Uncertainty

In this extension, we analyze the impact of return volume uncertainty. To do so, we model each volume parameter $d^{\pi(i)}$ as an independent random variable whose probability distribution is known to the producers. We refer to an RN with these probabilistic parameters as an RN instance under return volume uncertainty. Accordingly, we generalize the definition of design outcomes under EPR as follows: The optimal design profile induced by an individual system is the design profile that minimizes producers' expected recycling costs plus their design investments. The equilibrium design profile in a collective implementation is a design profile such that no producer can reduce the sum of its expected recycling cost and its design investment by unilateral deviation.

We show that given an RN instance under return volume uncertainty, whether a group incentive compatible allocation leads to superior design incentives for a producer critically hinges on the probability of the RN realization being design-reinforcing. To see this in the context of the two-producer setting, recall that superior design incentives (under no return volume uncertainty) require $d^{\pi(2)} \geq k_{r(1)}$.

This condition can be relaxed and still ensure superior design incentives for producer 1 under return volume uncertainty. That is, $d^{\pi(2)}$ can take a value smaller than $k_{r(1)}$ as long as the associated probability is sufficiently low. To see this, note that in the deterministic case, the rate of change of producer 1's dual-based cost allocation can be strictly higher than the rate of change of its stand-alone cost under a design-reinforcing RN (e.g., in the first RN instance considered in Example 1; see Table 2). Hence, under return volume uncertainty, whether producer 1's expected cost allocation changes at a faster or slower rate in design depends on the relative probability of a design-reinforcing realization of the RN versus a non-design-reinforcing realization. In fact, for a two-producer RN instance, we can show that a sufficient condition to ensure superior design incentives for producer

1 under the dual-based cost allocation is $\frac{Pr(d^{\pi(2)}<k_{r(1)})}{Pr(d^{\pi(2)}\geq k_{r(1)})} \leq \frac{E(d^{\pi(1)})}{k_{r(1)}}$, where $E(d^{\pi(1)})$ is the expectation of $\pi(1)$'s return volume (see Appendix A.4.1 for the derivation).

The above discussion indicates that uncertainty in RN parameters may increase the likelihood of some producers choosing a superior design in a collective implementation compared to under an individual system. Nevertheless, in general, if superior design incentives are required for all producers in conjunction with group incentive compatibility, then we need the condition that for all $i > 2$, the sample space of the random variable $d^{\pi(i)}$ is contained in the interval $[\sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}, \infty)$ (see Proposition 8 in Appendix A.4.1). This generalizes Theorem 1, implying that the fundamental insights regarding the design-stability tradeoff continue to apply under RN uncertainty.

8. Conclusions

Providing design incentives is a major goal of extended producer responsibility. Yet whether EPR is as well suited as proponents have claimed remains an open question both in practice and in theory. At the center of the debate is whether a collective implementation of EPR, which is widely adopted for its cost efficiency, can provide strong design-for-recyclability incentives, especially compared to an individual system benchmark where every producer directly reaps the benefits of its design improving investments. Recognizing that the above problem is closely related to the way a collective implementation is operated, this paper proposes a conceptual framework with two novel features: (i) it is network-based and thus captures the effect of capacity sharing in a collective system; (ii) it offers a biform game formulation of the interplay between product design and cost allocation used in a collective implementation. By a rigorous analysis of the proposed framework, this paper brings clarity to the design implications collective EPR.

We show that the capacity of and demand for different processing technologies in a recycling infrastructure should be a key input for measuring the design effectiveness of collective EPR implementations. This is because of two key factors: First, different recycling technologies determine the recycling cost reduction in response to design improvements in different ways. In particular, high processing efficiency may weaken or strengthen the cost savings due to a better product design. Examples of the two cases are when products are designed to facilitate manual dismantling and are designed with higher value materials, respectively. Second, under a collective implementation, the product-process technology interaction is further complicated by network effects, i.e., synergy derived from sharing the capacity of different recycling technologies, which is determined by the capacity configuration of the RN. One of our main contributions is to show that these two factors combined leads to a characterization of RNs that are design-reinforcing. Fundamentally, if design improvements lead to higher cost savings at less efficient processors, an RN is design-reinforcing if efficient capacity is limited and only low network synergy can be derived. Otherwise, a high-synergy capacity configuration is required. We show different refinements of this key characterization of the design-reinforcing property under different operating complexities.

We prove that if the RN is design-reinforcing, then the design-stability tradeoff can be resolved, and a win-win outcome can be attained. This can be achieved by an intuitive design-based cost allocation mechanism which charges producers according to the recyclability levels of their products and distributes side-payments. The key insight from this characterization is that in order for a cost allocation to be design-effective, it needs to properly capture how the available processing technologies respond to design improvements in the collective RN (e.g., through either the dual cost associated with the optimal product allotment or producers' marginal contributions). On the other hand, if the RN is not design-reinforcing, a collective implementation has to compromise between coalitional stability and design improvement; that is, there is a fundamental conflict between design and stability that cannot be overcome. Nevertheless, we show that superior design incentives can be guaranteed in a collective system via an individually rational cost allocation. However, if individual rationality is not sufficient to maintain the stability of a collective system, one has to accept collective EPR as an enabler of cost-efficiency at the expense of design for recyclability, and find means other than cost allocation to provide such incentives. We further show that the above insights apply when additional operational complexities are taken into account.

From a practical point of view, our results imply that a win-win EPR implementation solution requires matching the right type of product technology and network configuration, and especially understanding what a design-reinforcing collective RN structure is, as it relates to the technology and capacity availability of service provision in the EPR marketplace. In particular, these infrastructural properties are expected to evolve over time, especially as recycling technologies advance and service provision in the EPR industry matures and stabilizes. Hence, it is important that policy makers as well as lobbying groups associated with producers and NGOs realize the role of this operational or infrastructure related effect on the efficiency of EPR implementations, and set appropriate goals regarding design incentives accordingly. In particular, our results indicate that collective implementations are likely to enhance design incentives for product attributes that lead to high cost savings under the prevalent recycling technology. The more general message is that it is critical to understand the tensions between different policy objectives, and how they interact with the operational level implementation decisions in order to make informed policy decisions. Failing to account for the operational characteristics of collective implementations may lead to unintended and ineffective implementation outcomes.

Acknowledgments

This research was supported by the National Science Foundation [NSF-DMI Grant 1031167].

References

Agrawal, Vishal V, Sezer Ülkü. 2012. The role of modular upgradability as a green design strategy. *Manufacturing & Service Operations Management* **15**(4) 640–648.

Alev, Isil, Vishal Agrawal, Atalay Atasu. 2014. Extended producer responsibility, secondary markets, and export restrictions. *Georgetown McDonough School of Business Research Paper* (2477943).

Anupindi, Ravi, Yehuda Bassok, Eitan Zemel. 2001. A general framework for the study of decentralized distribution systems. *Manufacturing & Service Operations Management* **3**(4) 349–368.

Atasu, A., R. Subramanian. 2012. Extended producer responsibility for e-waste: Individual or collective producer responsibility? *Production and Operations Management* **21**(6) 1042–1059.

Atasu, A., L.N. Van Wassenhove. 2012. An operations perspective on product take-back legislation for e-waste: Theory, practice and research needs. *Forthcoming, Production and Operations Management* .

Branzei, Rodica, Dinko Dimitrov, Stef Tijs. 2003. Convex fuzzy games and participation monotonic allocation schemes. *Fuzzy sets and systems* **139**(2) 267–281.

Chen, Chialin. 2001. Design for the environment: a quality-based model for green product development. *Management Science* **47**(2) 250–263.

Chod, Jiri, Nils Rudi. 2006. Strategic investments, trading, and pricing under forecast updating. *Management Science* **52**(12) 1913–1929.

Dempsey, M., C. Van Rossem, R. Lifset, J. Linnell, J. Gregory, A. Atasu, J. Perry, A. Sverkman, L.N. Van Wassenhove, M. Therkelsen, et al. 2010. Individual producer responsibility: A review of practical approaches to implementing individual producer responsibility for the WEEE Directive.

Department for Business Innovation & Skills. 2013. Implementation of the WEEE recast directive 2012/19/EU and changes to the UK WEEE system: consultation. `https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/190207/bis-13-762-implementation-of-weee-recast-directive-and-changes-to-the-uk-weee-system-consultation.pdf`.

Drake, D., P. R. Kleindorfer, L. N. Van Wassenhove. 2015. Technology choice and capacity portfolios under emissions regulation .

Environmental Leader. 2014. E-waste to exceed 93.5 million tons annually. `http://www.environmentalleader.com/2014/02/24/e-waste-to-exceed-93-5-million-tons-annually/`.

Esenduran, G., E. Kemahlıoğlu-Ziya. 2015. A comparison of product take-back compliance schemes. *Production and Operations Management* **24**(1) 71–88.

European Recycling Platform. 2012. The european recycling platform. `http://www.erp-recycling.org/index.php?content=12`.

Fleischmann, M., P. Beullens, J. Bloemhof-Ruwaard, L.N. Van Wassenhove. 2001. The impact of product recovery on logistics network design. *Production and Operations Management* **10** 156–173.

Fullerton, D., W. Wu. 1998. Policies for green design. *Journal of Environmental Economics and Management* **36** 131–148.

Granot, D., F. Granot, G. Sošic, H. Cui. 2014. Allocation of greenhouse gas emissions in supply chains. Tech. rep., Working paper, University of British Columbia.

Granot, Daniel, Greys Sošić. 2003. A three-stage model for a decentralized distribution system of retailers. *Operations Research* **51**(5) 771–784.

Greenberg, Joseph. 1985. Cores of convex games without side payments. *Mathematics of operations research* **10**(3) 523–525.

Greenpeace. 2007. Joint statement by a group of industry and ngos on producer responsibility for Waste Electrical and Electronic Equipment. `http://www.greenpeace.org/international/PageFiles/24472/joint-statement-by-a-group-of.pdf`, Accessed April 2015.

Gui, L., A. Atasu, Ö. Ergun, B. Toktay. 2015. Efficient implementation of collective extended producer responsibility legislation. *Management Science*, Forthcoming.

Immorlica, Nicole, Mohammad Mahdian, Vahab S Mirrokni. 2008. Limitations of cross-monotonic cost-sharing schemes. *ACM Transactions on Algorithms (TALG)* **4**(2) 24.

IPR Working Group. 2012. Waste electrical and electronic equipment (WEEE) regulations: Individual producer responsibility (IPR) in a UK context. `http://www.bis.gov.uk/assets/biscore/business-sectors/docs/w/12-1007-waste-electrical-and-electronic-weee-regulations-individual-producer-ipr-responsibility`.

IPR Works. 2012. `http://iprworks.org/`. Last accessed Nov 2012.

Islegen, O., Erica L. Plambeck, T. Taylor. 2015. Variability in emissions cost: Implications for facility location, production and trade .

Kalimo, H., R. Lifset, C. van Rossem, L. van Wassenhove, A. Atasu, K. Mayers. 2012. Greening the economy through design incentives: Allocating extended producer responsibility. *European Energy and Environmental Law Review* **21** 274–305.

Lifset, R., T. Lindhqvist. 2008. Producer responsibility at a turning point? *Journal of Industrial Ecology* **12**(2) 144–147.

Maine Revised Statutes. 2007. Electronic waste. `http://www.mainelegislature.org/legis/statutes/38/title38sec1610.html`. Last accessed October 2014.

Maschler, M., B. Peleg, L. S. Shapley. 1971. The kernel and bargaining set for convex games. *International Journal of Game Theory* **1**(1) 73–93.

Mayers, K., R. Lifset, K. Bodenhoefer, L.N. Van Wassenhove. 2013. Implementing individual producer responsibility for Waste Electrical and Electronic Equipment through improved financing. *Journal of Industrial Ecology* **17** 186198.

Moulin, Hervé. 1999. Incremental cost sharing: Characterization by coalition strategy-proofness. *Social Choice and Welfare* **16**(2) 279–320.

Nagurney, A., F. Toyasaki. 2005. Reverse supply chain management and electronic waste recycling: A multitiered network equilibrium framework for e-cycling. *Transportation Research Part E* **41** 1–28.

Owen, G. 1975. On the core of linear production games. *Mathematical Programming* **9**(3) 358–370.

Plambeck, E., Q. Wang. 2009. Effects of e-waste regulation on new product introduction. *Management Science* **55**(3) 333–347.

Plambeck, Erica L, Terry A Taylor. 2005. Sell the plant? the impact of contract manufacturing on innovation, capacity, and profitability. *Management Science* **51**(1) 133–150.

Raz, G., A. Ovchinnikov. 2015. Coordinating pricing and supply of public interest goods using government rebates and subsidies. *Engineering Management, IEEE Transactions on* **62**(1) 65–79.

Roughgarden, Tim, Mukund Sundararajan. 2006. New trade-offs in cost-sharing mechanisms. *Proceedings of the thirty-eighth annual ACM symposium on Theory of computing*. ACM, 79–88.

Sander, Knut, Naoko Tojo, Jan Vernon. 2007. The producer responsibility principle of the weee directive. http://reach-consultants.com/documents/J572_WEEEPRPReportFinal.pdf.

Shapley, Lloyd S. 1952. A value for n-person games. Tech. rep., Defense Technical Information Center Document.

Shapley, Lloyd S. 1953. A value for n-person games. *Annals of Mathematical Studies* .

Shapley, Lloyd S. 1971. Cores of convex games. *International journal of game theory* **1**(1) 11–26.

Subramanian, Ravi, Sudheer Gupta, Brian Talbot. 2009. Product design and supply chain coordination under extended producer responsibility. *Production and Operations Management* **18**(3) 259–277.

Sundararajan, Mukund. 2009. Trade-offs in cost sharing. Ph.D. thesis, Stanford University.

Tian, F., G. Sošić, L. G. Debo. 2014. Manufacturers' competition and cooperation in sustainability: Stable recycling alliances. *Working paper. Available at SSRN 2459656* .

Walls, Margaret. 2006. Extended producer responsibility and product design: Economic theory and selected case studies .

Washington State Legislature. 2006. Electronic product recycling. http://app.leg.wa.gov/rcw/default.aspx?cite=70.95N.010. Last accessed October 2014.

WEEE. 2012. Directive 2012/19/eu of the european parliament and of the council of 4 July 2012 on waste electrical and electronic equipment (WEEE) (recast). http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32012L0019.

WMMFA. 2012. http://www.wmmfa.net/.

Zuidwijk, R., H. Krikke. 2008. Strategic response to EEE returns: Product eco-design or new recovery processes? *European Journal of Operational Research* **191** 1206–1222.

Appendix A: Proofs and Technical Analysis

For convenience, in this appendix (except for in the proof of Proposition 6 where the processor efficiency is product-differentiated), we assume that the indices of the producers are arranged according to the efficiency levels of the capacities that they have access to in an individual system. Specifically, $i < j$ implies that $\tau_{r(i)} < \tau_{r(j)}$, i.e., $r(i)$ is more efficient.

A.1. Technical Details in §4

In this section, we present the equilibrium analysis of the design outcome under the dual-based cost allocation. We first study RNs with two producers, and explain the results in Example 1. We then extend the analysis to RNs with n producers and prove Proposition 1. For brevity, we include certain technical details in an online companion document.

A.1.1. RNs with Two Producers In an RN with two producers, we can prove that the design-reinforcing condition $d^{\pi(2)} \geq k_{r(1)}$ is a sufficient and necessary one for a dual-based cost allocation to induce a superior equilibrium design profile compared to the individual system benchmark.

Proposition 7. *Consider an RN instance with two producers. Under the dual-based cost allocation mechanism x_d,*

1. *when $d^{\pi(2)} \geq k_{r(1)}$, i.e., the capacity at $r(1)$ is no larger than producer 2's return volume, there exists a unique equilibrium design profile Λ_d^{ne} that is superior to the design profile induced by an individual system, i.e., $\Lambda_d^{ne} \leq \Lambda_{ind}$.*
2. *when $d^{\pi(2)} < k_{r(1)}$, if an equilibrium design profile exists, it is inferior to the design profile induced by an individual system, i.e., $\Lambda_d^{ne} \geq \Lambda_{ind}$; otherwise, there exists a mixed-strategy equilibrium under which any design profile that can occur with a positive probability is inferior to Λ_{ind}.*

Proof of Proposition 7. The proof of Proposition 7 is based on the following three-step procedure.

• Step 1 (compute the best response functions): For each producer $i = 1, 2$, we calculate the recyclability level of $\pi(i)$ that minimizes the sum of i's dual-based cost allocation and its design investment, i.e., $x_d^i + Q^i$. We calculate this design choice for i as a function of the other producer's design strategy. This function, denoted as $\lambda_d^{*\pi(i)}$, is called producer i's *best response function* to j's design choice.

• Step 2 (find the equilibrium Λ_d^{ne}): We calculate the equilibrium design profile by solving the two best response functions simultaneously. This can be done by finding the intersection point of the two functions.

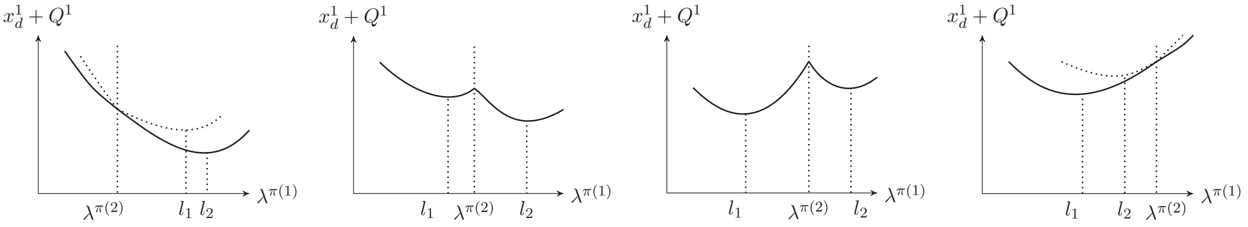
• Step 3 (compare Λ_d^{ne} with Λ_{ind}^*): We compare the equilibrium design profile with the design profile induced by an individual system (i.e., Λ_{ind}^*).

Below we explain the calculation details in each step under the two RN cases described in Proposition 7.

Case 1: When the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$

• Step 1: To compute the best response functions, we first analyze the dual-based cost allocation to each producer. This cost allocation is calculated based on the optimal dual solution of the centralized allotment problem (C), and thus depends on the socially optimal allotment f^*. It can be observed that when $d^{\pi(2)} \geq$

Figure 4 **This figure shows producer 1's total cost function $x_d^1 + Q^1$ when $d^{\pi(2)} \geq k_{r(1)}$. In the figure, l_1 and l_2 are as defined in (12), and $\lambda_0^{\pi(2)}$ is a constant in (l_1, l_2), calculated based on the given RN instance.**



(a) When $\lambda^{\pi(2)} < l_2$ (b) When $l_1 \leq \lambda^{\pi(2)} \leq \lambda_0^{\pi(2)}$ (c) When $\lambda_0^{\pi(2)} < \lambda^{\pi(2)} \leq l_2$ (d) When $\lambda^{\pi(2)} > l_2$

$k_{r(1)}$, f^* varies depending on the relative recyclability of the products (as depicted in Figure 3(b) and (c)). Accordingly, we can calculate producer 1's dual-based cost allocation x_d^1 as follows.

$$x_d^1 = \begin{cases} d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} - k_{r(1)} \cdot (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}) & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(1)} \cdot [c_{r(1)}^{\pi(1)} + (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)})] - k_{r(1)} \cdot (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}) & \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{11}$$

Recall that in our main model, $c_r^\pi = \lambda^\pi \cdot \tau_r + \bar{c}^\pi$ for each pair of product π and processor r. Hence, it can be observed that given producer 2's product design $\lambda^{\pi(2)}$, formula (11) is a piecewise linear function of producer 1's own design variable $\lambda^{\pi(1)}$, and the break point occurs when $\lambda^{\pi(1)} = \lambda^{\pi(2)}$. Since the investment function Q^1 is convex with respect to $\lambda^{\pi(1)}$, we can show that producer 1's total cost $x_d^1 + Q^1$ is a piecewise convex function of $\lambda^{\pi(1)}$, whose shape depends on the given $\lambda^{\pi(2)}$ and is as depicted in Figure 4. It can be observed based on the figure that the minimizer of $x_d^1 + Q^1$ (which is by definition producer 1's best response $\lambda_d^{*\pi(1)}$) can occur at two different values defined as below.

$$l_1 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(2)}\right); \quad l_2 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(1)}\right) \tag{12}$$

It can be observed that $l_1 < l_2$ since $\tau_{r(1)} < \tau_{r(2)}$ (i.e., $r(1)$ is the more cost efficient processor) and the investment function Q^1 is convex decreasing. Further calculation indicates that producer 1's best response $\lambda_d^{*\pi(1)}$ is a two-piece step function based on l_1 and l_2.

$$\lambda_d^{*\pi(1)} = \begin{cases} l_2 & \text{if } \lambda^{\pi(2)} \leq \lambda_0^{\pi(2)} \\ l_1 & \text{if } \lambda^{\pi(2)} > \lambda_0^{\pi(2)} \end{cases} \tag{13}$$

where $\lambda_0^{\pi(2)}$ is a constant defined by the formula $\frac{Q^1(l_2) + d^{\pi(1)} \cdot \tau_{r(2)} \cdot l_2 - Q^1(l_1) - d^{\pi(1)} \cdot \tau_{r(1)} \cdot l_1}{d^{\pi(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})}$. It can be verified that $\lambda_0^{\pi(2)}$ is a constant between l_1 and l_2.

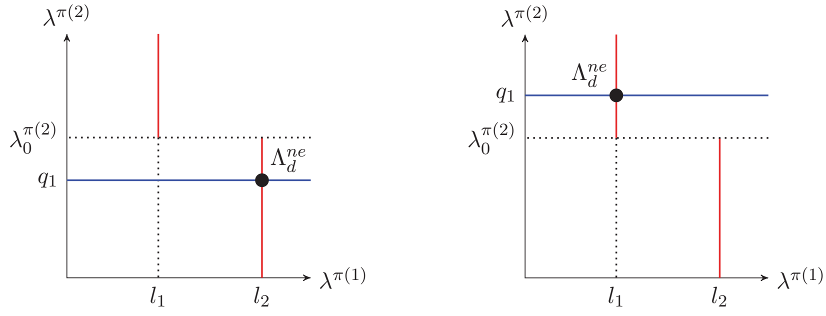
We continue to compute the best response function for producer 2, which is more straightforward. This is because we can calculate that producer 2's dual-based cost allocation equals

$$x_d^2 = d^{\pi(2)} \cdot c_{r(2)}^{\pi(2)} \tag{14}$$

under both patterns of the socially optimal allotment f^*. Hence, we conclude that when the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$, x_d^2 is independent of producer 1's design choice. Accordingly, it can be calculated that producer 2's best response equals a constant q_1 defined as

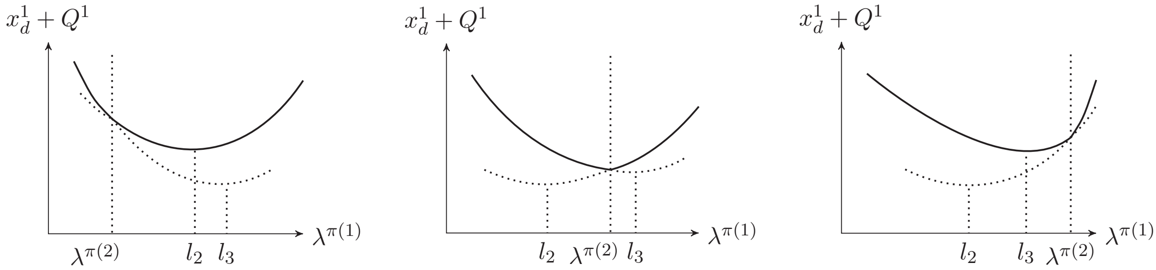
$$q_1 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(2)}\right). \tag{15}$$

Figure 5 This figure shows the equilibrium design profile Λ_d^{ne} under the dual-based allocation when $d^{\pi(2)} \geq k_{r(1)}$. In this figure, l_1, l_2 and q_1 are as defined in (12) and (15), and $\lambda_0^{\pi(2)}$ is the same constant as shown in Figure 4. The vertical and horizontal lines represent the best response functions of producer 1 and 2 respectively.



(a) When $q_1 \leq \lambda_0^{\pi(2)}$ (b) When $q_1 > \lambda_0^{\pi(2)}$

Figure 6 This figure shows producer 1's total cost function $x_d^1 + Q^1$ when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$. In the figure, l_2 and l_3 are as defined in (17).



(a) When $\lambda^{\pi(2)} \leq l_2$ (b) When $l_2 < \lambda^{\pi(2)} < l_3$ (c) When $\lambda^{\pi(2)} \geq l_3$

- Step 2: We locate the equilibrium design profile Λ_d^{ne} at the intersection the two best response functions calculated above (Figure 5). Based on this figure, we conclude that when the RN satisfies $d^{\pi(2)} \geq k_{r(1)}$, there exists a unique equilibrium design profile under the dual-based cost allocation, and it equals either $\{l_1, q_1\}$ or $\{l_2, q_1\}$ depending on the specification of the RN considered.
- Step 3: We have shown in §3.2 that for each producer i, the product design induced by an individual system is $\lambda_{ind}^{\pi(i)} = (Q^{i\prime})^{-1}\left(-d^{\pi(i)} \cdot \tau_{r(i)}\right)$. Comparing this formula with (12) and (15), we immediately obtain that $l_2 = \lambda_{ind}^{\pi(1)}$ and $q_1 = \lambda_{ind}^{\pi(2)}$. Moreover, as we have shown $l_1 < l_2$, we know that $l_1 < \lambda_{ind}^{\pi(1)}$, indicating the possibility of a strictly superior design of the product $\pi(1)$ under the dual-based cost allocation compared to that in an individual system. This completes the proof of the first result in Proposition 7.

Case 2: When the RN satisfies $d^{\pi(2)} < k_{r(1)}$ We continue to prove the second result in Proposition 7, i.e., the dual-based cost allocation leads to inferior product designs than those induced in an individual system when $d^{\pi(2)} < k_{r(1)}$. We analyze two sub-cases as follows.

Case 2(a): We consider the sub-case where the capacity at $r(1)$ is sufficient to process the return volume of $\pi(2)$ but not all products returned, i.e., $\underline{d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}}$. We follow the three-step procedure introduced at the beginning of the proof to analyze this sub-case.

• Step 1: We first observe that when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, similar to Case 1, there are two possible patterns of the socially optimal allotment depending on the relative recyclability of the products (Figure 3(d) and (e)). Accordingly, we can calculate producer 1's dual-based cost allocation x_d^1 as follows.

$$x_d^1 = \begin{cases} d^{\pi(1)} \cdot c_{r(2)}^{\pi(1)} - k_{r(1)} \cdot (c_{r(2)}^{\pi(1)} - c_{r(1)}^{\pi(1)}) & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(1)} \cdot [c_{r(1)}^{\pi(1)} + c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}] - k_{r(1)} \cdot (c_{r(2)}^{\pi(2)} - c_{r(1)}^{\pi(2)}) & \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{16}$$

It is easy to see that x_d^1 is a piecewise linear function of producer 1's design variable $\lambda^{\pi(1)}$ given any value of $\lambda^{\pi(2)}$, just as in Case 1. However, the difference is that x_d^1 defined in (16) is convex in $\lambda^{\pi(1)}$ as opposed to being concave in Case 1. This results in a different functional structure of producer 1's total cost $x_d^1 + Q^1$ (Figure 6), whose minimum may be attained at the following two values of $\lambda^{\pi(1)}$.

$$l_2 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(1)}\right); \quad l_3 \doteq (Q^{1\prime})^{-1}\left(-d^{\pi(1)} \cdot \tau_{r(2)} + k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)})\right). \tag{17}$$

Since we can calculate that $d^{\pi(1)} \cdot \tau_{r(2)} - k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) \leq d^{\pi(1)} \cdot \tau_{r(1)}$, we conclude that $l_3 > l_2$ as Q^1 is convex decreasing. Hence, it can be shown that the best response of producer 1, $\lambda_d^{*\pi(1)}$, is no longer a step function as in Case 1. Instead, it equals

$$\lambda_d^{*\pi(1)} = \begin{cases} l_2 & \text{if } \lambda^{\pi(2)} \leq l_2 \\ \lambda^{\pi(2)} & \text{if } \lambda^{\pi(2)} \in (l_2, l_3) \\ l_3 & \text{if } \lambda^{\pi(2)} \geq l_3 \end{cases} \tag{18}$$

We continue to compute the best response of producer 2 in this sub-case. We can calculate that in this case, producer 2's dual-based cost allocation equals

$$x_d^2 = \begin{cases} d^{\pi(2)} \cdot [c_{r(1)}^{\pi(2)} + c_{r(2)}^{\pi(1)} - c_{r(1)}^{\pi(1)}] & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ d^{\pi(2)} \cdot c_{r(2)}^{\pi(2)} & \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{19}$$

Following the same argument that we use to derive producer 1's best response in Case 1, we can show that $\lambda_d^{*\pi(2)}$ is the following two-piece step function:

$$\lambda_d^{*\pi(2)} = \begin{cases} q_2 & \text{if } \lambda^{\pi(1)} \leq \lambda_0^{\pi(1)} \\ q_1 & \text{if } \lambda^{\pi(1)} > \lambda_0^{\pi(1)} \end{cases} \tag{20}$$

where q_1 and q_2 equal

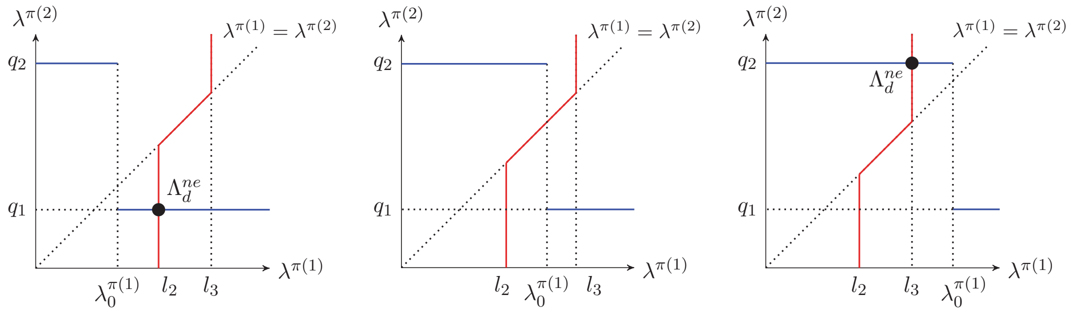
$$q_1 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(2)}\right); \quad q_2 \doteq (Q^{2\prime})^{-1}\left(-d^{\pi(2)} \cdot \tau_{r(1)}\right). \tag{21}$$

and $\lambda_0^{\pi(1)}$ is a constant defined by the formula $\frac{Q^2(q^1)+d^{\pi(2)}\cdot\tau_{r(2)}\cdot q^1 - Q^2(q_2) - d^{\pi(2)}\cdot\tau_{r(1)}\cdot q_2}{d^{\pi(2)}\cdot(\tau_{r(2)}-\tau_{r(1)})}$. It can be verified that $\lambda_0^{\pi(1)} \in (q_1, q_2)$ (it is easy to show that $q_1 < q_2$ as $\tau_{r(2)} > \tau_{r(1)}$ and Q^2 is convex decreasing).

• Step 2: We find the equilibrium design profile at the intersection of the two best response functions calculated above (Figure 7). Note that in this sub-case, a pure-strategy equilibrium does not exist if the RN instance leads to the situation shown in Figure 7(b). In that situation, we show that a mixed-strategy equilibrium always exists (see Claim 2 presented at the end of Step 3).

• Step 3: Based on the calculations already presented in this proof, we have $l_3 > l_2 = \lambda_{ind}^{\pi(1)}$ and $q_2 > q_1 = \lambda_{ind}^{\pi(2)}$. Hence, we conclude that when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$, all possible values of the best response function of each producer i shown in formula (18) and (20) are no better than i's design choice induced by an individual system. Hence, x_d leads to an inferior equilibrium design profile if such an equilibrium exists

Figure 7 This figure shows the equilibrium design profile Λ_d^{ne} under the dual-based allocation when $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$. In this figure, l_2, l_3, q_1 and q_2 are as defined in (17) and (21) respectively, and $\lambda_0^{\pi(1)}$ is the constant used in formula (20). The best response function of producer 1 is represented by the horizontal lines, and that of producer 2's is represented by the vertical and the diagonal lines. Note that in panel (b), no pure-strategy equilibrium design profile exists.



(a) When $\lambda_0^{\pi(1)} \leq l_2$ (b) When $l_2 < \lambda_0^{\pi(1)} < l_3$ (c) When $\lambda_0^{\pi(1) \geq l_3}$

(Figure 7(a) and (c)). We further show that this result can be extended to the mixed-strategy equilibrium in the situation shown in Figure 7(b). Specifically, to define a mixed strategy equilibrium, we treat each $\lambda^{\pi(i)}$ as a random variable. A mixed-strategy equilibrium is then defined as a collection of probability distributions $\{f^i(\lambda^{\pi(i)})\}$ of the corresponding $\lambda^{\pi(i)}$ variables. We prove the following result (see the online technical companion for the proof).

Lemma 2. *Given an RN instance such that $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$ and $l_1 < \lambda_0^{\pi(1)} < l_3$, there always exists a mixed-strategy equilibrium design profile $\{f^1(\lambda^{\pi(1)}), f^2(\lambda^{\pi(2)})\}$. Moreover, under this equilibrium, $\forall i = 1,2$ and $\forall \lambda^{\pi(i)}$ such that $f^i(\lambda^{\pi(i)}) > 0$, the inequality $\lambda^{\pi(i)} \geq \lambda_{ind}^{\pi(i)}$ is satisfied.*

The above lemma indicates that under the mixed-strategy equilibrium $\{f^1, f^2\}$, any product design outcome that can occur with positive probability is inferior to the one induced by an individual system.

Case 2(b): We finally consider the other sub-case where $r(1)$ has enough capacity to process the entire volume, i.e., $\underline{k_{r(1)} \geq d^{\pi(1)} + d^{\pi(2)}}$. In this case, the socially optimal allotment f^* sends all products to the cheaper processor $r(1)$ (Figure 3(f)) regardless of the design profile. Because of this, the optimal design choices of the two producers are not correlated in the collective system, and thus the equilibrium analysis becomes much simpler. Specifically, we can calculate that the dual-based cost allocation of each producer i equals

$$x_d^i = d^{\pi(i)} \cdot c_{r(1)}^{\pi(i)}. \tag{22}$$

Accordingly, the design choice for i that minimizes its total cost $x_d^i + Q^i$ equals $\lambda_d^{*\pi(i)} = (Q^{i\prime})^{-1}\left(-d^{\pi(i)} \cdot \tau_{r(1)}\right)$, which is no smaller than $\lambda_{ind}^{\pi(i)}$. Moreover, for producer 2, we can calculate that $\lambda_d^{*\pi(2)} > \lambda_{ind}^{\pi(2)}$ since $\tau_{r(1)} < \tau_{r(2)}$ and the investment function Q^2 is convex decreasing. That is, participating in a collective system induces producer 2 to adopt a strictly worse design in this case of the RN. Combining the above results, we conclude that Proposition 7 holds.

A.1.2. RNs with n Producers

Proof of Proposition 1 To prove Proposition 1, the major technical challenge is the following: With n producers, direct calculation of producers' best response functions as in the two-producer case shown in the proof of Proposition 7 becomes generally intractable. We overcome this issue by an algorithm approach. That is, we develop an algorithm, and show that this algorithm computes the equilibrium design profile when the RN satisfies the low-synergy condition. To present this algorithm, we denote an RN instance as G, and introduce a key structure that will be used, i.e., a sub-network G^i that consists of producer i and all other producers j such that $r(j)$ is less efficient than $r(i)$. Recall that in this appendix, we assume that a larger producer index indicates more efficient capacity, i.e., $i<j$ if $\tau_{r(i)} < \tau_{r(j)}$. Hence, the sub-network G^i consists of all producers $j \geq i$. The detailed steps of this algorithm are presented in Algorithm 1, shown on the next page.

Algorithm 1: Calculating the equilibrium design profile under the dual-based allocation in an n-producer RN

Input: An RN instance G and the investment function Q^i of each producer i
Output: A product design profile $\hat{\Lambda}$
Let $t=n$. Let $\hat{\Lambda}$ be a design profile.
while $t>0$ **do**
1. Consider the sub-network G^t. Compute producer t's dual-based cost allocation in G^t given that all other producers adopt the design decision specified by $\hat{\Lambda}$. Denoted this cost allocation as $(x^t)_d^{G^t}$. Compute producer t's design choice that minimizes $(x^t)_d^{G^t} + Q^t$ and assign the value to $\hat{\lambda}^{\pi(t)}$.
2. Let $t=t-1$.

end
Output the design profile $\hat{\Lambda}$.

Lemma 3. *Given an RN instance that satisfies $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i \in N$, a product design profile is an equilibrium under the dual-based cost allocation if and only if it is the solution to Algorithm 1.*

Proof of Lemma 3 The key observation that leads to Lemma 3 is the following.

Observation 1 *Under the low-synergy condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i \in N$, each producer i's dual-based cost allocation in the original RN instance G is equivalent to that in a sub-network G^i that consists of producer i and all other producers whose capacity is less efficient than $r(i)$.*

If Observation 1 holds, then the $\hat{\lambda}^{\pi(i)}$ value computed in Algorithm 1, which is producer i's best response in G^i when all other producers follow $\hat{\Lambda}$, is also guaranteed to be its best response in the original RN G. By definition, we know that $\hat{\Lambda}$ is an equilibrium design profile and vise versa.

In the rest of the proof of Lemma 3, we show that Observation 1 holds in two steps. In Step 1, we provide an algorithm to compute the socially optimal allotment in any RN with a given design profile of the products; this enables us to come up with a closed-form expressions of the optimal dual solution $[\beta^{*\pi}, \alpha_r^*]$ as functions of the design profile. In Step 2, we show that under the low-synergy condition, the optimal dual solution that corresponds to i's product and capacity is independent of any other producer with better capacity (i.e., any producer $j<i$). Below we illustrate the main idea in each step; the complete technical proof is presented in the online technical companion.

STEP 1: The algorithm we propose to compute the socially optimal allotment is a greedy one, described in Algorithm 2 shown on the next page.

Algorithm 2: A greedy product allotment algorithm in an n-producer RN

Input: An RN instance G

Output: A product allotment solution $\tilde{f}$

Let $\Pi_0=\Pi$ and $R_0=R$ equal the set of products and processors in G, respectively. Let $\tilde{f}_{[\pi,r]}=0$ $\forall r$ and $\forall \pi$. Let $t=0$.

while $\Pi_t \neq \emptyset$ **do**

1. Let $\pi_{max} \doteq \arg\max_{\pi\in\Pi_t} \lambda^\pi$ be the product with the worst design in Π_t.
   Let $r_{min} \doteq \arg\min_{r\in R_t} \tau_r$ be the most efficient processor in R_t.
2. Let $\tilde{f}_{[\bar{\pi},\bar{r}]}=\min\{d^{\bar{\pi}},k_{\bar{r}}\}$. Let $d^{\bar{\pi}}=d^{\bar{\pi}}-\tilde{f}_{[\bar{\pi},\bar{r}]}$ and $k_{\bar{r}}=k_{\bar{r}}-\tilde{f}_{[\bar{\pi},\bar{r}]}$.
3. Let $\Pi_{t+1}\doteq\Pi_t\setminus\{\pi\in\Pi_t: d^\pi=0\}$, and $R_{t+1}\doteq R_t\setminus\{r\in R_t: k_r=0\}$. Let $t=t+1$.

end

Output the product allotment $\tilde{f}$.

The optimality of the $\tilde{f}$ solution generated by Algorithm 2 can be proven based on duality theory. That is, given any RN instance as modeled in §3, we can identify a feasible dual solution to the centralized allotment problem (C) that satisfies complementary slackness with respect to the $\tilde{f}$ solution computed by Algorithm 2 (please refer to the online technical companion for the proof). This indicates that both $\tilde{f}$ and the dual solution identified are optimal. Below we present the closed-form expression of this optimal dual solution. Some additional notation are used in the formulas.

- We use $\bar{r}_\pi$ and $\underline{r}_\pi$ to denote the most and the least cost efficient processor that handles product π under the optimal allotment.
- We use $\bar{\pi}_r$ to denote the most recyclable product processed at r under the optimal allotment[2].

The optimal dual solution to the centralized allotment problem (C) can be computed as follows.

$$\beta^{*\pi}=c^\pi_{\underline{r}_\pi}+\sum_{\pi':\lambda^{\pi'}<\lambda^\pi}(c^{\pi'}_{\underline{r}_{\pi'}}-c^{\pi'}_{\bar{r}_{\pi'}})\quad \forall\pi\in\Pi \tag{23}$$

$$\alpha^*_r=(c^{\bar{\pi}_r}_r-c^{\bar{\pi}_r}_{\underline{r}_{\bar{\pi}_r}})+\sum_{\pi':\lambda^{\pi'}<\lambda^{\bar{\pi}_r}}(c^{\pi'}_{\bar{r}_{\pi'}}-c^{\pi'}_{\underline{r}_{\pi'}})\quad \forall r\in R. \tag{24}$$

Intuitively, $\beta^{*\pi}$ calculates the additional cost incurred when another unit of product π is to be processed in the RN: First, this unit of π will be routed to processor $\underline{r}_\pi$ under the optimal allotment, resulting in a cost of $c^\pi_{\underline{r}_\pi}$. Moreover, for each product π' that is more recyclable than π (and therefore is allotted after π in the greedy algorithm), a unit of π' will be effectively rerouted from $\bar{r}_{\pi'}$ to $\underline{r}_{\pi'}$, resulting in a cost increase of $c^{\pi'}_{\underline{r}_{\pi'}}-c^{\pi'}_{\bar{r}_{\pi'}}$. Similarly, α^*_r calculates the potential cost saving if the capacity at processor r were to be increased by one: A unit of product $\bar{\pi}_r$ will be effectively rerouted from $\underline{r}_{\bar{\pi}_r}$ to r, which saves $c^{\bar{\pi}_r}_r-c^{\bar{\pi}_r}_{\underline{r}_{\bar{\pi}_r}}$. This

[2] This definition of $\bar{\pi}_r$ applies in the case where the socially optimal allotment is non-degenerate. In degenerate cases, it is defined as the product that will be recycled at r if the capacity at r were to be increased by one.

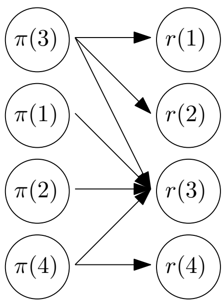


Figure 8 An RN example

- For product $\pi(3)$, due to the low-synergy condition $d^{\pi(3)} > k_{r(1)} + k_{r(2)}$, its last unit will be allotted to $r(3)$. This is also the case for product $\pi(4)$: Since $d^{\pi(4)} > k_{r(1)} + k_{r(2)} + k_{r(3)}$, the last unit of $\pi(4)$ will be allotted to $r(4)$.
- For product $\pi(1)$ and $\pi(2)$, we can see that $r(1)$ and $r(2)$ are already saturated by $\pi(3)$ when these two products are allotted in the greedy algorithm. Moreover, due to the modeling assumption, we know that $r(1), r(2), r(3)$ combined have sufficient capacity to process $\pi(1), \pi(2), \pi(3)$. Hence, $\pi(1)$ and $\pi(2)$ are both alloted to $r(3)$ only.

also results in a unit of each product π' that is more recyclable than $\bar{\pi}_r$ being rerouted from $\underline{r}_{\pi'}$ to $\bar{r}_{\pi'}$, which accounts for a cost saving of $c^{\pi'}_{\bar{r}_{\pi'}} - c^{\pi'}_{\underline{r}_{\pi'}}$.

STEP 2: In this step, we prove that, under the low-synergy condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i \in N$, for each producer i, the optimal dual solution $[\beta^{*\pi(i)}, \alpha^{*\pi(i)}]$ is independent of any producer $j < i$.

We first show this for $\beta^{*\pi(i)}$. For convenience, we rewrite formula (23) that calculates $\beta^{*\pi(i)}$ as follows.

$$\beta^{*\pi(i)} = \underbrace{c^{\pi(i)}_{\underline{r}_{\pi(i)}}}_{①} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)} < \lambda^{\pi(i)} \text{ and } j>i} \left(c^{\pi(j)}_{\underline{r}_{\pi(j)}} - c^{\pi(j)}_{\bar{r}_{\pi(j)}}\right)}_{②} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)} < \lambda^{\pi(i)} \text{ and } j<i} \left(c^{\pi(j)}_{\underline{r}_{\pi(j)}} - c^{\pi(j)}_{\bar{r}_{\pi(j)}}\right)}_{③} \tag{25}$$

In the following points $(i)-(iii)$, we show that each term in the above formula is independent of both the product and the capacity of any producer $j > i$, respectively.

(i) *Part ① in (25) is independent of producers $j < i$: This is due to the fact that for any i, $\underline{r}_{\pi(i)}$ corresponds to a processor $r(l)$ where $l \geq i$.* To illustrate this, consider an example RN instance with four producers that satisfies $d^{\pi(i)} > \sum_{j<i} k_{r(j)}$[3]. Assuming the design profile is such that $\lambda^{\pi(3)} > \lambda^{\pi(1)} > \lambda^{\pi(2)} > \lambda^{\pi(4)}$, the socially optimal allotment is as depicted in Figure 8. We can make the following observations.

In general RNs, we can show that for any product $\pi(i)$, either its last unit is allotted to $r(i)$ (as in the case of $\pi(3)$ and $\pi(4)$ in this example), or it is entirely allotted to some $r(j)$ where $j > i$ (as in the case of $\pi(1)$ and $\pi(2)$ in this example). Please see the online technical companion for full details.

(ii) *Part ② in (25) is independent of producers $j < i$: This is the case as we can show that for any $\pi(j)$ such that $\lambda^{\pi(j)} < \lambda^{\pi(i)}$, both $\underline{r}_{\pi(j)}$ and $\bar{r}_{\pi(j)}$ correspond to $r(l)$'s where $l \geq i$.* In other words, $\pi(j)$ is recycled at processors less efficient than $r(i)$ under the socially optimal allotment. To see this, note that such a product $\pi(j)$ will be alloted after $\pi(i)$ in the greedy algorithm and thus will only be alloted to processors that are less efficient than $\underline{r}_{\pi(i)}$, which we know from point (i) is less or equally efficient compared to $r(i)$.

(iii) *Part ③ in (25) equals zero: This is because for any $\pi(j)$ such that $\lambda^{\pi(j)} < \lambda^{\pi(i)}$ and whose index $j < i$, it must be recycled at only one processor under the socially optimal allotment, i.e., $\underline{r}_{\pi'} = \bar{r}_{\pi'}$.* We have shown this in point (i), illustrated by the allotment of product $\pi(1)$ and $\pi(2)$ in the RN in Figure 8.

[3] For convenience, we consider a non-degenerate version of the low-synergy condition. The argument can be generalized to degenerate cases as well.

Combining the above observations, we conclude that $\beta^{*\pi(i)}$ is independent of any producer $j<i$.

We continue to show that $\alpha^*_{r(i)}$ is also independent of any producer $j<i$. Similar to the above analysis to $\beta^{*\pi(i)}$, we also rewrite formula (24) that calculates $\alpha^*_{r(i)}$ as the summation of three parts.

$$\alpha^*_{r(i)} = \underbrace{(c^{\bar{\pi}_{r(i)}}_{r(i)} - c^{\bar{\pi}_{r(i)}}_{\underline{r}_{\bar{\pi}_{r(i)}}})}_{①} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)}<\lambda^{\bar{\pi}_{r(i)}} \text{ and } j>i} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}})}_{②} + \underbrace{\sum_{\pi(j):\lambda^{\pi(j)}<\lambda^{\bar{\pi}_{r(i)}} \text{ and } j<i} (c^{\pi(j)}_{\bar{r}_{\pi(j)}} - c^{\pi(j)}_{\underline{r}_{\pi(j)}})}_{③}. \quad (26)$$

We also show that each part in this formula is independent of either the product or the capacity of any producer $j<i$. The proof is also based on the argument in (i)-(iii) above, plus some additional observations.

(iv) *Part ① in (26) is independent of producers $j<i$: This is because for any i, $\bar{\pi}_{r(i)}$ corresponds to a product $\pi(l)$ where $l \geq i$.* To see this, note that according to point (i) above, if there exists a product $\pi(j)$ where $j>i$ and that is less recyclable than $r(i)$ (i.e., allotted before $\pi(i)$ in the greedy algorithm), then due to the low-synergy condition, $r(i)$ will be saturated by $\pi(j)$, i.e., $\bar{\pi}_{r(i)} = \pi(j)$; this is the case for $i=1,2$ in the example shown in Figure 8. Otherwise, we show that the last unit of $\pi(i)$ is allotted to $r(i)$, and the remaining capacity at $r(i)$ is sufficient to process the products that follow $r(i)$ in the greedy algorithm until a product with a larger index appears; this is the case for $i=3$ in the example, and thus $\bar{\pi}_{r(3)} = 4$.

(v) *Part ② in (26) is independent of producers $j<i$ and part ③ there equals zero.* This can be shown by combining points (iv) with the arguments in points (ii) and (iii), respectively.

Combining the above observations, we conclude that $\alpha^*_{r(i)}$ is independent of any producer $j<i$. This completes the proof of Lemma 3. □

Continuing the proof of Proposition 1, our next step is to compare the design outcome $\hat{\Lambda}$ computed by Algorithm 1 with the design profile induced by an individual system. To do so, we can first calculate that for each producer i,

$$\hat{\lambda}^{\pi(i)} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{\bar{r}_{\pi(i)}}). \quad (27)$$

Recall τ_r is the efficiency level of processor r. To see how formula (27) is derived, note that $\hat{\lambda}^{\pi(i)}$ is the global minimum of the function $(x^i)^{G^i}_d + Q^i$. Hence, it is the solution of the equation $\frac{\partial (x^i)^{G^i}_d + Q^i}{\partial \lambda^{\pi(i)}} = 0$, where $(x^i)^{G^i}_d$ is the dual-based cost allocation on the sub-network G^i. According to Observation 1, $(x^i)^{G^i}_d$ equals the dual-based cost allocation in the original RN, and can be computed as $(x^i)^{G^i}_d = \beta^{*\pi(i)} \cdot d^{\pi(i)} - |\alpha^*_{r(i)}| \cdot k_{r(i)}$. According to formulas (23) and (24), we know that

$$\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} = \tau_{\bar{r}_{\pi(i)}}; \quad \frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} = 0 \quad \Rightarrow \quad \frac{\partial (x^i)^{G^i}_d}{\partial \lambda^{\pi(i)}} = d^{\pi(i)} \cdot \tau_{\bar{r}_{\pi(i)}} \quad (28)$$

Plugging this into the equation $\frac{\partial (x^i)^{G^i}_d + Q^i}{\partial \lambda^{\pi(i)}} = 0$, we obtain formula (27). Based on the observation in (i), we know that $\bar{r}_{\pi(i)}$ corresponds to some processor $r(l)$ where $l \geq i$. Based on the assumption about producer indices, we know that $\tau_{r(l)} \geq \tau_{r(i)}$. Since the investment functions Q^i is convex decreasing, we conclude that $\hat{\lambda}^{\pi(i)} = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{\bar{r}_{\pi(i)}}) = (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(l)}) \leq (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(i)}) = \lambda^{\pi(i)}_{ind}$. Combining this observation (which holds for each producer $i \in N$) with Lemma 3, we complete the proof of Proposition 1.

A.2. Technical Details in §5

In this section, we provide the technical details in the analysis of the marginal contribution-based cost allocation x_m. Recall that in this appendix, we assume that a smaller producer index indicates a more efficient processor associated with this producer, i.e., $i < j$ indicates $\tau_{r(i)} < \tau_{r(j)}$. Under this assumption, we can rewrite the allocation x_m as $x_m^i = v(i \cup \{j : j > i\}) - v(\{j : j > i\})$ $\forall i \in N$.

Proof of Proposition 2 We first present an algorithm that computes the equilibrium design profile under the marginal contribution-based allocation x_m. We then show that according to the algorithm, the design profile computed is superior than the one induced by an individual system.

STEP 1: The algorithm we present in this proof (Algorithm 3 shown on the next page) is similar to Algorithm 1 shown in the proof of Proposition 1: It also computes the equilibrium design profile in a sequential manner, starting from producer n and following the decreasing order of producer indices. This can be done under the cost allocation x_m, as the cost allocated to each producer i only depends on the design choices of producers in the set $\{j : j \geq i\}$.

Algorithm 3: Calculating the equilibrium design profile under the marginal contribution based allocation x_m in an n-producer RN

Input: An RN instance G and the investment function Q^i of each producer i
Output: A product design profile $\hat{\Lambda}$
Let $t = n$. Let $\hat{\Lambda}$ be a design profile.
while $t > 0$ **do**

1. Compute producer t's marginal contribution based cost allocation x_m^t given that all other producers adopt the design decision specified by $\hat{\Lambda}$. Compute producer t's design choice that minimizes $x_m^t + Q^t$ and assign the value to $\hat{\lambda}^{\pi(t)}$.
2. Let $t = t - 1$.

end
Output the design profile $\hat{\Lambda}$.

STEP 2: We next show that the design profile computed by Algorithm 3 is superior to the one induced by an individual system. We first derive a closed-form expression of the global minimum of the function $x_m^i + Q^i$. To do this, we decompose each producer i's cost allocation x_m^i as shown in formula (29). In the formula, we use f^{*S} to denote the optimal product allotment solution within a coalition S.

$$x_m^i = \underbrace{\sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot c^{\pi(i)}_{r(t)}}_{\substack{\text{processing cost of } \pi(i) \\ \text{in the coalition } \{j\,:\,j\geq i\}}} + \underbrace{\sum_{l=i+1}^{n} \sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(l),r(t)]} \cdot c^{\pi(l)}_{r(t)} - v(\{j : j > i\})}_{\substack{\text{recycling cost reduction for products } \pi(l)\ \forall l > i \\ \text{after producer } i \text{ joins the coalition}}} \tag{29}$$

Note that given the optimal product allotment solution $f^{*\{j:j\geq i\}}$, only the first term in (29) is a function of producer i's design variable. Hence, within an interval of $\lambda^{\pi(i)}$ where $f^{*\{j:j\geq i\}}$ is unchanged, we can calculate that the local minimum must satisfy the equation below:

$$\frac{\partial(x_m^i + Q^i)}{\partial \lambda^{\pi(i)}} = \sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \frac{\partial c^{\pi(i)}_{r(t)}}{\partial \lambda^{\pi(i)}} + Q^{i\prime} = \sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)} + Q^{i\prime} = 0 \tag{30}$$

and thus equals $(Q^{i\prime})^{-1}(-\sum_{t=i}^{n} f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$.

We continue to show that the global minimizer of the function $x_m^i + Q^i$ is achieved at such a local minimum under some $f^{*\{j:j\geq i\}}$ solution. Assume this is not the case, and the global minimizer of $x_m^i + Q^i$ equals some other value $\lambda_0^{\pi(i)}$. Then we know that there exist intervals $[c_1, \lambda_0^{\pi(i)}]$ and $[\lambda_0^{\pi(i)}, c_2]$, where the function $x_m^i + Q^i$ strictly decreases and increases, respectively. Assume c_1 and c_2 are sufficiently close to $\lambda_0^{\pi(i)}$ such that when producer i's design choice varies within each of the two intervals, the optimal product allotment in the coalition $\{j : j \geq i\}$ is not changed. Denote the corresponding optimal product allotment solutions as $f_1^{*\{j:j\geq i\}}$ and $f_2^{*\{j:j\geq i\}}$. Since $x_m^i + Q^i$ strictly decreases on $[c_1, \lambda_0^{\pi(i)}]$, we know that $\lambda_0^{\pi(i)} < (Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. Similarly, we can conclude that $\lambda_0^{\pi(i)} > (Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. This leads to the inequality $(Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)}) > (Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. However, this cannot be the case: The optimal allotment changes from $f_2^{*\{j:j\geq i\}}$ to $f_1^{*\{j:j\geq i\}}$ after producer i improves its own design (i.e., the value of $\lambda^{\pi(i)}$ is smaller in the interval $[\lambda_0^{\pi(i)}, c_2]$ compared to in the interval $[c_1, \lambda_0^{\pi(i)}]$). According to the greedy allotment algorithm, product $\pi(i)$ will be allotted to less efficient processors under $f_1^{*\{j:j\geq i\}}$ than $f_2^{*\{j:j\geq i\}}$. This means that $\sum_{t=i}^n f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)} \geq \sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)}$, and thus $(Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_1^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$ should be *no larger than* $(Q^{i\prime})^{-1}(-\sum_{t=i}^n (f_2^{*\{j:j\geq i\}})_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$. This leads to a contradiction. Thus we conclude that the global minimum of $x_m^i + Q^i$ equals $(Q^{i\prime})^{-1}(-\sum_{t=i}^n f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)})$ under some $f^{*\{j:j\geq i\}}$ solution.

Finally, we show that this global minimizer is superior than $\lambda_{ind}^{\pi(i)}$. This is because $r(i)$ is the most cost efficient processor in the sub-coalition $\{j : j \geq i\}$, and thus we have $\sum_{t=i}^n f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)} \geq d^{\pi(i)} \cdot \tau_{r(i)}$. Hence we know that $(Q^{i\prime})^{-1}(-\sum_{t=i}^n f^{*\{j:j\geq i\}}_{[\pi(i),r(t)]} \cdot \tau_{r(t)}) \leq (Q^{i\prime})^{-1}(-d^{\pi(i)} \cdot \tau_{r(i)}) = \lambda_{ind}^{\pi(i)}$. This completes the proof of Proposition 2. $\square$

Proof of Proposition 3 In order to show that x_m is individually rational (i.e., $x_m^i = v(\{j : j \geq i\}) - v(\{j : j > i\}) \geq v(i)$ holds for each producer i), we show that the PA game is sub-additive. Recall that we use f^{*S} to denote the socially optimal allotment within the sub-coalition S. Let S_1 and S_2 denote any two mutually exclusive sub-coalitions in N. Consider their union $S_1 \cup S_2$ and the following product allotment within this combined coalition: For all products manufactured by producers in S_1 and S_2, it is allotted according to f^{*S_1} and f^{*S_2}, respectively. This is obviously a feasible allotment solution and the total cost incurred equals $v(S_1) + v(S_2)$. Since $v(S_1 \cup S_2)$ is the minimum total cost that can be achieved in $S_1 \cup S_2$, we know that $v(S_1 \cup S_2) \leq v(S_1) + v(S_2)$. Since this inequality holds for any mutually exclusive sub-coalitions S_1 and S_2, by definition, the PA game is sub-additive. This means that for any producer i, $v(i \cup \{j : \tau_{r(j)} > \tau_{r(i)}\}) \leq v(\{j : \tau_{r(j)} > \tau_{r(i)}\}) + v(i)$. Since x_m^i is calculated as the difference, $v(i \cup \{j : \tau_{r(j)} > \tau_{r(i)}\}) - v(\{j : \tau_{r(j)} > \tau_{r(i)}\})$, we conclude that $x_m^i \leq v(i)$ holds for each producer i, i.e., x_m is by definition individually rational.

Proof of Lemma 1. Given a weakly convex PA game under any product design profile Λ, consider an arbitrary sub-coalition S. We arrange the members in S in increasing order of their indices, and re-label them as $i_1, i_2, ..., i_{|S|}$. To show that the cost allocation x_m is group incentive compatible, we prove that $\sum_{t=1}^{|S|} x_m^{i_t} \leq v(S)$ holds, i.e., S is not charged a higher cost compared to operating alone under x_m. First, we observe that since the PA game is weakly convex, we know that $\forall t = 1, ..., |S| - 1$, $v(\{j : j \geq i_t\}) - v(\{j : j >$

$i_t\}) \le v(\{j : j \ge i_t\} \cap S) - v(\{j : j > i_t\} \cap S) = v(\{i_l : t \le l \le |S|\}) - v(\{i_l : t < l \le |S|\})$ holds. Rearranging the terms, we obtain

$$v(\{j : j \ge i_t\}) \le v(\{i_l : t \le l \le |S|\}) - v(\{i_l : t < l \le |S|\}) + v(\{j : j > i_t\}) \quad \forall t = 1, ..., |S| - 1\,. \tag{31}$$

Moreover, since the PA game has been shown to be sub-additive in the proof of Proposition 3, we have

$$v(\{j : j \ge i_{|S|}\}) \le v(\{i_{|S|}\}) + v(\{j : j > i_{|S|}\}) \tag{32}$$

Adding up inequality (31) over all $t = 1, ..., |S| - 1$ and inequality (32), we can derive the following inequality

$$\begin{aligned}
\sum_{t=1}^{|S|} v(\{j : j \ge i_t\}) \le & \sum_{t=1}^{|S|-1} [v(\{i_l : t \le l \le |S|\}) - v(\{i_l : t < l \le |S|\}) + v(\{j : j > i_t\})] + v(\{i_{|S|}\}) + v(\{j : j > i_{|S|}\}) \\
= & \; v(S) - \sum_{t=1}^{|S|-1} [v(\{i_l : t < l \le |S|\}) - v(\{i_l : t+1 \le l \le |S|\})] + \sum_{t=1}^{|S|} v(\{j : j > i_t\}) \\
= & \; v(S) + \sum_{t=1}^{|S|} v(\{j : j > i_t\})
\end{aligned} \tag{33}$$

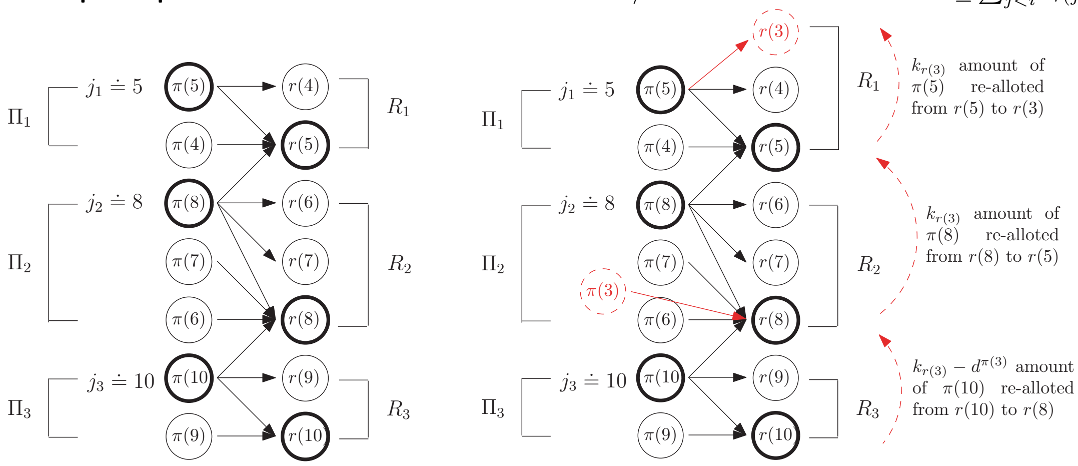
By rearranging the terms in (33), we derive that $\sum_{t=1}^{|S|} [v(\{j : j \ge i_t\}) - v(\{j : j > i_t\})] \le v(S)$. Note that on the left-hand-side, $v(\{j : j \ge i_t\}) - v(\{j : j > i_t\})$ is by definition $x_m^{i_t}$, i.e., the marginal contribution-based cost allocation to producer i_t. Hence, we obtain $\sum_{t=1}^{|S|} x_m^{i_t} \le v(S)$. Since this inequality holds for all sub-coalitions, the allocation mechanism x_m is group incentive compatible. $\square$

Proof of Proposition 4. In this proof, we consider an RN instance that satisfies the low-synergy condition $d^{\pi(i)} \ge \sum_{j<i} k_{r(j)}$ $\forall i > 2$. We show that given an arbitrary design profile Λ, the PA game is weakly convex, i.e., for any individual producer i, the inequality $v(T \cup \{i\}) - v(T) \le v(S \cup \{i\}) - v(S)$ holds $\forall S \subset T \subset \{i+1, i+2, ..., n\}$. The complete proof involves analyzing two cases for the coalition T depending on whether producer 2, whose return volume is not constrained in the low-synergy condition in this proposition, belongs to T. In this appendix, we present the proof for the case where producer 2 is not in T, that is, for each producer $i \in T$, the low-synergy condition $d^{\pi(i)} \ge \sum_{j<i} k_{r(j)}$ is satisfied. This assumption enables us to demonstrate the main idea of the proof while ensuring technical brevity. The proof for the other case where producer 2 belongs to T is based on similar arguments and is present in the online technical companion.

Hence, for the rest of this proof, we consider a sub-coalition T where $2 \notin T$. The proof consists of two steps: In the first step, we derive a closed-form expression for the difference $v(T \cup \{i\}) - v(T)$. Based on this formula, we prove the convexity inequality $v(T \cup \{i\}) - v(T) \le v(S \cup \{i\}) - v(S)$ in the second step.

STEP 1: Recall that in the proof of Proposition 1, we have shown that the optimal allotment in any RN can be computed by a greedy algorithm, and based on this algorithm, we have concluded some features of this allotment solution if the RN satisfies the low-synergy condition (refer to point (i)-(v) on page 36-37). Here, in order to derive a closed-form expression for $v(T \cup \{i\}) - v(T)$, we introduce the following notation that helps to mathematically characterize the structure of the optimal allotment in a low-synergy RN given an arbitrary design profile Λ.

Figure 9 **Optimal product allotment in a coalition T where $2 \notin T$ under the condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i > 2$.**



(a) Optimal product allotment in $T = \{4,5,6,7,8,9,10\}$

(b) Optimal product allotment when $i = 3$ joins $T = \{4,5,6,7,8,9,10\}$

- We define $J(T)$ as a subset of products such that $J(T) = \{\pi(j) : \lambda^{\pi(l)} < \lambda^{\pi(j)}, \forall l > j\}$. In words, for a product $\pi(j) \in J(T)$, any other product with a larger index $l > j$ has better design than $\pi(j)$. We rank the indices corresponding to the products in $J(T)$ in ascending order and relabel them as $j_1, j_2, ...j_t$.
- We then divide the set of products and processors into t subsets based on $j_1, j_2, ...j_t$:

—Product subsets: For all $h = 1, ..., t-1$, define $\Pi_h = \{\pi : \lambda^\pi \in [\lambda^{\pi(j_h)}, \lambda^{\pi(j_{h+1})})\}$. Define $\Pi_t = \{\pi : \lambda^\pi \in [\lambda^{\pi(j_t)}, \infty)\}$.

—Processor subsets: Define $R_1 = \{r(l) : l \leq j_1\}$. For all $h = 2, ..., t$, define $R_h = \{r(l) : l \in (j_{h-1}, j_h]\}$.

We illustrate the above notation by an RN example where $T = \{4,5,6,7,8,9,10\}$, and the design profile is such that $\lambda^{\pi(5)} > \lambda^{\pi(4)} > \lambda^{\pi(8)} > \lambda^{\pi(7)} > \lambda^{\pi(6)} > \lambda^{\pi(10)} > \lambda^{\pi(9)}$. In the RN figure, we arrange the product nodes from top down in descending order of $\lambda^{\pi(i)}$, and processor nodes in ascending order of $\tau_{r(i)}$ (recall that in this appendix, we assume $i < j$ means that $\tau_{r(i)} < \tau_{r(j)}$.) That is, the least recyclable product and the most efficient processor are placed at the top of the network. We implement the greedy allotment algorithm (Algorithm 2) and obtain the optimal product allotment f^{*T}, shown in Figure 9(a). According to the notation introduced above, in this example, we have $j_1 = 5$, $j_2 = 8$ and $j_3 = 10$. The product and the processor set Π and R are partitioned into $\Pi_1 = \{\pi(5), \pi(4)\}$, $\Pi_2 = \{\pi(8), \pi(7), \pi(6)\}$, $\Pi_3 = \{\pi(10), \pi(9)\}$, and $R_1 = \{r(4), r(5)\}$, $R_2 = \{r(6), r(7), r(8)\}$, $R_3 = \{r(9), r(10)\}$, respectively.

We can then characterize the structure of the optimal allotment f^{*T} in a low-synergy RN as follows. For each $h = 1, 2, ..., t$,

(a) The product $\pi(j_h)$ saturates all processors in $R_h \setminus \{r(j_h)\}$. The rest of its volume is allotted to $r(j_h)$.

(b) For all other products $\pi \in \Pi(h)$, its entire volume is allotted to $r(j_h)$.

This can be illustrated by the example in Figure 9(a). For instance, in Π_1, $\pi(5)$ saturates $r(4)$ and has its last unit routed to $r(5)$ due to its large volume under the low-synergy condition; $\pi(4)$ is entirely routed to $r(5)$. This is also the case in Π_2 and Π_3: $\pi(8)$ and $\pi(10)$ saturates all processors in $R_2 \setminus \{r(8)\}$ and $R_3 \setminus \{r(10)\}$, and their last units are allotted to $r(8)$ and $r(10)$, respectively. Meanwhile, $r(8)$ and $r(10)$ have sufficient capacity to absorb the volume of the other products in $\Pi(2)$ and $\Pi(3)$, respectively. The detailed proof of why the optimal allotment has these features in a low-synergy RN is provided in the online technical companion.

Now we analyze how the optimal allotment solution will change when i joins the coalition T. To demonstrate this, consider the case where producer 3 joins the collective RN shown in Figure 9(a), assuming $\lambda^{\pi(7)} > \lambda^{\pi(3)} > \lambda^{\pi(6)}$. Figure 9(b) shows the change in the optimal allotment in this example. We generalize this example as follows.

- For product $\pi(j_h)$ such that $\lambda^{\pi(j_h)} > \lambda^{\pi(i)}$, $k_{r(i)}$ amount of its volume is re-allotted from $r(j_h)$ to $r(j_{h-1})$. This is the case for product $\pi(5)$ and $\pi(8)$ in Figure 9(b).
- For product $\pi(j_h)$ such that $\lambda^{\pi(j_h)} < \lambda^{\pi(i)}$, $k_{r(i)} - d^{\pi(i)}$ amount of its volume is re-allotted from $r(j_h)$ to $r(j_{h-1})$. This is the case for product $\pi(10)$ in Figure 9(b).
- For other products, there is no change in the way they are alloted. This is the case for product $\pi(4)$, $\pi(7)$, $\pi(6)$ and $\pi(9)$ in Figure 9(b).

At the same time, it can also be observed that in this example, when producer 3 joins the coalition T, its product $\pi(3)$ is allotted to $r(8)$. This is because under the given design profile, $\pi(3)$ belongs to the product set $\Pi(2)$ when producer 3 joins T, and $j_2 = 8$ in this example. In the general case, let $\bar{h}$ be the largest index such that $\pi(j_{\bar{h}}) \in J(T)$ is less recyclable than $\pi(i)$. Then we know that when producer i joins T, product $\pi(i)$ belongs to the set $\Pi(\bar{h})$, and will be alloted to the processor $r(j_{\bar{h}})$. It is also possible that $\pi(i)$ is the least recyclable product in $T \cup \{i\}$. In this case, $\pi(i)$ will be processed at $r(i)$ when i joins T. Also by definition, the set $J(T \cup \{i\})$ has an additional element $\pi(i)$ besides those contained in $J(T)$; for notation convenience, we relabel i as j_0. Based on the above discussion, we derive a closed-form formula of $v(T \cup \{i\}) - v(T)$. For a cleaner mathematical representation, we present the formula of $v(T \cup \{i\}) - v(T) - v(i)$.

$$
\begin{aligned}
& v(T \cup \{i\}) - v(T) - v(i) \\
= {} & k_{r(i)} \cdot \sum_{h:\lambda^{\pi(j_h)} > \lambda^{\pi(i)}} \left(c^{\pi(j_h)}_{r(j_{h-1})} - c^{\pi(j_h)}_{r(j_h)}\right) + \left(k_{r(i)} - d^{\pi(i)}\right) \cdot \sum_{h:\lambda^{\pi(j_h)} < \lambda^{\pi(i)}} \left(c^{\pi(j_h)}_{r(j_{h-1})} - c^{\pi(j_h)}_{r(j_h)}\right) + d^{\pi(i)} \cdot \left(c^{\pi(i)}_{r(j_{\bar{h}})} - c^{\pi(i)}_{r(i)}\right) \\
= {} & \underbrace{d^{\pi(i)} \cdot \sum_{h:\lambda^{\pi(j_h)} > \lambda^{\pi(i)}} \left(\lambda^{\pi(j_h)} - \lambda^{\pi(i)}\right) \cdot \left(\tau_{r(j_{h-1})} - \tau_{r(j_h)}\right)}_{\Delta^1(T,i)} + \left(k_{r(i)} - d^{\pi(i)}\right) \cdot \underbrace{\sum_h \lambda^{\pi(j_h)} \cdot \left(\tau_{r(j_{h-1})} - \tau_{r(j_h)}\right)}_{\Delta^2(T,i)}
\end{aligned} \tag{34}
$$

STEP 2: In this step, we consider a subset $S \subset T$, and prove that $v(T \cup \{i\}) - v(T) - v(i) \leq v(S \cup \{i\}) - v(S) - v(i)$ (which immediately leads to the convexity inequality $v(T \cup \{i\}) - v(T) \leq v(S \cup \{i\}) - v(S)$).

First, we can observe that formula (34) only depends on products and processors associated with producers in the set $J(T)$. Hence, it follows that if $J(T) = J(S)$, then $v(T \cup \{i\}) - v(T) - v(i) = v(S \cup \{i\}) - v(S) - v(i)$ holds. Next, we show that the inequality also holds if $J(T) \neq J(S)$. To do this, it is sufficient to show $\Delta^1(T,i) \leq \Delta^1(S,i)$ and $\Delta^2(T,i) \leq \Delta^2(S,i)$, which we first demonstrate using the example in Figure 9(b).

In this example, let S be the set $\{4,7,9\}$. Then we know that $J(S) = \{4,7,9\}$, which is different from $J(T) = \{5,8,10\}$. We first compare $\Delta^1(T,i)$ and $\Delta^1(S,i)$.

$$\Delta^1(T,i) = (\lambda^{\pi(5)} - \lambda^{\pi(3)}) \cdot (\tau_{r(3)} - \tau_{r(5)}) + (\lambda^{\pi(8)} - \lambda^{\pi(3)}) \cdot (\tau_{r(5)} - \tau_{r(8)}) \tag{35}$$

$$\Delta^1(S,i) = (\lambda^{\pi(4)} - \lambda^{\pi(3)}) \cdot (\tau_{r(3)} - \tau_{r(4)}) + (\lambda^{\pi(7)} - \lambda^{\pi(3)}) \cdot (\tau_{r(4)} - \tau_{r(7)}) \tag{36}$$

In order to compare these two terms, we rewrote them as below.

$$
\begin{aligned}
(35) \;=\; & \underbrace{\big(\lambda^{\pi(5)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(3)}-\tau_{r(4)}\big)}_{I_1^1(T)}+\underbrace{\big(\lambda^{\pi(5)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(4)}-\tau_{r(5)}\big)}_{I_2^1(T)}+\underbrace{\big(\lambda^{\pi(8)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(5)}-\tau_{r(7)}\big)}_{I_3^1(T)}\\
&+\underbrace{\big(\lambda^{\pi(8)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(7)}-\tau_{r(8)}\big)}_{I_4^1(T)} \qquad (37)\\
(36) \;=\; & \underbrace{\big(\lambda^{\pi(4)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(3)}-\tau_{r(4)}\big)}_{I_1^1(S)}+\underbrace{\big(\lambda^{\pi(7)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(4)}-\tau_{r(5)}\big)}_{I_2^1(S)}+\underbrace{\big(\lambda^{\pi(7)}-\lambda^{\pi(3)}\big)\cdot\big(\tau_{r(5)}-\tau_{r(7)}\big)}_{I_3^1(S)} \qquad (38)
\end{aligned}
$$

We know that in this example, the products that are more recyclable than $\pi(3)$ satisfy $\lambda^{\pi(5)}>\lambda^{\pi(4)}>\lambda^{\pi(8)}>\lambda^{\pi(7)}>\lambda^{\pi(3)}$. Since $\tau_{r(3)}-\tau_{r(4)}<0$ and $\tau_{r(4)}-\tau_{r(5)}<0$, we have $I_1^1(T)<I_1^1(S)$ and $I_2^1(T)<I_2^1(S)$. Similarly, since $\tau_{r(5)}-\tau_{r(7)}<0$, we have $I_3^1(T)<I_3^1(S)$. Moreover, we have $\tau_{r(7)}-\tau_{r(8)}<0$ and thus $I_4^1(T)<0$. Hence, we conclude that $\Delta^1(T,i)=I_1^1(T)+I_2^1(T)+I_3^1(T)+I_4^1(T)<I_1^1(S)+I_2^1(S)+I_3^1(S)=\Delta^1(S,i)$.

We can show that $\Delta^2(T,i)<\Delta^2(S,i)$ in this example in a similar way. The calculations are shown below.

$$
\begin{aligned}
\Delta^2(T,i) \;&=\; \lambda^{\pi(5)}\cdot\big(\tau_{r(3)}-\tau_{r(5)}\big)+\lambda^{\pi(8)}\cdot\big(\tau_{r(5)}-\tau_{r(8)}\big)+\lambda^{\pi(10)}\cdot\big(\tau_{r(8)}-\tau_{r(10)}\big)\\
&=\; \underbrace{\lambda^{\pi(5)}\cdot\big(\tau_{r(3)}-\tau_{r(4)}\big)}_{I_1^2(T)}+\underbrace{\lambda^{\pi(5)}\cdot\big(\tau_{r(4)}-\tau_{r(5)}\big)+\lambda^{\pi(8)}\cdot\big(\tau_{r(5)}-\tau_{r(7)}\big)}_{I_2^2(T)}\\
&\quad+\underbrace{\lambda^{\pi(8)}\cdot\big(\tau_{r(7)}-\tau_{r(8)}\big)+\lambda^{\pi(10)}\cdot\big(\tau_{r(8)}-\tau_{r(9)}\big)}_{I_3^2(T)}+\underbrace{\lambda^{\pi(10)}\cdot\big(\tau_{r(9)}-\tau_{r(10)}\big)}_{I_4^2(T)}\\
&<^1\; \underbrace{\lambda^{\pi(4)}\cdot\big(\tau_{r(3)}-\tau_{r(4)}\big)}_{I_1^2(S)}+\underbrace{\lambda^{\pi(7)}\cdot\big(\tau_{r(4)}-\tau_{r(7)}\big)}_{I_2^2(S)}+\underbrace{\lambda^{\pi(9)}\cdot\big(\tau_{r(7)}-\tau_{r(9)}\big)}_{I_3^2(S)}=\Delta^2(S,i) \qquad (39)
\end{aligned}
$$

The inequality $<^1$ holds as we can show that $I_1^2(T)<I_1^2(S)$, $I_2^2(T)<I_2^2(S)$, $I_3^2(T)<I_3^2(S)$, and $I_4^2(T)<0$. Hence, we conclude that $v(T\cup\{i\})-v(T)-v(i)=\Delta^1(T,i)+\Delta^2(T,i)<\Delta^1(S,i)+\Delta^2(S,i)=v(S\cup\{i\})-v(S)-v(i)$ in this example.

We can extend the decomposition and the rearrangement techniques illustrated in the above example to show $\Delta^1(T,i)\le\Delta^1(S,i)$ and $\Delta^2(T,i)\le\Delta^2(S,i)$ in the general case, which leads to the inequality $v(T\cup\{i\})-v(T)-v(i)<v(S\cup\{i\})-v(S)-v(i)$ by formula (34). Please refer to the online technical companion for details. □

A.3. Technical Details in §6

A.3.1. Examples of the Linear Design-based Allocation We first show that the dual-based allocation x_d, the marginal contribution-based allocation x_m, and the allocation by return share are all piecewise linear functions of the design variables.

Dual-based allocation x_d: Based on duality theory, the dual optimal solution is a linear function of the objective function coefficients of the primal problem given the optimal basis[4]. Hence, we know that both $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ are piecewise linear in the objective function coefficients of the centralized allotment problem (C), which are the unit recycling cost parameters $\{c_r^\pi\}$. Since each c_r^π parameter is linear in the design variables, we know that $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ are also piecewise linear the in design variables, and so is each producer i's dual-based allocation x_d^i according to its definition in (6).

[4] See page 148 in *Introduction to Linear Optimization* by Bertsimas D. and Tsitsiklis J. N.

Marginal contribution-based allocation x_m: Based on the way that the formula of x_m is written in (29), we know that each x_m^i is linear in the unit recycling cost parameters $\{c_r^\pi\}$ under a given product allotment solution within the coalition $\{j : j \geq i\}$. Thus x_m is a piecewise linear function in the design variables.

Allocation by return share: The allocation by return share to each producer i can be calculated as

$$x_r^i = \frac{d^{\pi(i)}}{\sum_{i\in N} d^{\pi(i)}} \cdot v(N) \tag{40}$$

Since $v(N) = \sum_{\pi\in\Pi}\sum_{r\in R} c_r^\pi \cdot f^*_{[\pi,r]}$, we know that $v(N)$ is linear in the design variables under a given optimal allotment f^*. Hence, x_r^i is a piecewise linear function in the design variables.

A.3.2. Technical Details of Theorem 1

Proof of Theorem 1. We need to prove that the three statements presented in Theorem 1 are equivalent. The following has already been proven.

- Proposition 4 indicates that statement 1 implies statement 2.
- Lemma 1 and Proposition 2 combined indicate that statement 2 implies statement 3.

In this proof, we further show that statement 3 implies statement 1, that is, assuming there exists a linear design-based mechanism x_l that is group incentive compatible and guarantees superior design incentives, we show that the RN instance is a low-synergy one. For convenience, the proof below focuses on the case where the base cost to recycle each product π is zero, i.e., $\bar{c}^\pi = 0$. In the general case, we can include the base cost as part of the side payment parameter b_u^i, and apply the same argument.

STEP 1: Consider any producer i. Since the allocation x_l guarantees superior design incentives, we know that a unit design improvement in $\pi(i)$ leads to at least as large a cost reduction in x_l^i as in i's stand-alone recycling cost. This indicates that for each linear piece in the interval I_u,

$$\frac{\partial x_l^i}{\partial \lambda^{\pi(i)}} = a_u^i \geq \frac{\partial v(i)}{\partial \lambda^{\pi(i)}} = d^{\pi(i)} \cdot \tau_{r(i)} \tag{41}$$

holds. This implies that $a_u^i \cdot \lambda^{\pi(i)} \geq d^{\pi(i)} \cdot \tau_{r(i)} \cdot \lambda^{\pi(i)} = v(i)$. Hence, for any sub-coalitions S and T such that $S \cup T = N$ and $S \cap T = \{i\}$, we can calculate that $\forall u = 1, 2, ..., U$,

$$v(i) + b_u^i \leq a_u^i \cdot \lambda^{\pi(i)} + b_u^i = \sum_{j\in S} x_l^j + \sum_{j\in T} x_l^j - \sum_{j\in N} x_l^j \leq^1 v(S) + v(T) - v(N) \tag{42}$$

The inequality $\leq^1$ in the above formula is due to (i) the group incentive compatibility of the allocation mechanism x_l, i.e., $\sum_{j\in S} x_l^j \leq v(S)$ and $\sum_{j\in T} x_l^j \leq v(T)$, and (ii) the fact that x_l allocates all the cost, i.e., $\sum_{j\in N} x^j = v(N)$. By rearranging the terms, we know that the inequality $b_u^i \leq v(S) + v(T) - v(N) - v(i)$ must hold. We further rewrite the inequality as follows: For every coalition T, let $cost_T^i$ denote the part of $v(T)$ that depends on $\lambda^{\pi(i)}$. Hence, $b_u^i \leq v(S) + v(T) - v(N) - v(i)$ can be written as

$$cost_S^i + cost_T^i - cost_N^i - v(i) \geq b_u^i - (v(S) - cost_S^i) - (v(T) - cost_T^i) - (v(N) - cost_N^i) \tag{43}$$

Note that while the left-hand-side of (43) is a piecewise linear function of $\lambda^{\pi(i)}$ (since for each coalition T, $v(T)$ is a piecewise linear function of $\lambda^{\pi(i)}$), the right-hand-side of the formula is independent of $\lambda^{\pi(i)}$. We next show that if (43) is guaranteed, then the inequality $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ must hold under any design profile. To see this, assume that there exists a design profile Λ_0 under which $cost_S^i + cost_T^i - cost_N^i - v(i) < 0$.

Consider another design profile Λ_h where i's design choice equals $\lambda_h^{\pi(j)} = h \cdot \lambda_0^{\pi(j)}$, and other producers' design choices equal $\lambda_h^{\pi(j)} = \lambda_0^{\pi(j)}$ $\forall j \neq i$. It can be observed that under Λ^h, the left-hand-side of (43) is h times the value calculated under Λ_0, i.e., it linearly decreases in h. Meanwhile, the right-hand-side of (43) equals the value calculated under Λ_0. In this case, we can always find a sufficiently large h value under which (43) is violated. Thus we conclude that $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ must hold in order to ensure the inequality in (43).

STEP 2: Next we show that if $\forall i \in N$, $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ holds for any two coalitions S and T such that $S \cup T = N$ and $S \cap T = \{i\}$, then the RN instance must satisfy the condition $d^{\pi(i)} \geq \sum_{j<i} k_{r(j)}$ $\forall i > 2$. To do this, consider any producer $i \geq 3$, $S = \{1, 2, ..., i-2, i\}$, and $T = \{i-1, i, i+1, ..., n\}$. Consider a product design profile where $\pi(i)$ is the least recyclable, i.e., $\lambda^{\pi(i)}$ is the largest among all products. According to the greedy allotment algorithm (Algorithm 2 in the proof of Proposition 1), we know that $\pi(i)$ has the priority to use the most cost efficient capacity. Hence, we know that

- Comparing to producer i's operating alone, the cost to recycle $\pi(i)$ is reduced in the sub-coalition T if $\pi(i)$ is allotted to processor $r(i-1)$, which is more efficient than $r(i)$.
- The cost savings in recycling $\pi(i)$ in the grand coalition N versus in the sub-coalition S is also derived from re-allotting the portion of the $\pi(i)$ volume that is recycled at $r(i)$ when $i \in S$ to $r(i-1)$ when $i \in N$. Hence,

—when $d^{\pi(i)} \leq \sum_{j=1}^{i-2} k_{r(j)}$, the entire volume of $\pi(i)$ is already recycled at more efficient processors in S. Thus zero cost saving is generated when i participates in the grand coalition N.

—when $\sum_{j=1}^{i-2} k_{r(j)} < d^{\pi(i)} < \sum_{j=1}^{i-1} k_{r(j)}$, $d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}$ amount of $\pi(i)$ is recycled at $r(i)$ in S and will be re-allotted to $r(i-1)$ when i participates in N.

—when $d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)}$, $d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}$ amount of $\pi(i)$ is recycled at $r(i)$ in S. However, this volume exceeds the capacity at $r(i-1)$, out of which $k_{r(i-1)}$ amount of $\pi(i)$ will be re-allotted to $r(i-1)$ when i participates in N.

Accordingly, we can calculate that

$$cost_T^i - v(i) = \min\{d^{\pi(i)}, k_{r(i-1)}\} \cdot (\tau_{r(i-1)} - \tau_{r(i)}) \cdot \lambda^{\pi(i)} \tag{44}$$

$$cost_S^i - cost_N^i = \begin{cases} 0 \quad \text{if } d^{\pi(i)} \leq \sum_{j=1}^{i-2} k_{r(j)} \\ (d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}) \cdot (\tau_{r(i)} - \tau_{r(i-1)}) \cdot \lambda^{\pi(i)} \quad \text{if } \sum_{j=1}^{i-2} k_{r(j)} < d^{\pi(i)} < \sum_{j=1}^{i-1} k_{r(j)} \\ k_{r(i-1)} \cdot (\tau_{r(i)} - \tau_{r(i-1)}) \cdot \lambda^{\pi(i)} \quad \text{if } d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)} \end{cases} \tag{45}$$

Since we assume that $\tau_{r(i-1)} < \tau_{r(i)}$, it is obvious that when $d^{\pi(i)} \leq \sum_{j=1}^{i-2} k_{r(j)}$, $cost_S^i + cost_T^i - cost_N^i - v(i) = cost_T^i - v(i) < 0$. In the second situation where $\sum_{j=1}^{i-2} k_{r(j)} < d^{\pi(i)} < \sum_{j=1}^{i-1} k_{r(j)}$, since $d^{\pi(i)} - \sum_{j=1}^{i-2} k_{r(j)}$ is strictly smaller than both $d^{\pi(i)}$ and $k_{r(i-1)}$, we can calculate that $cost_S^i + cost_T^i - cost_N^i - v(i) < 0$ as well. In the third situation, since $d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)}$, we have $\min\{d^{\pi(i)}, k_{r(i-1)}\} = k_{r(i-1)}$, and thus we can calculate that $cost_S^i + cost_T^i - cost_N^i - v(i) = 0$. Hence, we conclude that $cost_S^i + cost_T^i - cost_N^i - v(i) \geq 0$ only holds in the third situation, i.e., where $d^{\pi(i)} \geq \sum_{j=1}^{i-1} k_{r(j)}$. Since this inequality must hold $\forall i \geq 3$, we conclude that the low synergy condition should be satisfied. This completes the proof that statement 3 implies statement 1 in Theorem 1. Combining this result with Proposition 4, Lemma 1 and Proposition 2, we derive Theorem 1. $\square$

The proof of Theorem 1 provides insights into the tradeoff between design incentives and group incentive compatibility. Specifically, we can derive the following corollary from the proof.

COROLLARY 1. *Given an RN instance, if the PA game is not guaranteed to be weakly convex, then for any linear design-based allocation x_l that is group incentive compatible, there must exist a producer i and an interval I_u in which the linear piece of the allocation satisfies $a_u^i < \tau_{r(i)} \cdot d^{\pi(i)}$.*

The corollary indicates that when a PA game is not weakly convex, group incentive compatibility of a linear design-based allocation requires an additional condition on at least one parameter a_u^i, i.e., it should be smaller than a threshold. This observation echoes the existing studies on convex cooperative games, which show that group incentive compatibility is harder to achieve without convexity of the cooperative game. In the problem context considered in this paper, this additional constraint means that there must be at least one producer i who is charged a strictly smaller "design cost" than when i operates individually. This undermines producer i's design incentives. This provides a technical explanation of the root cause of the tradeoff between design incentives and coalitional stability in a collective implementation.

A.4. Technical Details in §7

Proof of Proposition 5. Consider an RN instance where design improvement leads to higher cost savings at more efficient processors (as it is modeled in Assumption 2). We first consider the case where the dual-based cost allocation is adopted. We prove that under the condition $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \le k_{r(i)}\ \forall i \in N$, any pure-strategy equilibrium design profile Λ_d^{ne} is superior to the optimal design profile induced by an individual system.

STEP 1: We analyze the socially optimal allotment f^* given a design profile under the network condition $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \le k_{r(i)}$. We first show that in such an RN, the total volume of less recyclable products than $\pi(i)$ (i.e., $\sum_{j:\lambda^{\pi(j)}>\lambda^{\pi(i)}} d^{\pi(j)}$) does not exceed the total capacity at processors that are at least as efficient as $r(i)$ (i.e., $\sum_{l:\tau_{r(l)}\le\tau_{r(i)}} k_{r(l)}$). To show this, let J_1 and J_2 be two sets where $J_1 = \{j : \lambda^{\pi(j)} > \lambda^{\pi(i)}, \tau_{r(j)} > \tau_{r(i)}\}$ and $J_2 = \{j : \lambda^{\pi(j)} > \lambda^{\pi(i)}, \tau_{r(j)} < \tau_{r(i)}\}$. We can calculate that

$$\sum_{j:\lambda^{\pi(j)}>\lambda^{\pi(i)}} d^{\pi(j)} = \sum_{j\in J_1} d^{\pi(j)} + \sum_{j\in J_2} d^{\pi(j)} \le \sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} + \sum_{j\in J_2} d^{\pi(j)} \le^1 k_{r(i)} + \sum_{j\in J_2} k_{r(j)} \le \sum_{l:\tau_{r(l)}\le\tau_{r(i)}} k_{r(l)}\,. \tag{46}$$

The inequality $\le^1$ in the above formula is due to the condition that $\sum_{j:\tau_{r(j)}>\tau_{r(i)}} d^{\pi(j)} \le k_{r(i)}$, and the assumption that $d^{\pi(j)} \le k_{r(j)}\ \forall j$. According to the greedy allotment algorithm (Algorithm 2 introduced in the proof of Proposition 1), we know that under the socially optimal allotment f^*, all products less recyclable than $\pi(i)$ will be recycled at processors that are at least as efficient as $r(i)$. Accordingly, we know that only one of the following two situations can happen under f^*.

Situation (a) $\pi(i)$ is recycled at either processors that are more efficient than $r(i)$ or at $r(i)$ itself;

Situation (b) $\pi(i)$ is recycled at a processor less efficient than $r(i)$, and if the capacity at $r(i)$ is increased by 1 unit, this additional unit of capacity will be used to process $\pi(i)$.

STEP 2: We continue to analyze the equilibrium design profile when the dual-based cost allocation is used. Let f^*_{ne} denote the socially optimal allotment under the equilibrium design profile. Based on a similar argument as in the proof of Proposition 1, we can show that the design choice for producer i under a pure strategy equilibrium design profile equals $(Q^{i\prime})^{-1}(-\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}}) = (Q^{i\prime})^{-1}(-\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} - \frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot k_{r(i)})$, where $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ are optimal dual solutions corresponding to f^*_{ne}. Furthermore, based on equations (23)-(24), we can calculate the derivatives of $\beta^{*\pi(i)}$ and $\alpha^*_{r(i)}$ with respect to $\lambda^{\pi(i)}$ as follows.

- If situation (a) occurs under f^*_{ne}, then

1. there exists $r(j_0)$ more or equally efficient as $r(i)$ such that $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} = \frac{\partial c^{\pi(i)}_{r(j_0)}}{\partial \lambda^{\pi(i)}}$. Hence, according to Assumption 2, we know that $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \geq \frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}}$.
2. $\frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} = 0$.

Hence, the equilibrium design choice for producer i satisfies

$$(\lambda^{ne}_d)^{\pi(i)} = (Q^{i\prime})^{-1}(-\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}}) = (Q^{i\prime})^{-1}(-\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) \leq^2 (Q^{i\prime})^{-1}(-\frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) = \lambda^{\pi(i)}_{ind} \tag{47}$$

The inequality $\leq^2$ in the above formula is derived due to the fact that Q^i is a convex decreasing function.

- If situation (b) occurs under f^*_{ne}, then there exists $r(j_0)$ less efficient than $r(i)$ such that

1. $\frac{\partial \beta^{*\pi(i)}}{\partial \lambda^{\pi(i)}} = \frac{\partial c^{\pi(i)}_{r(j)}}{\partial \lambda^{\pi(i)}}$;
2. $\frac{\partial \alpha^*_{r(i)}}{\partial \lambda^{\pi(i)}} = \frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}}$

Hence, we can calculate that

$$\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}} = \frac{\partial c^{\pi(i)}_{r(j)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} + \frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}} \cdot k_{r(i)} \geq^3 \frac{\partial c^{\pi(i)}_{r(j)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} + \frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} = \frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} \tag{48}$$

The inequality $\geq^3$ is due to the fact that (i) $r(j)$ is less efficient than $r(i)$, thus according to Assumption 2, $\frac{\partial \left(c^{\pi(i)}_{r(i)} - c^{\pi(i)}_{r(j)}\right)}{\partial \lambda^{\pi(i)}} \geq 0$, and (ii) $k_{r(i)} \geq d^{\pi(i)}$. Hence, we can conclude that in this situation, producer i's equilibrium design choice satisfies

$$(\lambda^{ne}_d)^{\pi(i)} = (Q^{i\prime})^{-1}(-\frac{\partial x^i_d}{\partial \lambda^{\pi(i)}}) \leq (Q^{i\prime})^{-1}(-\frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}) = \lambda^{\pi(i)}_{ind} \tag{49}$$

Since (47) - (49) hold for each producer i, we conclude that $\Lambda^{ne}_d \leq \Lambda_{ind}$. This completes the proof of the first result in Proposition 5.

Next we consider the marginal contribution-based allocation $\bar{x}_m$ proposed in Proposition 5. Recall that in this appendix, we assume that $i < j$ indicates $\tau_{r(i)} < \tau_{r(j)}$. Hence, the allocation can be rewritten as $\bar{x}^i_m = v(i \cup \{j : j < i\}) - v(\{j : j < i\})\ \forall i \in N$.

The individual rationality of the allocation is due to the sub-additivity of the PA game, which is shown in the proof of Proposition 3. To show that $\bar{x}_m$ induces superior design incentives, we adopt the same arguments used in the proof of Proposition 2. That is, we define $f^{*\{j:j\leq i\}}$ as the optimal product allotment in the sub-coalition $\{j : j \leq i\}$, and rewrite producer i's cost allocation x^i_m as

$$\bar{x}^i_m = \underbrace{\sum_{t=1}^{i} f^{*\{j:j\leq i\}}_{[\pi(i),r(t)]} \cdot c^{\pi(i)}_{r(t)}}_{\substack{\text{processing cost of } \pi(i) \\ \text{in the coalition } \{j:j\leq i\}}} + \underbrace{\sum_{l=1}^{i-1}\sum_{t=1}^{i} f^{*\{j:j\leq i\}}_{[\pi(l),r(t)]} \cdot c^{\pi(l)}_{r(t)} - v(\{j : j < i\})}_{\substack{\text{recycling cost reduction for products } \pi(l)\ \forall l<i \\ \text{after producer } i \text{ joins the coalition}}} \tag{50}$$

Under Assumption 2, we know that $\forall t < i$, processor $r(t)$ is more efficient than $r(i)$ and $\frac{\partial c^{\pi(i)}_{r(t)}}{\partial \lambda^{\pi(i)}} > \frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}}$ holds. Hence, for each i, we have

$$\begin{aligned}\frac{\partial \bar{x}^i_m}{\partial \lambda^{\pi(i)}} &= \sum_{t=1}^{i} f^{*\{j:j\le i\}}_{[\pi(i),r(t)]} \cdot \frac{\partial c^{\pi(i)}_{r(t)}}{\partial \lambda^{\pi(i)}} \ge \frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)} \\ \Rightarrow \quad & (\lambda^{ne}_m)^{\pi(i)} = (Q^{i\prime})^{-1}\left(-\frac{\partial \bar{x}^i_m}{\partial \lambda^{\pi(i)}}\right) \le (Q^{i\prime})^{-1}\left(-\frac{\partial c^{\pi(i)}_{r(i)}}{\partial \lambda^{\pi(i)}} \cdot d^{\pi(i)}\right) = \lambda^{\pi(i)}_{ind} \end{aligned} \tag{51}$$

That is, the equilibrium design profile under the allocation $\bar{x}^i_m$ is superior than that induced by an individual system. This completes the proof of the second result in Proposition 5.

Proof of Proposition 6. The proof is based on similar arguments used in the proof of Theorem 1. Please see the online technical companion for details. □

A.4.1. Technical Details in §7.3 In this section, we present the detailed analysis of the extension to return volume uncertainty. First, we introduce the additional notation used in this study. Let P_i and S_i denote the probability density function and the sample space of each random variable $d^{\pi(i)}$, respectively. The expected volume of $\pi(i)$ is denoted by $E(d^{\pi(i)})$. Each producer is assumed to make the design choice based on the *expected* recycling cost under EPR implementation.

- For each producer i, its optimal design choice induced by an individual system is defined as the value of $\lambda^{\pi(i)}$ that minimizes its expected total cost $c^{\pi(i)}_{r(i)} \cdot E(d^{\pi(i)}) + Q^i(\lambda^{\pi(i)})$. This optimal choice can be calculated as $\lambda^{\pi(i)}_{ind} = (Q^{i\prime})^{-1}\left(-E(d^{\pi(i)}) \cdot \tau_{r(i)}\right)$.
- In a collective system with a cost allocation mechanism x, due to return volume uncertainty, the cost allocation to each producer i is a random variable as well, whose expectation is denoted as $E(x^i)$. The equilibrium design profile under the mechanism x is defined as the set of design choices $\{(\lambda^{ne}_x)^{\pi(i)}\}$ such that $E_d\left(x^i\left(\Lambda^{ne}_x\right)\right) + Q^i\left((\lambda^{ne}_x)^{\pi(i)}\right) \le E_d\left(x^i\left(\lambda^{\pi(i)} \cup (\Lambda^{ne}_x)^{-i}\right)\right) + Q^i\left(\lambda^{\pi(i)}\right)$ holds $\forall \lambda^{\pi(i)}$.

Based on the above notation, we first show that given an RN instance under volume uncertainty, producer 1 is guaranteed to have superior design incentives in a collective system with the dual-based allocation if the condition $\frac{Pr(d^{\pi(2)} < k_{r(1)})}{Pr(d^{\pi(2)} \ge k_{r(1)})} \le \frac{E(d^{\pi(1)})}{k_{r(1)}}$ is satisfied. Note that based on the proof of Proposition 7, we know that whether superior design incentives can be achieved is determined by the rate of change of the cost allocation in design. Based on a similar argument, we can show that under return volume uncertainty, this holds true if we consider the expected cost allocation. Hence, in the following, we first calculate the rate at which the expected dual-based cost allocation to producer 1 $E(x^1_d)$ changes with respect to the design variable $d^{\pi(1)}$. According to the proof of Proposition 7, we know that the formula that calculates x^1_d depends on whether the RN realization satisfies $d^{\pi(2)} \ge k_{r(1)}$ (which defines a low-synergy RN), $d^{\pi(2)} < k_{r(1)} < d^{\pi(1)} + d^{\pi(2)}$ or $k_{r(1)} \ge d^{\pi(1)} + d^{\pi(2)}$ (which define the two situations of a high-synergy RN). Hence, for convenience, we denote the dual-based allocation to producer 1 under these three situations as $(x^1_d)_l$, $(x^1_d)^1_h$ and $(x^1_d)^2_h$. We also divide the sample space $S_1 \times S_2$ into three parts that correspond to these three situations, and denote them as $\boldsymbol{s}_l$, $\boldsymbol{s}^1_h$ and $\boldsymbol{s}^2_h$. Thus we can calculate that

$$E\left(x^1_d\right) = \int_{\boldsymbol{s}_l} (x^1_d)_l \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}^1_h} (x^1_d)^1_h \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}^2_h} (x^1_d)^2_h \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) \tag{52}$$

Since $(x_d^1)_l$, $(x_d^1)_h^1$ and $(x_d^1)_h^2$ are all piecewise linear functions of $\lambda^{\pi(1)}$ whose formulas are as shown in equations (11), (16) and (22), we can calculate that

$$\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}} = \int_{\boldsymbol{s}_l} \frac{\partial (x_d^1)_l}{\partial \lambda^{\pi(1)}} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}_h^1} \frac{\partial (x_d^1)_h^1}{\partial \lambda^{\pi(1)}} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) + \int_{\boldsymbol{s}_h^2} \frac{\partial (x_d^1)_h^2}{\partial \lambda^{\pi(1)}} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)}) \tag{53}$$

By replacing $(x_d^1)_l$, $(x_d^1)_h^1$ and $(x_d^1)_h^2$ with equations (11), (16) and (22), we can derive the following (please refer to the online technical companion for calculation details).

$$\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}} = \begin{cases} (\bar{d}_l + \bar{d}_h^1) \cdot \tau_{r(2)} - p_h^1 \cdot k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \bar{d}_h^2 \cdot \tau_{r(1)} & \text{if } \lambda^{\pi(1)} < \lambda^{\pi(2)} \\ E(d^{\pi(1)}) \cdot \tau_{r(1)} \quad \text{if } \lambda^{\pi(1)} \geq \lambda^{\pi(2)} \end{cases} \tag{54}$$

where the notation $\bar{d}_l$ is defined as $\int_{\boldsymbol{s}_l} d^{\pi(1)} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)})$, i.e., the expected volume of product $\pi(1)$ if the realization of the RN is a low-synergy one. Similarly, we denote $\bar{d}_h^t \doteq \int_{\boldsymbol{s}_h^t} d^{\pi(1)} \cdot P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)})$ $t = 1, 2$ as the expected volume of $\pi(1)$ in each of the two scenarios of a high-synergy RN. The notation p_h^1 denotes the probability that the RN realization belongs to the first scenario of the high-synergy case, i.e., $\int_{\boldsymbol{s}_h^1} P_1 P_2 \boldsymbol{d}(d^{\pi(1)})\boldsymbol{d}(d^{\pi(2)})$.

Based on the equilibrium analysis in §A.1.1, we know that in order for producer 1's equilibrium design outcome to be superior than its optimal design in an individual system, i.e., $(Q^{i\prime})^{-1}\left(-E(d^{\pi(1)}) \cdot \tau_{r(1)}\right)$, the rate at which its expected cost allocation changes in $\lambda^{\pi(1)}$, i.e., $\frac{\partial E\left(x_d^1\right)}{\partial \lambda^{\pi(1)}}$, should be at least as large as $E(d^{\pi(1)}) \cdot \tau_{r(1)}$. Accordingly, since it can be calculated that $\bar{d}_l + \bar{d}_h^1 + \bar{d}_h^2 = E(d^{\pi(1)})$, we derive the following condition based on formula (54):

$$(\bar{d}_l + \bar{d}_h^1) \cdot \tau_{r(2)} - p_h^1 \cdot k_{r(1)} \cdot (\tau_{r(2)} - \tau_{r(1)}) + \bar{d}_h^2 \cdot \tau_{r(1)} \geq E(d^{\pi(1)}) \cdot \tau_{r(1)} \quad \Rightarrow \quad p_h^1 \leq \frac{\bar{d}_l + \bar{d}_h^1}{k_{r(1)}} \tag{55}$$

Hence, we conclude that producer 1 is guaranteed to have superior design incentives in a collective implementation under the dual-based cost allocation than its operating alone if and only if the probability associated with the first scenario of low-synergy RNs is smaller than a threshold, i.e., $p_h^1 \leq \frac{\bar{d}_l + \bar{d}_h^1}{k_{r(1)}}$. To better demonstrate this insight, we next consider a sufficient condition that ensures the above inequality. That is, since p_h^1 is at most as large as the total probability of high-synergy RNs, i.e., $Pr\left(d^{\pi(2)} < k_{r(1)}\right)$, $p_h^1 \leq \frac{\bar{d}_l + \bar{d}_h^1}{k_{r(1)}}$ holds if $Pr\left(d^{\pi(2)} < k_{r(1)}\right) \leq \frac{\bar{d}_l}{k_{r(1)}}$. Moreover, note that the sample space of the volume parameters that correspond to the low-synergy RNs equals $s_l = S_1 \times [k_{r(1)}, \infty)$. Thus we can calculate that $\bar{d}_l = \int_{S_1} d^{\pi(1)} \cdot P_1 d(d^{\pi(1)}) \int_{[k_{r(1)}, \infty)} P_2 d(d^{\pi(2)}) = E(d^{\pi(1)}) \cdot Pr\left(d^{\pi(2)} \geq k_{r(1)}\right)$. Hence, the inequality $Pr\left(d^{\pi(2)} < k_{r(1)}\right) \leq \frac{\bar{d}_l}{k_{r(1)}}$ is equivalent to $Pr\left(d^{\pi(2)} < k_{r(1)}\right) \leq \frac{E(d^{\pi(1)}) \cdot Pr\left(d^{\pi(2)} \geq k_{r(1)}\right)}{k_{r(1)}}$, that is,

$$\frac{Pr\left(d^{\pi(2)} < k_{r(1)}\right)}{Pr\left(d^{\pi(2)} \geq k_{r(1)}\right)} \leq \frac{E(d^{\pi(1)})}{k_{r(1)}} \tag{56}$$

However, note that if the probability of high-synergy RNs is strictly larger than zero, a superior design profile cannot be ensured in a two-producer RN. This is because producer 2 will make a strictly inferior design choice based on its expected dual-based cost allocation. To see this, note that with fixed return volumes, the rate at which producer 2's cost allocation x_d^2 changes in design is smaller or equal to the rate of change of its stand-alone cost (see Table 2 and the proof of Proposition 7). Hence, producer 2 will at best choose the same design outcome as in an individual system, and this is only ensured if the RN is design-reinforcing (e.g., in the first RN instance considered in Example 1). Hence, in the probabilistic case, even a small probability associated with non-design-reinforcing RNs undermines the rate at which producer 2's expected cost allocation changes in design, and thus leads to strictly inferior design incentives. We generalize this finding to RNs with any number of producers in the next proposition.

PROPOSITION 8. *Given an RN instance under return volume uncertainty and the probability distribution* P_i *of each volume parameter* $d^{\pi(i)}$*, the following statements are equivalent.*

1. *The probability that the RN realization is design reinforcing is one, i.e., the sample space of* $d^{\pi(i)}$ *is contained in the interval* $[\sum_{j:\tau_{r(j)}<\tau_{r(i)}} k_{r(j)}, \infty)$ $\forall i > 2$.
2. *There exists a linear design-based allocation mechanism* x_l *such that (i)* x_l *is group incentive compatible, and (ii) any equilibrium design profile under* x_l *is superior to the design profile induced by an individual system, i.e.,* $\Lambda_l^{ne} \leq \Lambda_{ind}$.

Proof of Proposition 8. The proof is based on similar arguments used in the proof of Theorem 1. Please see the online technical companion for details. □