The high cost of lenient return policies force consumer electronics OEMs to look for ways to recover value from lightly used consumer returns, which constitute a substantial fraction of sales and cannot be re-sold as new products. Refurbishing to remarket or to fulfill warranty claims are the two common disposition options considered to unlock the value in consumer returns, which present the OEM with a challenging problem: How should an OEM dynamically allocate consumer returns between fulfilling warranty claims and remarketing refurbished products over the product’s life-cycle? We analyze this dynamic allocation problem and find that when warranty claims and consumer returns are jointly taken into account, the remarketing option is generally dominated by the option of refurbishing and earmarking consumer returns to fulfill warranty claims. Over the product’s life-cycle, the OEM should strategically emphasize earmarking of consumer returns at the early stages of the life-cycle to build up earmarked inventory for the future warranty demand, whereas it should consider remarketing at the later stages of the life-cycle after enough earmarked inventory is accumulated or most of the warranty demand uncertainty is resolved. These findings show that, for product categories with significant warranty coverage and refund costs, remarketing may not be the most profitable disposition option even if the product has strong remarketing potential and the OEM has the pricing leverage to tap into this market. We also show that the optimal dynamic disposition policy is a price-dependent base-stock policy where the earmarked quantity is capacitated by the new and refurbished product sales quantities. We compare with the myopic policy and show that it is a good heuristic for the optimal dynamic disposition policy.

**Key words:** consumer returns, consumer electronics, warranty, refurbishing, closed-loop supply chains

1. *Introduction*

Consumer returns are products that are purchased by the consumer from the manufacturer or a retailer and then returned for a refund within the time window allowed by the return policy. In
the U.S. market, consumer returns have been estimated at $200 billion per year and average 8.2 percent of total retail sales (Greve, 2011). In the consumer electronics sector, which is the focus of this paper, consumer returns require testing prior to the disposition decision, and are typically returned to the OEM for full credit for this purpose. Despite the fact that the majority of these returns are found to have no defects in their intended functionality (Accenture 2008, Ferguson et al. 2006), litigation concerns prevent OEMs from returning them to the new product distribution channel. Thus, consumer returns represent a significant cost to consumer electronics manufacturers (King 2013); yet, prevalent industry practice reinforces the notion that full-refund return policies will continue to be offered due to competitive pressures (Shang et al. 2014, 2015). These companies view no-defect-found consumer returns as a necessary cost of doing business, and are increasingly focusing on the ways to recapture value from them.

Refurbishing consumer returns provides the OEM with the possibility of recapturing value in two ways: savings in the cost of warranty claims or revenues from remarketing (selling as refurbished product). To honor warranty agreements, OEMs are obliged to either repair the failed product, which is usually not cost effective for most failure categories of consumer electronics, or replace it with a functional product, which can be a refurbished product. When refurbishing is cheaper than manufacturing, fulfilling a warranty claim with a refurbished product instead of a new product generates savings in cost. However, refurbishing a consumer return to fill a warranty demand has an opportunity cost: the potential margin that can be earned by remarketing it. On the other hand, remarketing cannibalizes new product sales, and the price of refurbished products should be set in relation to the new product price and in coordination with the amount of expected warranty claims. As such, identifying the best dynamic disposition strategy in face of consumer returns and warranty claims received during the product’s life-cycle is a challenging but important decision for consumer electronics OEMs.

A significant body of academic literature provides guidelines on the profitability of remarketing when warranty claims are ignored and remarketing is considered as the only disposition option (e.g., Debo et al. 2005, Ferguson and Toktay 2006, Ferrer and Swaminathan 2006, Atasu et al. 2008). Other independently developed research streams (cost minimizing disposition strategies and inventory management under warranty service) do not bring together pricing and refurbishing for the dual purposes of remarketing and fulfilling warranty claims. For consumer electronics OEMs, however, the disposition decision lies at the intersection of pricing new and remarkehed products

1 For example, Apple states clearly in the warranty policy that the failed Apple product can be replaced with a device that is “formed from new and/or previously used parts that are equivalent to new in performance and reliability” (https://www.apple.com/legal/warranty/).
and stocking refurbished consumer returns to meet future warranty demand. While prior research concludes that remarketing is a valuable disposition option when new product cannibalization concerns are not significant, there is little guidance about how warranty claims and money-back guarantees jointly affect the profitability of remarketing as well as the OEM’s dynamic disposition strategy. With these motivations, our goal in this paper is to address the following question: How should an OEM dynamically allocate consumer returns between fulfilling warranty claims and remarketing refurbished products over the product’s life-cycle? While answering this question, we also study how the OEM’s dynamic disposition strategy is shaped by the inter-temporal changes in the consumer return rate and warranty demand.

We begin our analysis by studying the problem in a single-period setting where, at the beginning of each period, the OEM decides the prices of the new and refurbished products along with the quantity to be refurbished and earmarked to fill uncertain warranty demand\(^2\). By analyzing the problem in a single-period setting, we can partially characterize the optimal disposition policy in closed-form, provide comparative static analysis and generate insights about key trade-offs. Our main finding is that when warranty claims and consumer returns are taken into account, the profitability of remarketing requires a stricter parameter condition than the one suggested by the earlier literature. Moreover, our numerical analysis shows that remarketing is dominated by earmarking in most cases. This counterintuitive result is driven by the fact that each remarketed product can potentially generate a future warranty coverage cost or refund cost, and therefore, when earmarking is economically attractive, the OEM optimally allocates some consumer returns to the earmarking option to reduce these costs. Interestingly, the cost reduction effect of earmarking can enable the OEM to remarket returned products more aggressively than when solely remarketing. Our analytical comparisons reveal that, in certain cases, the OEM sells more refurbished products when earmarking some of the returns than without it.

We next consider the multi-period problem as a natural extension of the single-period problem such that, in every period, the surplus earmarked inventory is available to meet warranty demand in subsequent periods, and the OEM jointly decides the quantity to be earmarked in that period together with the prices of new and refurbished products. We show that the optimal dynamic disposition policy in each period is a price-dependent base-stock policy where the earmarked quantity is capacitated by the new and refurbished product sales quantities, which are endogenously determined by the OEM’s pricing decisions. Via numerical analysis, we study the behavior of the optimal

\(^2\)For brevity, in the rest of the paper, we refer to the disposition option of refurbishing and earmarking consumer returns to fulfill warranty claims shortly as the *earmarking* option.
dynamic disposition policy with respect to the inter-temporal changes in the consumer return rate and failure rate. We find that if the consumer return rate is decreasing over time, the optimal earmarking quantity is also decreasing while the optimal refurbished product sales are increasing. This is because a decreasing consumer return rate implies an increasing warranty demand (as more consumers keep their products) and a decreasing refurbishing capacity, and therefore favor building up earmarked inventory at the early stages in the life-cycle. On the other hand, if the consumer return rate is increasing, the OEM faces a relatively high warranty demand with a relatively low refurbishing capacity at the beginning of the life-cycle, and the optimal dynamic policy again favors emphasizing earmarking at the early stages in the life-cycle to fulfill the immediate warranty claims. When the product’s failure rate decreases over time, we observe a similar behavior in the optimal dynamic policy. As such, although the behavior of the optimal dynamic policy can vary depending on the underlying inter-temporal changes, its overall pattern prescribes a consistent disposition strategy.

We conduct an extensive numerical study to further investigate the dynamic allocation of consumer returns between remarketing and earmarking options. We find that, in the majority of the instances, a larger percentage of the consumer returns are allocated to the earmarking option, and throughout the life-cycle, the percentage allocated to earmarking decreases while the percentage allocated to remarketing increases. Thus, our earlier result regarding the behavior of the optimal dynamic policy generalizes to the majority of our practical cases. We also observe that the percentage of consumer returns allocated to the earmarking option is quite robust, since earmarking dominates both remarketing and salvaging options. To better understand the value of the optimal dynamic disposition policy, we numerically compare its profit with the profit of the myopic disposition policy, when the problem parameters are stationary. Our numerical results show that the myopic policy is generally an efficient heuristic for the optimal dynamic policy. The optimal dynamic policy is most beneficial when the consumer return rate, warranty demand uncertainty, remarketing potential, and the manufacturing cost are high while the refurbishing cost and salvage value are low.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature and position our paper. In, Sections 3 and 4, we present and analyze the single-period and multi-period problems. In Section 5, we report our numerical study. In Section 6, we conclude. We refer the reader to the Online Appendix for all proofs.
2. Literature Review

Our paper draws on three different research streams: Closed-loop supply chains, consumer return policies, and inventory management under warranty service. Below, we review the literature on these three research streams and highlight our contribution to each.

The two research streams of the closed-loop supply chains literature that are closest to our problem consider the disposition decision for returned products and market-related issues. The research on the disposition decision focuses on the allocation of the limited amount of returned products to appropriate recovery options such as disposal, dismantling for parts and remanufacturing to sell. The papers considering a single disposition option (typically dismantling) focus on spare parts recovery either for usage throughout the life cycle (e.g., Fleischmann et al. 2003, Ferguson et al. 2011) or to reduce the final order/buy quantities (e.g., Teunter and Fortuin 1999). More complex models involve multiple remanufacturing options (e.g., Inderfurth et al. 2001). These papers focus on the cost side of the disposition decision rather than the value of the recovered products in the refurbished product market. The two exceptions are Ferguson et al. (2011) and Calmon and Graves (2015). Ferguson et al. (2011) consider a scenario where a returned (electronics) product can be dismantled for parts to meet the uncertain spare parts demand, or remanufactured to be sold at an exogenous price under demand uncertainty. Similarly, Calmon and Graves (2015) study the inventory system of refurbished consumer electronics products which are used to serve warranty claims or sold through side-sales channel at an exogenous price. Nevertheless, all papers in the disposition decision literature have two common assumptions: i) the return and demand streams, as well as product prices, are exogenous and independent, and ii) consumers do not differentiate between new and remanufactured products. As such, our model differs from this literature in that we endogenize the OEM’s pricing decisions for new and remanufactured products and take into account consumer preferences over these products.

The papers on market-related issues in closed-loop supply chains essentially focus on the profitability of remarketing under different operational and market conditions (see e.g., Guide and Van Wassenhove 2001, Debo et al. 2005, Ferrer and Swaminathan 2006, Ferguson and Toktay 2006, Atasu et al. 2008, and for a comprehensive review, Souza 2013). Similar to these papers, we also assume a heterogenous consumer base and use a vertical market segmentation model to capture the impact of cannibalization and consumer valuations on the OEM’s remanufacturing strategy. In contrast, however, we consider an additional disposition option, refurbishing consumer returns to fulfill warranty claims, which is also influenced by the pricing decisions of the OEM. Moreover,
a significant portion of this literature focuses on the end-of-use or end-of-life returns that are received after long durations of use, resulting in high levels of variability in their quality (e.g., Guide and Van Wassenhove 2001, Debo et al. 2005, Ferguson and Toktay 2006). In contrast, we study consumer returns, the vast majority of which are barely used. Other papers considering lightly used consumer returns in the closed-loop supply chains context focus on different aspects, such as coordination mechanisms to reduce returns (Ferguson et al. 2006), time-sensitive products (Guide et al. 2006), returns processing (Ketzenberg and Zuidwijk 2009), and profitability of money-back guarantees (Akçay et al. 2013). To our knowledge, the dynamic joint pricing and stocking decisions of the OEM have not been investigated for consumer returns.

Outside the closed-loop supply chains context, the literature on consumer returns focuses on the issue of how, and to what extent, the seller should refund product returns arising from buyers’ remorse or from the lack of fit between product attributes and consumer expectations. Davis et al. (1995), Che (1996) and Moorthy and Srinivasan (1995) analyze the benefits of a full refund policy when consumers are not opportunistic. In case of opportunistic consumers, Davis et al. (1998) and Chu et al. (1998) show that full refund policies are suboptimal. In a series of papers, Shulmann et al. (2009, 2010, 2011) study the impact of the provision of product fit information, competition and reverse channel structure on the form of the return policy. Su (2009) investigates the impact of return policies on supply chain performance and proposes coordination mechanisms taking into account consumer returns. Using transactions data from a major U.S. consumer electronics retailer, Shang et al. (2014) propose an econometric model to estimate consumers’ experience duration and the probability of a return, whereas Shang et al. (2015) empirically investigate the value of money-back-guarantee policies in online retailing. The majority of this literature is devoted to analyzing the trade-off between enforcing stricter return policies (via restocking fees) versus increasing sales by a more service-oriented sales policy. In practice, and counter to the recommendations of this literature stream, the major consumer electronics OEMs and retailers still offer free return policies, at least in the U.S. market (Shang et al. 2014, 2015). In our paper, we take the return policy as given (full refund) and, rather than focusing on the cost versus customer service trade-off, we explore how OEMs can more effectively utilize the products that are returned.

Finally, the literature on inventory management under warranty demand focuses on the effects of future warranty claims on production and stocking decisions of an OEM providing warranty service. As such, the focus of this literature is on operational issues such as the optimal inventory-warranty policy, quality uncertainty of returned products, and the impact of production lot sizes on quality (e.g., Khawam et al. 2007, Huang et al. 2008, Djamaludin et al. 1994). Although the
papers in this literature stream consider the impact of warranty service on the OEM’s stocking decisions, we approach this problem from a more integrated perspective and show how the OEM’s stocking decision under warranty service is influenced by the OEM’s pricing decisions of new and refurbished products.

3. The Single-Period Problem

Our focus is on consumer electronics, which typically have short product life cycles and high depreciation in market value. Consequently, consumer returns (and not end-of-use or end-of-life returns) requiring low-touch refurbishing are the main option for selling refurbished products or meeting warranty demand with anything other than new products. To understand the underlying drivers of the OEM’s joint pricing and stocking problem, we begin our analysis in a single-period setting, where at the beginning of the period, the OEM sets the new and refurbished product prices to determine the sales volumes, and decides the quantity of the consumer returns that will be refurbished and earmarked to fulfill the warranty demand. In other words, the OEM first makes the planning for the whole period in expectation of the consumer returns and warranty claims that it will receive during the period, and then allocates the arriving consumer returns to one of the disposition options (refurbishing to remarket or refurbishing to earmark for warranty demand) based on the initially planned allocation quantities. As such, the single-period framework provides a convenient starting point to analyze the OEM’s complex disposition decisions at an aggregate and strategic level, and it is commonly used in the context of consumer returns and closed-loop supply chains (e.g., Akçay et al. 2013, Ferguson et al. 2006, Su 2009). In Section 4, we relax the single-period assumption and analyze the problem in a multi-period setting.

For consumer electronics products, there is a significant difference between the return time windows of the consumer returns and warranty claims. The consumer returns typically depend only on recent sales due to short time windows of money-back guarantees (14 or 30 days), whereas the warranty claims depend on a longer history of sales since warranty agreements span a significant portion of the product life-cycle (1 or 2 years). This implies that, for the majority of the life-cycle, the warranty claims depend on a larger number of sales compared to the consumer returns, and therefore the relative uncertainty in total warranty demand is typically much higher than the uncertainty in the number of consumer returns. We discussed this point with the CEO of a third party refurbisher and learned that the consumer return rates are consistently in the 8–12% range across all brands, while the warranty demand rates can range from 2–30% (Francis 2012). Moreover, to reduce the consumer returns by increasing the retailers’ sales efforts, consumer electronics OEMs
usually offer target rebate contracts to retailers (Ferguson et al. 2006); on the other hand, the warranty demand can be reduced mainly by design and manufacturing improvements which require longer periods of time (e.g., over successive product generations). Thus, to maintain tractability while capturing the primary drivers of the optimal disposition strategy of a consumer electronics OEM, in our model we attribute all the variability to the warranty demand.

To model the pricing decisions, we assume that consumers are heterogenous according to their willingness-to-pay and that a consumer’s willingness-to-pay for the new product $\theta$ is uniformly distributed within the interval $[0, 1]$. Furthermore, we assume that a consumer’s willingness-to-pay for the refurbished product is a known fraction of its willingness-to-pay for the new product, i.e., $\delta \theta$ with $\delta \in (0, 1)$. Let, $p_n$ and $p_r$ denote the prices for the new and refurbished products, respectively. These assumptions lead to the inverse demand functions $p_n = 1 - D_n - \delta D_r$ and $p_r = \delta(1 - D_n - D_r)$, with $D_n$ and $D_r$ denoting the demand (sales) for new and refurbished products. This demand model derivation is frequently used in the closed-loop supply chain literature (e.g., Agrawal et al. 2012, Atasu et al. 2008, Ferguson and Toktay 2006, Debo et al. 2005). The OEM incurs a unit cost of $c_n$ to produce a new product and a unit cost of $c_r$ to refurbish a consumer return. To eliminate trivial cases, we let $0 < c_r < c_n$.

Consumer returns are a fraction ($\alpha$) of the total sales, i.e., $R_c(D_n, D_r) = \alpha(D_n + D_r)$ with $\alpha \in (0, 1)$. We refer to $\alpha D_n$ as the new-product consumer returns and $\alpha D_r$ as the refurbished-product consumer returns. For each type of consumer returns, the OEM refunds the selling price of the product to the customer; thus, the total refund cost is equal to $p_n \alpha D_n + p_r \alpha D_r$. Warranty demand (warranty claims), on the other hand, form a separate stream from consumer returns given by $R_w(D_n, D_r, \xi) = \gamma(1 - \alpha)(D_n + D_r) + \xi$, where $\gamma \in (0, 1)$ is the (known) base product failure rate, $1 - \alpha$ is the fraction of sales not returned as consumer returns, and $\xi \in [0, \bar{\xi}]$ is a nonnegative continuous random variable distributed according to $F(\cdot)$. For analytical convenience, we assume that $F(\cdot)$ is strictly increasing in the interval $[0, \bar{\xi}]$, and therefore has an inverse. $R_w(D_n, D_r, \xi)$ is similar to the additive demand function in the price-setting newsvendor models, where the objective is to jointly decide on the replenishment quantity and price of a product to meet stochastic price-dependent demand (e.g., Petruzzi and Dada 1999, Dana and Petruzzi 2001).

The OEM can meet the warranty demand either by using new products or refurbishing consumer returns. Let $Q_r$ denote the earmarked quantity of consumer returns, which is refurbished to satisfy warranty demand during the period. We assume that warranty demand not met by refurbished products is met by new products at the end of the period\(^3\). Thus, $c_n E(R_w(D_n, D_r, \xi) - Q_r)^+$ is the

\(^3\)In the multi-period setting, we relax this assumption and allow warranty demand to be backlogged until enough
expected cost of covering the surplus warranty demand (warranty demand exceeding the earmarked quantity) by using new products. Similarly, there is an overage cost \( h \) incurred per unit of leftover earmarked products. Hence, \( hE(Q_r - R_w(D_n, D_r, \xi))^+ \) is the expected overage cost of the surplus earmarked quantity.

Consumer returns that are not refurbished for either remarketing or warranty demand coverage are salvaged (e.g., recycled or cannibalized for parts) at the end of the period. To keep the model tractable, we assume that the OEM refurbishes a product only once and all refurbished-product consumer returns are salvaged. This also reflects the most common policy from practice, as cores are rarely refurbished more than once. Because the total number of refurbished products cannot be larger than the number of new-product consumer returns, there is a refurbishing capacity constraint \( D_r + Q_r \leq \alpha D_n \). Consequently, the total salvage revenue earned at the end of the period is equal to \( s(\alpha D_n - D_r - Q_r + \alpha D_r) \), where \( s \) is the unit salvage value, \( \alpha D_n - D_r - Q_r \) is the amount of new-product consumer returns that are not refurbished by the end of the period, and \( \alpha D_r \) is the amount of the consumer returns from the refurbished products.

The failed products returned as warranty claims typically lose most of their recoverable value before they are repaired or allocated to a disposition option due to short product life-cycles (Guide et al. 2006, Blackburn et al. 2004). As such, adding a small salvage value for the products returned due to warranty claims in our model would make the warranty demand coverage less expensive but would not alter the qualitative nature of our findings. Thus, in our model, we set the salvage value of the returned products due to warranty claims to zero.

The OEM’s single-period disposition problem is given as follows:

\[
\begin{align*}
\max \Pi(D_n, D_r, Q_r) &= ((1-\alpha)p_n - c_n)D_n + ((1-\alpha)p_r - c_r)D_r - c_rQ_r \\
&\quad - hE(Q_r - R_w(D_n, D_r, \xi))^+ - c_n E(R_w(D_n, D_r, \xi) - Q_r)^+ \\
&\quad + s(\alpha D_n - (1-\alpha)D_r - Q_r),
\end{align*}
\]

subject to \( D_r + Q_r \leq \alpha D_n \) and \( D_n, D_r, Q_r \geq 0 \). In the profit function given above, the first two terms are the net profit from selling new and refurbished products after the refunds for consumer returns are deducted, the third term is the refurbishing cost of the earmarked quantity, the fourth term is the expected overage cost incurred for the earmarked quantity left at the end of the period, the fifth term is the expected cost of covering the surplus warranty demand by new products, and the sixth term is the total salvage revenue.

consumer returns are available.
It is straightforward to show that (1) is jointly concave in \((D_n, D_r, Q_r)\). The following lemma provides the characterization of the optimal earmarking quantity for a given new and refurbished product demand.

**Lemma 1.** For a given \((D_n, D_r)\), the optimal earmarking quantity is given by

\[
Q^*_r = \min\left(\gamma(1 - \alpha)(D_n + D_r) + \tilde{z}, \alpha D_n - D_r\right)
\]

with \(\tilde{z} = F^{-1}\left(\frac{c_n - c_r - s}{c_n + h}\right)\).

Lemma 1 states that the optimal earmarking quantity either attains its interior solution, which is equal to the base warranty demand \((\gamma(1 - \alpha)(D_n + D_r))\) plus the safety stock \((\tilde{z})\) against warranty demand uncertainty, or is found at its boundary, which is equal to the new-product consumer returns minus the quantity allocated to the remarketing option \((\alpha D_n - D_r)\). We will refer to this boundary as the *earmarking capacity*. When the earmarking capacity is binding, all new-product consumer returns are allocated to the remarketing and earmarking options, and no new-product consumer returns are salvaged, whereas in the interior solution some of those returns are salvaged. Moreover, in the interior solution, the optimal safety stock is determined by a critical fractile, where \(c_n - c_r - s\) is the marginal savings from filling a warranty demand by refurbishing (or equivalently, the underage cost of not filling a warranty demand by refurbishing), and \(c_r + s\) is the marginal cost of filling a warranty demand by refurbishing, where \(c_r\) and \(s\) represent the direct and opportunity costs of refurbishing for warranty demand, respectively. Thus, for the rest of the analysis, we assume that the marginal saving from filling a warranty demand by refurbishing is positive \((c_n - c_r - s > 0)\); otherwise, the earmarking option is not economically attractive.

### 3.1 Unconstrained Earmarking

Next, we focus on the case where the OEM sets new and refurbished product prices such that the optimal earmarked quantity has an interior solution.

**Proposition 1.** If it is optimal to set new and refurbished product prices such that the optimal earmarking quantity has an interior solution, the optimal policy is characterized as follows:

<table>
<thead>
<tr>
<th>Condition</th>
<th>(D^*_n)</th>
<th>(D^*_r)</th>
<th>(Q^*_r)</th>
<th>(p^*_n)</th>
<th>(p^*_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_r &lt; M_n &lt; \frac{M_r}{\delta})</td>
<td>(\frac{M_n - M_r}{2(1 - \alpha)(1 - \delta)})</td>
<td>(\frac{M_r - \delta M_n}{2(1 - \alpha)(1 - \delta)})</td>
<td>(\frac{\gamma M_r}{2\delta} + \tilde{z})</td>
<td>(1 - \frac{M_n}{2(1 - \alpha)})</td>
<td>(\delta - \frac{M_r}{2(1 - \alpha)})</td>
</tr>
<tr>
<td>(M_n \geq \frac{M_r}{\alpha}, M_n &gt; 0)</td>
<td>(\frac{M_n}{2(1 - \alpha)})</td>
<td>0</td>
<td>(\frac{\gamma M_n}{2\delta} + \tilde{z})</td>
<td>(1 - \frac{M_n}{2(1 - \alpha)})</td>
<td>–</td>
</tr>
</tbody>
</table>

\(M_n := 1 - \alpha - c_n + \alpha s - (c_r + s)\gamma(1 - \alpha), \ M_r := \delta(1 - \alpha) - c_r - (1 - \alpha)s - (c_r + s)\gamma(1 - \alpha)\)
The functions of parameters $M_n$ and $M_r$, appearing in Proposition 1, represent the maximum marginal values of a new and remarketed product, respectively. In $M_n$, the term $1 - \alpha - c_n + \alpha s$ is the maximum net marginal revenue earned from selling a new product, where $1 - \alpha - c_n$ is the maximum direct profit from a new product, considering that $\alpha$ portion of each new product sold is returned and refunded, and $\alpha s$ is the salvage revenue generated by the return. On the other hand, each new product sold contributes to the base warranty demand, and therefore $M_n$ includes the marginal cost of filling the base warranty demand by refurbishing, which is given by the term $(c_r + s)\gamma(1 - \alpha)$. Similarly, in $M_r$, the term $\delta(1 - \alpha) - c_r - (1 - \alpha)s$ is the net maximum marginal revenue from selling a refurbished product, where $\delta(1 - \alpha) - c_r$ is the maximum direct profit from a remarkeeted product after the refund cost is deducted. Because consumer returns from remarkeeted products are only salvaged, each remarkeeted product generates a salvage revenue of $\alpha s$. On the other hand, each remarkeeted product incurs an opportunity cost of $s$, since it prevents a consumer return from being salvaged before refurbishing. Thus, $(1 - \alpha)s$ is the net marginal opportunity cost of remarketing a returned product. Since selling a refurbished product also contributes to the base warranty demand, $M_r$ includes the term $(c_r + s)\gamma(1 - \alpha)$.

Proposition 1 prescribes two different disposition strategies in the interior solution: If $M_n$ is sufficiently low ($M_n < M_r/\delta$), remarketing is profitable. Thus, consumer returns are allocated to both remarketing and earmarking options, and the rest are salvaged without being refurbished. The optimal earmarking quantity in this case is the base warranty demand ($\gamma M_r/2\delta$) plus the safety stock ($\hat{z}$). On the other hand, if $M_n$ is sufficiently high, remarketing is not profitable and the consumer returns are either earmarked or salvaged. In this case, the earmarked safety stock remains the same but the base warranty demand depends on the failure rate and maximum marginal value of a new product ($\gamma M_n/2$).

When remarketing is profitable, by Proposition 1, the optimal fill rate for warranty demand fulfilled by earmarking is $1 - \frac{2\delta L(z)}{\gamma M_r + 2\delta \mu}$, where $L(z) := E(\xi - z)^+$ and $\mu := E(\xi)$ denote the loss function and expected value of $\xi$, respectively. The optimal fill rate is increasing in $\gamma$, $\mu$, $M_r$ and $\delta$, since an increase in these parameters suggests a higher warranty demand and larger optimal earmarking quantity. Thus, although earmarking and remarketing are seemingly competing disposition options, when the optimal earmarking quantity is unconstrained, more profitable remaketing conditions imply less salvaging rather than less earmarking.

**Corollary 1.** When it is optimal to set new and refurbished product prices such that the optimal
earmarking quantity has an interior solution, remarketing is profitable if

\[ c_r < \delta c_n - s(1 - \alpha + \alpha \delta) - (c_r + s)\gamma(1 - \alpha)(1 - \delta). \]  

(3)

Earlier literature on closed-loop supply chains (e.g., Atasu et al. 2008) concludes that when the refurbishing cost structure is linear and there are no fixed costs (for collecting and processing the returned products), remarketing is profitable only if the unit refurbishing cost is sufficiently lower than the unit manufacturing cost \((c_r < \delta c_n)\). Corollary 1 generalizes this result and shows that when consumer returns, warranty demand and salvaging are taken into account, the profitability of remarketing not only depends on the remarketing potential, refurbishing cost and manufacturing cost but also the consumer return rate, failure rate and salvage value. As such, for products with relatively high consumer return rates, salvage values, or warranty coverage costs, the classical profitability condition for remarketing is incomplete, and, as (3) shows, a stricter condition is required for remarketing to be profitable for such products. Moreover, (3) reveals a substitution effect between \(c_r, \gamma\) and \(\alpha\): As refurbishing becomes more costly, not only does the profit margin of the remarketed products decrease but also the warranty coverage cost increases. Thus, to keep the refurbished products in the market, the OEM needs to either reduce the failure rate or receive more consumer returns, since a lower failure rate or higher consumer return rate imply a lower base warranty demand rate \((\gamma(1 - \alpha))\).

To generate more insights about how earmarking affects the OEM’s remarketing decisions, we compare the interior solution presented in Proposition 1 with the interior solution of the benchmark scenario where the OEM covers all warranty demand with new products \((Q_r = 0)\). The optimal policy under this benchmark scenario shows a similar structure to the one presented in Proposition 1 with slightly different definitions of \(M_n\) and \(M_r\). In both scenarios, the interior solution means that some fraction of the new-product consumer returns are optimally salvaged. The following corollary highlights the impact of earmarking on remarketing as a result of this comparison.

**Corollary 2.** When the optimal solution is in the interior and remarketing is profitable, earmarking consumer returns for warranty coverage increases the refurbished product sales by the amount \(\gamma(c_n - c_r - s)/2\delta\).

Corollary 2 shows that earmarking can benefit remarketing due to its warranty cost reduction effect. That is, compared to the benchmark scenario, each earmarked consumer return decreases the marginal warranty coverage cost by the amount \(c_n - c_r - s\), and consequently, allows more refurbished products to be profitably sold. A bigger cost reduction in unit warranty coverage
cost or a higher product failure rate makes the earmarking option more valuable due to a larger increase in the refurbished product sales. On the other hand, a higher remarketing potential makes the earmarking option less valuable. When remarketing is not profitable, earmarking affects new product sales in a similar way.

### 3.1.1 Comparative Statics

To better understand the drivers behind the optimal disposition strategy, we conduct a comparative static analysis based on the solution provided in Proposition 1. Below, we report two key findings about the impact of consumer return rate and refurbishing cost on the optimal policy parameters.

**Corollary 3.** A higher consumer return rate decreases the optimal new product sales and optimal earmarking quantity, but increases the optimal refurbished product sales.

As the consumer returns become more abundant, the OEM incurs higher refund costs and optimally charges more for new and refurbished products. Since refunding a new product is more expensive than refunding a refurbished product, when remarketing is profitable, the new product price increases in the consumer return rate faster than the refurbished product price. Thus, although both products become more expensive in absolute terms, a higher consumer return rate makes the new products relatively more expensive compared to the refurbished products and favors the refurbished product sales. The optimal earmarking quantity is decreasing in the consumer return rate because when the earmarking quantity is unconstrained, a higher consumer return rate implies a lower warranty demand due to a lower total sales quantity.

**Corollary 4.** When remarketing is profitable, a higher refurbishing cost increases the optimal new product sales. On the other hand, when remarketing is not profitable, a higher refurbishing cost decreases the optimal new product sales. The optimal refurbished product sales and optimal earmarking quantity are decreasing in the refurbishing cost.

When remarketing is not profitable, the optimal new product sales are decreasing in the refurbishing cost, since a higher refurbishing cost implies more expensive marginal warranty coverage cost (or lower marginal warranty savings by earmarking), and the warranty demand can only be lowered by reducing the new product sales. The optimal earmarking quantity also decreases due

---

4 This effect can be seen more clearly from the gap between the optimal new and refurbished product prices, i.e., when remarketing is profitable, by Proposition 1, $p_n^* - p_r^* = (c_n - c_r - s)/(1 - \alpha) + (1 - \delta)/2$. Thus, a higher $\alpha$ increases the gap between new and refurbished products and makes the new products relatively more expensive.

5 The optimal total sales quantity ($D_n^* + D_r^*$) is given by $M_r/(1 - \alpha)\delta$, and it can be easily verified that it is decreasing in $\alpha$. The same can also be easily verified when remarketing is not profitable ($D_r^* = 0$).
to the decrease in the warranty demand. On the other hand, when remarketing is profitable, a higher refurbishing cost not only increases the marginal warranty coverage cost but also decreases the marginal profit from a remarketed product. Consequently, the optimal refurbished product sales decreases rapidly in the refurbishing cost, whereas the optimal new product sales increases to partially offset this decrease. The net effect of these changes is a decrease in the optimal total sales quantity, and therefore, there is a decrease in the warranty demand and optimal earmarking quantity.

3.2 Optimal Single-Period Policy

As discussed in the previous section, when the earmarking quantity is unconstrained, the optimal policy can be characterized by closed-form expressions. When the earmarking quantity is constrained, however, the analytical expressions are tedious and not amenable to comparative static analysis. Thus, in this section, we use representative numerical examples to obtain insights about the overall optimal disposition strategy in a single-period setting including the constrained cases (Figure 1). For these examples, we choose parameter values that demonstrate typical shifts in the dominant strategy and are anchored in realistic ranges, which are developed for the numerical study in Section 5. As such, although the discussion is based on a limited data set, it captures the main dynamics in the single-period setting that are worthy of discussion and shed light on the dynamics of the multi-period problem. For brevity, in the figures, we do not differentiate between the cases where new-product consumer returns are salvaged or not (i.e., unconstrained and constrained earmarking cases) and refer to the optimal policy with the names of the disposition options it includes.

**Consumer return rate vs. failure rate.** We first focus on the consumer return rate and failure rate to understand how the interaction between the refurbishing capacity and warranty demand shape the optimal disposition strategy. Figure 1 shows that for low consumer return rates, the dominant policy is earmarking. This is because a low consumer return rate implies a high base warranty demand rate (i.e., more consumers keep and use their product until the end of the period) and low earmarking capacity. Thus, to avoid high warranty coverage costs, the optimal policy generally prioritizes earmarking over remarketing (Figures 1a–1b). In particular, when a low consumer return rate is combined with a high failure return rate, the OEM foregoes remarketing for any level of refurbishing cost and remarketing potential to obtain the maximum possible warranty savings, and therefore allocates all consumer returns to the earmarking option (Figure 1b). On
the other hand, when the consumer return rate and failure rate are both low and the remarketing potential is sufficiently high, the OEM can take a more balanced approach and allocate consumer returns to both the earmarking and remarketing options (Figure 1a). For high consumer return rates, the OEM allocates consumer returns to both disposition options in the majority of the cases (Figure 1c) since refurbishing capacity is abundant and, as discussed in Corollary 3, remarketed products are preferred over new products due to their relatively low refund costs\(^6\). Similar figures and insights are obtained when the failure rate is fixed and the warranty demand uncertainty is varied.

**Figure 1: Optimal Single-Period Policy** ($c_n = 0.30, \ s = 0.09, \ \bar{\xi} = 0.05, \ h = 0$)†

†$\alpha = \{0.05, 0.30\}, \ \gamma = \{0.01, 0.05\}, \ \xi$ is assumed to be uniformly distributed.

**Remarketing potential vs. refurbishing cost.** Next, we study the interaction between the remarketing potential and the refurbishing cost. For sufficiently high consumer return rates, as refurbishing becomes more expensive, the remarketing potential threshold above which remarketing is profitable is increasing (Figure 1c). It is important to note, however, that as the refurbishing cost increases, the OEM shuts down the refurbished product market but still refurbishes and earmarks consumer returns to cover warranty demand. This is because, as discussed in Corollary 1, the profitability of remarketing depends not only on the profit margins of the remarketed products but also on the warranty coverage costs generated by them. Thus, for sufficiently high refurbishing costs, the remarketing profit does not offset the warranty coverage cost generated by remarketing, and the optimal policy leans toward earmarking or salvaging. On the other hand, when the consumer return rate and failure rate are both low, for sufficiently low refurbishing costs, the remarketing potential

\(^6\)For high consumer return rates, the failure rate does not significantly change the general shape of the optimal policy regions but affects whether some new-product consumer returns are salvaged or not. Thus, for brevity, we provide a single figure for the high consumer return rate example.
threshold is either constant or very slowly decreasing (Figure 1a), implying that remarketing might have a slight advantage over earmarking for these parameter combinations.

We also observe that pure remarketing is not an optimal strategy by itself, i.e., some level of earmarking is always optimal even though the earmarked quantity can be relatively small. This is because when earmarking is economically attractive \( (c_n - c_r - s > 0) \), it can help offset the warranty costs generated by the remarkeared products.

4. The Multi-Period Problem

To investigate how the OEM’s disposition decision is affected by intertemporal changes, we extend the single-period model to a multi-period setting. To this end, the planning horizon is divided into \( T \) periods where the decision epochs are denoted by \( t = 0, 1, 2, ..., T - 1 \). At the beginning of each period, the OEM decides the quantity to be refurbished and earmarked to cover the warranty demand \( (Q_t^r) \), together with the new and refurbished product sales quantities \( (D^n_t, D^r_t) \). Analogous to the single-period case, the inverse demand functions for new and refurbished products at each period are given as \( p^n_t = 1 - D^n_t - \delta_t D^r_t \) and \( p^r_t = \delta_t (1 - D^n_t - D^r_t) \). In each period, sales takes place, consumer returns are received and random warranty demand is realized. The random warranty demand in period \( t \) is given by \( R^w_t(D^n_t, D^r_t, \xi_t) = \gamma_t(1 - \alpha_t)(D^n_t + D^r_t) + \xi_t \), where \( \gamma_t \) is the base failure rate in period \( t \), \( \alpha_t \) is the consumer return rate in period \( t \), and \( \xi_t \) is the random portion of warranty demand in period \( t \). Similar to the single-period model, \( \xi_t \in [0, \bar{\xi}_t] \) is a continuous nonnegative random variable with cumulative distribution function \( F_t(\cdot) \). If the earmarked quantity is larger than the realized warranty demand, the surplus earmarked inventory is carried to the next period. The state of the system \( (x_t) \) is the earmarked inventory level at the beginning of period \( t \), before the earmarking decision is taken. As such, the system state at the beginning of period \( t + 1 \) is given by \( x_{t+1} = x_t + Q_t^r - R^w_t(D^n_t, D^r_t, \xi_t) = y_t - R^w_t(D^n_t, D^r_t, \xi_t) \), where \( y_t \) is the earmarked inventory level at the beginning of period \( t \) after the earmarking decision is taken \( (y_t := x_t + Q_t^r) \). The consumer returns received during a period are either allocated to one of the disposition options or salvaged. Thus, the earmarked quantity in each period should satisfy \( 0 \leq Q_t^r \leq \alpha_t D^n_t - D^r_t \), or equivalently, \( x_t \leq y_t \leq x_t + \alpha_t D^n_t - D^r_t \). Feasible new and refurbished product sales quantities \( (D^n_t, D^r_t) \) are confined to the set \( \Omega = \{(D^n_t, D^r_t)| D^n_t \in [0, 1], D^r_t \in [0, \alpha_t D^n_t]\} \), and the state space is \( \mathbb{R} \). The OEM’s dynamic disposition problem is related to the single product joint dynamic pricing and replenishment problem under stochastic demand (e.g., Zabel 1972, Federgruen and Hetching 1999). Our model differs from those models, however, in that we consider pricing of two vertically
differentiated products and there is a capacity constraint limiting the maximum quantity that can be “ordered” (earmarked) in each period.

The OEM can backorder the unfilled warranty demand until there is enough earmarked inventory. Thus, at the end of the period, the backordering cost \( b_t \) is incurred for each backlogged warranty demand. The practical examples of backordering cost in warranty inventory systems are the loss of good will due to an increase in customer waiting time for replacement products as well as the additional production and transportation costs caused by congestion due to backlogged warranty demand (see e.g., Huang et al. 2008, Khwam 2007). If there are no backorders, the holding cost \( h_t \) is incurred per unit of surplus earmarked inventory kept in stock. In any period, all refurbished-product consumer returns (\( \alpha_t D^r_t \)) and the new-product consumer returns that are not earmarked or remarketed in that period (\( \alpha_t D^a_t - D^r_t - Q^r_t \)) are salvaged at the end of the period at a salvage value \( s \). For short life-cycled consumer electronics products, the changes in \( c_n, c_r \) and \( s \) are typically much slower compared to the changes in the consumer return rate and failure rate. Thus, for analytical convenience, we take these parameters as fixed throughout the life-cycle. The OEM’s profit in period \( t \) is given by:

\[
\Pi_t(y_t, D^a_t, D^r_t) = ((1 - \alpha_t)p_t^n - c_n)D^a_t + ((1 - \alpha_t)p_t^r - c_r)D^r_t - c_r(y_t - x_t)
- h_tE(y_t - R_t^w(D^a_t, D^r_t, \xi_t))^+ - b_tE(R_t^w(D^a_t, D^r_t, \xi_t) - y_t)^+
+ s(\alpha_t D^a_t - (1 - \alpha_t)D^r_t - (y_t - x_t)),
\]

where the first two terms are the net profit from selling new and refurbished products, the third term is the cost of refurbishing the period’s earmarking quantity, the fourth and fifth terms are the expected holding and backordering costs incurred for the earmarked inventory, and the last term is the total salvage revenue.

In the rest of the analysis, we suppress the time index unless necessary and use the notation defined in Section 3 whenever possible (e.g., \( D_n \) for \( D^n_t \)). Define the functions \( \pi_t(D_n, D_r) := ((1-\alpha)p_n - c_n + \alpha s)D_n + ((1-\alpha)p_r - c_r - (1-\alpha) s)D_r \) and \( G_t(y, D_n, D_r) := h E(y - R_w(D_n, D_r, \xi))^+ + b E(R_w(D_n, D_r, \xi) - y)^+ \). Reorganizing the terms yields the profit in period \( t \) as \( \Pi_t(y, D_n, D_r) = (c_r + s)x + \pi_t(D_n, D_r) - (c_r + s)y - G_t(y, D_n, D_r) \).

Let \( V_t(x) \) denote the maximum expected discounted profit in periods \( t, t + 1, \ldots, T \), if period \( t \) begins in state \( x \). At the end of the planing period, the unfilled warranty demand is covered by the new products and the surplus earmarked inventory is salvaged. Thus, \( V_T(x) = c_T(x) \) where \( c_T(x) = sx^+ - c_n x^- \) with the conventions \( x^+ = \max(0, x) \) and \( x^- = \max(0, -x) \). Note that \( c_T(x) \) is a concave increasing function since \( c_n > s \). For \( t = 0, 1, \ldots, T - 1 \), we can state the dynamic
programming formulation of the multi-period problem as follows:

\[ V_t(x) = (c_r + s)x + \max_{\{x \leq y \leq x + \alpha D_n - D_r, (D_n, D_r) \in \Omega\}} J_t(y, D_n, D_r), \]

with \( J_t(y, D_n, D_r) = \pi(t)(D_n, D_r) - (c_r + s)y - G_t(y, D_n, D_r) + \beta E(V_{t+1}(y - R_w(D_n, D_r, \xi))) \). A more convenient representation of this formulation can be obtained by defining the function \( V_t^+(x) = V_t(x) - (c_r + s)x \) and reorganizing the terms in \( J_t(y, D_n, D_r) \). Then, for \( t = 0, 1, ..., T - 1 \):

\[ V_t^+(x) = \max_{\{x \leq y \leq x + \alpha D_n - D_r, (D_n, D_r) \in \Omega\}} J_t(y, D_n, D_r), \tag{4} \]

with

\[ J_t(y, D_n, D_r) = W_t^+(y, D_n, D_r) + \beta E(V_{t+1}(y - R_w(D_n, D_r, \xi))) \],

\[ W_t^+(y, D_n, D_r) = \pi(t)(D_n, D_r) - \beta(c_r + s)(\gamma(1 - \alpha)(D_n + D_r) + E(\xi)) \]

\[-(c_r + s)(1 - \beta)y - G_t(y, D_n, D_r), \tag{6} \]

and \( V_T^+(x) = (c_n - c_r - s)x - (c_n - s)x^+ \). Without loss of generality, we assume that \((c_r + s)(1 - \beta) < b\). Otherwise, backordering is cheaper than covering a warranty demand by refurbishing and it would be optimal to backorder all warranty demand.

**Proposition 2.** For \( t = 0, 1, ..., T - 1 \), the following statements hold:

(a) The function \( J_t(y, D_n, D_r) \) is jointly concave in \((y, D_n, D_r)\) and the function \( V_t^+(x) \) is concave in \( x \).

(b) \( J_t(y, D_n, D_r) \) has a finite maximizer denoted by \((\hat{y}_t, \hat{D}_t^a, \hat{D}_t^r)\).

(c) Let \( K_t := \alpha \hat{D}_t^a - \hat{D}_t^r \),

(c.1) if \( x > \hat{y}_t \), it is optimal not to earmark any consumer returns, i.e., \( y_t^*(x) = x \), and sell the quantities \((\hat{D}_t^a, y_t^*(x)) = \max_{(D_n, D_r) \in \Omega} J_t(x, D_n, D_r) \).

(c.2) if \( \hat{y}_t - \hat{K}_t \leq x \leq \hat{y}_t \), it is optimal to earmark up to the level \( \hat{y}_t \) and sell the global optimal quantities, i.e., \((\hat{y}_t^*, x), (\hat{D}_t^a, y_t^*(x)) = (\hat{y}_t, \hat{D}_t^a, \hat{D}_t^r) \).

(c.3) if \( x < \hat{y}_t - \hat{K}_t \), the optimal earmark-up-to level and optimal sales quantities are given by \((y_t^*(x), \hat{D}_t^a, y_t^*(x)) = \max_{x \leq y \leq x + \alpha D_n - D_r, (D_n, D_r) \in \Omega} J_t(y, D_n, D_r) \), and \( y_t^*(x) < \hat{y}_t \).

Proposition 2 shows that the optimal policy is essentially a price-dependent base-stock policy where the earmarked quantity is capacitated by the new and refurbished product sales quantities, which are endogenously determined by the OEM’s pricing decisions. \( \hat{K}_t \) is the capacity level if the OEM can sell new and refurbished products at the global optimal levels \((\hat{D}_t^a, \hat{D}_t^r)\). If the earmarked inventory at the beginning of the period is sufficiently large \((x > \hat{y}_t)\), earmarking is not
a concern, and the new and refurbished product sales quantities (prices) are decided under this high level of protection against the warranty demand uncertainty. If the earmarked inventory at the beginning of the period is sufficiently low \(x < \hat{y}_t - \hat{K}_t\), the OEM can optimally set a new capacity for earmarking, denoted by \(K_t^*(x) := \alpha D_t^{n*}(x) - D_t^{r*}(x)\), by adjusting the new and refurbished product sales quantities accordingly. Even with an adjustment in the sales quantities, however, the optimal earmark up-to level \(y_t^*(x)\) does not exceed its global optimal level \((\hat{y}_t)\).

### 4.1 Optimal Dynamic Policy

In this section, we explore the inter-temporal behavior of the optimal dynamic disposition policy during the life-cycle of the product. In particular, we focus on the inter-temporal changes in the consumer return and failure rates to better understand how the evolution of these parameters affect the optimal policy. To study the dynamic behavior of the optimal new and refurbished product sales and earmarked quantity, we divide the life-cycle of the product into ten periods and compute the optimal policy for each period. We then simulate sample-paths of the state and decision variables and compute their averages. Representative numerical results showing the inter-temporal behavior of \(D_t^r\) and \(Q_t^r\) under different scenarios, capturing various characteristics of products, consumers, and business environment, are reported in Figures 2–4. The parameters used in these representative examples are drawn from the parameter set developed for the numerical study in Section 5. The behavior of the optimal new product sales \(D_t^n\) is relegated to the online Appendix A.2 since it is primarily driven by the interplay between \(D_t^r\) and \(Q_t^r\).

**Impact of the consumer return rate.** For new-generation products with a disruptive technology and design, the consumer return rates are typically higher in the earlier stages of the life-cycle due to a larger likelihood of mismatch between the product’s functionality and the consumers’ expectations. As the product reaches its maturity phase, consumers are better informed about the product’s functionality (e.g., due to the OEM’s or retailer’s efforts) and less likely to return the product. Consequently, the consumer return rates of such products often show a decreasing pattern throughout the life-cycle. Figure 2 shows that in such scenarios, the optimal policy emphasizes earmarking at the early stages of the life-cycle and gradually decreases the earmarked quantity towards the end of the life-cycle. This is because a decreasing consumer return rate implies an increasing number of warranty claims and a decreasing refurbishing capacity. Thus, it is optimal
to prioritize earmarking consumer returns early in the life-cycle in order to build up earmarked inventory to hedge against the large number of warranty claims that will arrive at the later stages in the life-cycle when refurbishing capacity is more constrained. Depending on the speed of the inventory buildup, which is driven by the warranty demand rate, warranty demand uncertainty and the speed of the decrease in the consumer return rate, remarketing may become more pronounced relatively later in the life-cycle. Interestingly, we observe that even when the consumer return rate is stationary ($\alpha_1$ in Figure 2), for sufficiently high warranty demand uncertainty, it is optimal to build up some level of earmarked inventory at the early stages in the life-cycle to protect against the demand uncertainty faced during the later stages of the life-cycle.

Figure 2: Dynamics under a Decreasing Consumer Return Rate ($\delta = 0.7$, $c_n = 0.3$, $c_r/c_n = 0.2$, $s/c_n = 0.1$, $\xi = 0.1$, $h/c_n = 0.02$, $b = 0.21$, $\beta = 1$, $\gamma = 0.05$)

For consumer electronics products that are less disruptive in their technology and design, consumer return rates can be primarily driven by the competitive product offerings that are launched during the product’s short life-cycle. For such products, the consumer return rate can be increasing over time since the alternative products launched at different points in the life-cycle can encourage consumers to try multiple products during the money-back return periods and increase the likelihood of a return. In such scenarios, the optimal earmarking quantity shows a concave intertemporal behavior (Figure 3). The reason is as follows: For sufficiently high failure rates, a low consumer return rate implies a relatively high base warranty demand, causing the optimal policy to allocate scarce consumer returns to cover warranty demand rather than remarketing them, since the latter option also generates warranty demand. As the consumer return rate increases, however, the warranty demand decreases and the refurbishing capacity increases. Consequently, the earmarking quantity decreases while the refurbished product sales increase. The increase in the refurbished product sales is driven by the ample refurbishing capacity but also the refund cost advantage of the refurbished products, i.e., as discussed in the single-period model, a high volume of consumer
returns favor remarketing since refunding a refurbished product is cheaper than refunding a new product.

Figure 3: Dynamics under an Increasing Consumer Return Rate ($\delta = 0.7$, $c_n = 0.3$, $c_r/c_n = 0.2$, $s/c_n = 0.1$, $\xi = 0.05$, $h/c_n = 0.02$, $b = 0.21$, $\beta = 1$, $\gamma = 0.05$)

(a) Consumer return rate ($\alpha_t$)  
(b) Earmarking Quantity ($Q^*_r$)  
(c) Refurbished Product Sales ($D^*_r$)

It is interesting to note that, for sufficiently high warranty demand uncertainty, the above observations point to a consistent overall disposition strategy. That is, at the early stages in the life-cycle, the OEM should emphasize earmarking of consumer returns to either fulfill the current warranty claims or build up earmarked inventory for the future warranty claims, whereas remarketing should be considered at the later stages in the life-cycle after enough earmarked inventory is accumulated.

**Impact of the failure rate.** Due to design and manufacturing problems, the OEMs can receive higher amounts of warranty claims at the early stages in the life-cycle. As these design and manufacturing problems are resolved throughout the life-cycle, the product’s failure rate drops. Figure 4 shows that under this scenario, the OEM should allocate most of the consumer returns to the earmarking option due to high warranty demand at the early stages of the life-cycle. As the failure rate decreases, the warranty demand decreases and, consequently, the refurbished product sales increase. As such, the inter-temporal behavior of the optimal refurbished product sales and optimal earmarking quantity in the face of a decreasing failure rate can be considered as qualitatively similar to the case of a decreasing consumer return rate. There is a difference, however, in that a steeper drop in the failure rate encourages a higher level of remarketing, whereas a steeper drop in the consumer return rate discourages it.
Figure 4: Dynamics under a Decreasing Failure Rate ($\delta = 0.7$, $c_n = 0.3$, $c_r/c_n = 0.2$, $s/c_n = 0.1$, $\xi = 0.05$, $h/c_n = 0.02$, $b = 0.21$, $\beta = 1$, $\alpha = 0.15$)

- (a) Failure Rate ($\gamma_t$)
- (b) Earmarking Quantity ($Q^*_r$)
- (c) Refurbished Product Sales ($D^*_r$)

5. Dynamic Allocation of Consumer Returns and Value of the Optimal Dynamic Policy

The previous section shows that the optimal disposition strategy prescribes emphasizing different disposition options at different stages in the product’s life-cycle. To better understand how the dynamic allocation of consumer returns between remarketing and earmarking options change throughout the life-cycle as well as when such a dynamic policy is most beneficial, we carry out a numerical study.

Parameter Development. To be consistent with our model development, we choose parameter ranges that are typically observed for consumer electronics having short product life-cycles. The manufacturing cost of a new product ($c_n$) is estimated by using the reported price and material costs of various consumer electronics products and normalized to the range of $[0, 1]$ for convenience (see online Appendix A.3 for details on estimation and normalization). The refurbishing cost ($c_r$) is taken within the range of 10% to 50% of the manufacturing cost since most consumer electronics returns are characterized as no-trouble-found returns and therefore can be brought back to almost new condition by simple buff-and-polish operations (Accenture 2008, Francis 2012, Gventer 2012). Reported consumer return rates ($\alpha$) for consumer electronics vary from 2% to 20% depending on the product category and geographical location of the market (e.g., Accenture 2008, Shang et al. 2013). Thus, in our experiments we vary $\alpha$ from 5% to 30% to capture the reported rates and possible high-return scenarios. Based on our discussions with industry experts, we learned that most OEMs try to keep their failure rate ($\gamma$) below 5%. However, due to the uncertainties involved in the production and distribution process, the realized warranty demand rates can be very high (Gventer...
Therefore, the OEMs suffer from a high upside risk of warranty demand, which we represent in our numerical experiments by a uniformly distributed $\xi$ in the interval $[0, \xi]$ in each period, and vary $\xi$ from 1% to 10%. For many products, the ratio of the new product price to the refurbished product price lies within the range of 30% to 100% (Subramanian and Subramanyam, 2012). This ratio can be taken as a proxy for the relative willingness-to-pay (remarketing potential) for the refurbished products ($\delta$). Accordingly, we vary $\delta$ from 50% to 85% to capture the reported ranges as well as some relatively low willingness-to-pay scenarios. The unit holding cost of earmarked inventory per period ($h$) is taken as 2% of the manufacturing cost of a new product, corresponding to the 20% annual inventory holding cost rate. The backordering cost per warranty claim per period ($b$) is approximated by the marginal saving of covering a warranty demand by refurbishing, or equivalently, the underage cost of not filling a warranty demand by refurbishing ($c_n - c_r - s$). Consumer electronics returns have relatively small salvage value compared to the potential value created by refurbishing and therefore the OEMs commonly consider salvaging (e.g., recycling, parts harvesting) as a fallback to decrease the congestion in the refurbishing facility (e.g., Geyer and Blass 2010, Guide et al. 2008). Thus, in our numerical experiments, we vary the salvage value ($s$) from 5% to 30% of the manufacturing cost of a new product. This range is in line with the values reported in previous work, and it captures different scenarios where salvaging is more or less valuable compared to refurbishing. The salvage value also affects the marginal cost of covering a warranty demand by refurbishing ($c_r + s$), which should be less than the manufacturing cost of a new product; otherwise, refurbishing for warranty coverage is not economically attractive and the problem trivially boils down to the one with a single disposition option ($Q_r = 0$). As such, in our experimental design, $c_r + s$ varies between 15% to 85% of the manufacturing cost of a new product, reflecting a relatively rich set of cases for the marginal warranty coverage cost. We set the per period discount factor ($\beta$) to 0.98 and 1 to capture scenarios with high discounting (20% annual cost of capital) and no discounting, respectively.

Table 1 provides a summary of the parameters used in the numerical experiments. While these parameter estimates are not directly based on data reported by firms, they are realistic as discussed above, and thus provide insights that are close to those that a firm, using their own proprietary data, should obtain. We generate a numerical set consisting of 1944 instances obtained from all possible combinations of this parameter set.

**Dynamic Allocation of Consumer Returns.** Figure 5 shows the optimal dynamic allocation of consumer returns among the remarketing, earmarking, and salvaging options. The allocation
Table 1: Parameter Values Used in Numerical Experiments

<table>
<thead>
<tr>
<th>c_n</th>
<th>c_r/c_n</th>
<th>α</th>
<th>δ</th>
<th>ξ</th>
<th>γ</th>
<th>s/c_n</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.1</td>
<td>0.05</td>
<td>0.50</td>
<td>0.01</td>
<td>0.01</td>
<td>0.05</td>
<td>0.98</td>
</tr>
<tr>
<td>0.30</td>
<td>0.5</td>
<td>0.15</td>
<td>0.70</td>
<td>0.05</td>
<td>0.03</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>0.30</td>
<td>0.85</td>
<td>0.10</td>
<td>0.05</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Quantities are presented in terms of cumulative percentages of consumer returns\(^9\) averaged over all instances at each time period. This is because the changes in the allocation percentages across periods are relatively small for the majority of the life-cycle and the allocation of total consumer returns over time is observed more clearly by cumulative percentages. As such, the percentage allocation of the total consumer returns quantity received during the life-cycle is given by the last column in each figure.

Over all the experiment instances, we find that the majority of the consumer returns are allocated to the earmarking option throughout the life-cycle. On average, about 27% of all returns are allocated to the remarketing option, about 65% of all returns are allocated to the earmarking option, and the rest are salvaged (see last column in Figure 5). These ratios change in favor of remarketing for low warranty demand uncertainty and high consumer return rates. For example, our numerical results show that when \(ξ = 0.01\) or \(α = 0.3\), about 46% of all returns are allocated to remarketing and 40% are allocated to earmarking. Yet, even for such parameter combinations where the remarketing option has an advantage over the earmarking option (due to low warranty uncertainty and high consumer return rate) almost half of the consumer returns are still earmarked.

On the other hand, for the parameter combinations where the earmarking option has an advantage (e.g., high warranty uncertainty and low consumer return rate), the fraction of returns allocated to the remarketing option decreases significantly. For example, when \(ξ = 0.1\) or \(α = 0.05\), less than 10% of all consumer returns are allocated to the remarketing option, and more than 87% of returns are allocated to the earmarking option. These findings confirm our earlier intuition, developed in Section 3.2, and show that even in a multi-period setting, earmarking is generally more dominant than remarketing since earmarked consumer returns can offset the warranty claim and refund costs generated by new and refurbished products.

We also observe from Figure 5 that over time, the percentage of consumer returns allocated to the earmarking option is decreasing while the percentage of consumer returns allocated to the remarketing and salvaging options is increasing, and this overall inter-temporal behavior is consistent under different parameter combinations (Figure 6). Thus, our earlier observations that the

\(^9\)Cumulative quantity of consumer returns allocated to a disposition option by period \(t\) divided by the cumulative consumer returns received by period \(t\).
OEM should strategically emphasize earmarking at the early stages in the life-cycle and postpone remarketing to the later stages appear to be robust.

Intuitively, it is expected that the percentage of returns allocated to the earmarking option would be lower when the remarketing potential or the refurbishing cost is high. Figure 6 shows, however, that an increase in the remarketing potential or refurbishing cost does not significantly affect the fraction of consumer returns allocated to the earmarking option but instead changes the allocation between the remarketing and salvaging options. For example, as $\delta$ increases, about 65% of all returns are consistently allocated to the earmarking option, while the percentage of returns allocated to salvaging shifts to remarketing. Similarly, as $c_r/c_n$ increases, the percentage allocated to earmarking is preserved (about 65%) and the rest is reallocated in favor of salvaging. This is because the remarketing and salvaging options are generally dominated by the earmarking option (Figure 5). Consequently, when parameters change in favor of remarketing or salvaging, the optimal policy shifts the allocation of returns beginning from the least valuable disposition option rather than the earmarking option.

Comparison with the Myopic Policy. To shed some light into the value of the dynamic disposition policy, we benchmark its performance vis-à-vis the myopic policy. In line with the previous literature on inventory theory (e.g., Zipkin 2000), the myopic policy in period $t$ is found by maximizing (6) over $(y,D_n,D_r)$ subject to the original constraint set $x \leq y \leq x + \alpha D_n - D_r$ and $(D_n,D_r) \in \Omega$. We define the performance measure as the percentage profit penalty incurred by the myopic policy ($\Delta_M \%$). Table 2 reports the frequency distribution of the percentage profit penalty among all experiment instances.

Our results show that the myopic policy performs quite well compared to the optimal policy. Over all the experiment instances, the mean and median $\Delta_M \%$ are found to be 0.91% and 0.23%,
respectively. We observe from Table 2 that for 96.2% of all instances, the percentage profit penalty is less than or equal to 5%. The maximum profit penalty is 15% and there are 12 instances (out of 1944 instances) where the percentage profit penalty can be considered as high (between 10% and 15%).

Table 2: Frequency Distribution of Profit Penalty due to Myopic Policy

<table>
<thead>
<tr>
<th>Profit penalty ($\Delta M%$)</th>
<th>Number of instances</th>
<th>Cumulative percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 0.5%$</td>
<td>1277</td>
<td>65.7%</td>
</tr>
<tr>
<td>$\leq 1%$</td>
<td>1482</td>
<td>76.2%</td>
</tr>
<tr>
<td>$\leq 3%$</td>
<td>1786</td>
<td>91.9%</td>
</tr>
<tr>
<td>$\leq 5%$</td>
<td>1870</td>
<td>96.2%</td>
</tr>
<tr>
<td>$\leq 10%$</td>
<td>1932</td>
<td>99.4%</td>
</tr>
<tr>
<td>$\leq 15%$</td>
<td>1944</td>
<td>100%</td>
</tr>
</tbody>
</table>

To better understand which parameters drive the performance of the myopic policy, in Table 3 we provide an overview of the differences in the average values of the system parameters between the high ($\Delta M\% \leq 3\%$) and low ($\Delta M\% > 3\%$) performing instances of the myopic policy. We find that for all measured parameters except $\beta$ and $\gamma$, the average parameter values for the high performing instances is significantly different than the average parameter values for the low per-
forming instances. In particular, we find that the myopic policy performs better for smaller values of $\xi$, $\alpha$, $\delta$, $c_n$, and larger values of $c_r/c_n$, $s/c_n$, $c_r+s$. A small $\xi$ positively impacts $\Delta M\%$ since lower demand variability requires less strategic buildup of the earmarked stock and hence favors the myopic policy (e.g., for 1296 instances with $\bar{\xi} \leq 0.05$, the mean and max $\Delta M\%$ are found to be 0.4% and 3.7%, respectively). Similarly, under scarce refurbishing capacity (small $\alpha$), it is optimal to allocate most of the returns for warranty coverage immediately without keeping them as earmarked stock, implying less benefit from strategic earmarking. Higher $c_r$, $s$, and a lower $\delta$ improve the performance of the myopic policy since they encourage more salvaging of consumer returns instead of remarketing, and therefore give more weight to the immediate salvaging revenues in the overall profit. Finally, a smaller $c_n$ implies less marginal saving from warranty coverage by refurbishing, which benefits the myopic policy. We conclude that the myopic policy can be used with confidence in practice when the consumer return rate, remarketing potential, manufacturing cost, and warranty demand uncertainty are low, and refurbishing cost and salvage value are high.

Table 3: Tests for Parametric Differences

<table>
<thead>
<tr>
<th>Parameter means</th>
<th>$\Delta M% \leq 3%$</th>
<th>$\Delta M% &gt; 3%$</th>
<th>$H_1$</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi$</td>
<td>0.050</td>
<td>0.096</td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>0.990</td>
<td></td>
<td>0.3705</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.164</td>
<td>0.202</td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.030</td>
<td>0.031</td>
<td></td>
<td>0.1561</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.677</td>
<td>0.753</td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>$c_r/c_n$</td>
<td>0.274</td>
<td>0.283</td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>$s/c_n$</td>
<td>0.170</td>
<td>0.127</td>
<td></td>
<td>0.0000</td>
</tr>
<tr>
<td>$c_r+s$</td>
<td>0.133</td>
<td>0.071</td>
<td></td>
<td>0.0000</td>
</tr>
</tbody>
</table>

***P-Value < 0.01

6. Conclusion

The high cost of lenient return policies force consumer electronics OEMs to look for ways to recover value from lightly used returned products, known as consumer returns. Refurbishing these returns to remarket them or fulfill warranty claims are two common disposition options considered for consumer returns. These options, however, present the OEM with a challenging dynamic allocation problem that lies at the intersection of pricing new and refurbished products and stocking refurbished consumer returns to meet future warranty demand. Since consumer electronics are sold in rapidly changing markets and have short-life cycles, the allocation of refurbished products between remarketing and warranty demand is also influenced by inter-temporal changes in critical
parameters such as the consumer return rate and the product’s failure rate. In this paper, we analyze this dynamic allocation problem and provide managerial insights that can be helpful to consumer electronics OEMs.

Earlier research on closed-loop supply chains concluded that remarketing can be profitable when the new product cannibalization is low. Our study, on the other hand, reveals that when warranty claims and consumer returns are jointly taken into account, refurbishing and earmarking consumer returns to fulfill warranty claims generally dominates the remarketing option, and the OEM should strategically emphasize earmarking consumer returns at the early stages in the life-cycle while switching to remarketing at the later stages in the life-cycle. These results are driven by the fact that remarkeeted products also generate warranty coverage costs, and the OEM is exposed to much larger total warranty demand uncertainty at the early stages in the life-cycle than at the later stages, when most of the warranty demand uncertainty is resolved. Thus, the option of refurbishing to fulfill warranty claims gives the OEM the opportunity to build up earmarked inventory of refurbished consumer returns that can be used to reduce the cost of warranty claims and hedge against future warranty demand uncertainty. Our analytical results reveal that, in certain cases, refurbishing consumer returns to fulfill warranty claims can even increase the refurbished product sales due to this warranty cost reduction effect. These findings contribute to the previous academic literature on closed-loop supply chains by showing that, for product categories with significant warranty coverage and refund costs, remarketing may not be the most profitable disposition option even if the product has strong remarketing potential and the OEM has the pricing leverage to tap into this market. Also, they provide an alternative explanation to the question of why most OEMs are reluctant to sell refurbished products. As such, the predictions of our model can be used to develop testable hypotheses about the impact of warranty service on OEMs remarketing activities.

To obtain more granular insights about the OEM’s dynamic allocation strategy, we study the impact of inter-temporal changes in the consumer return rate and the failure rate on the optimal disposition policy. Our numerical results show that if the consumer return rate is decreasing over time, the optimal earmarked quantity is decreasing while the optimal refurbished product sales are increasing. This happens because a decreasing consumer return rate implies an increasing warranty demand and a decreasing refurbishing capacity, making it optimal to refurbish and earmark most of the consumer returns early in the life-cycle and remarket more aggressively after enough earmarked inventory is accumulated. On the other hand, if the consumer return rate is increasing, the OEM faces a relatively high warranty demand with a relatively low refurbishing capacity at the early stages in the life-cycle compared to warranty demand and refurbishing capacity it faces at the later
stages of the life-cycle. Thus, to bridge this gap between the warranty demand and the consumer returns supply, the OEM optimally allocates the majority of the consumer returns to fulfill the immediate warranty demand and postpones remarketing to later stages in the life-cycle. Similar behavior of the optimal disposition policy (originating from the inter-temporal mismatch between the warranty demand and consumer returns supply) is also observed for a decreasing failure rate. These observations provide a useful managerial insight: when the warranty demand uncertainty is sufficiently high, regardless of the inter-temporal changes in the consumer return and failure rates, the OEM should allocate the majority of the consumer returns to the earmarking option at the early stages in the life-cycle and consider remarketing only after enough earmarked inventory is built up or most of the warranty demand uncertainty is resolved.

To our knowledge, this paper is the first to analyze a dynamic joint pricing and stocking problem with dual disposition options in a closed-loop supply chain setting. We show that the optimal dynamic disposition policy for this problem is a price-dependent base-stock policy where the maximum quantity that can be earmarked is limited by the new and refurbished product sales quantities, which are endogenously determined by the OEM’s pricing decisions. To better understand the operating conditions where the optimal dynamic disposition policy is most valuable, we numerically quantify the value of the optimal dynamic disposition policy by comparing its profit with the profit obtained under a myopic disposition policy, when all parameters are stationary. We find that, overall, the profit penalty incurred by the myopic policy is low and therefore it can be considered as a good heuristic for the optimal dynamic policy. The optimal dynamic policy is most valuable for product categories with high warranty demand uncertainty, high consumer return rates and high manufacturing cost since these parameters imply higher benefits from strategically earmarking at the early stages of the life-cycle. Similarly, high remarketing potential, low refurbishing cost and low salvage values imply less salvaging and more remarketing, and so these conditions also benefit more from a dynamic disposition policy.

References

Manage. Sci. 54(10): 1731–1746.


Calmon A.P., S.C. Graves. 2015. Inventory management in a consumer electronics closed-loop 
supply chain. INSEAD Working Paper Series, 2015/44/TOM.


Chu, W., E. Gerstner, J.D. Hess. 1998. Managing dissatisfaction: How to decrease customer 


Davis, S., M. Hagerty, E. Gerstner. 1998. Return policies and the optimal level of “Hassle”. J. 


Djamaludin, I., D.N.P. Murthy, R.J. Wilson. 1994. Quality control through lot sizing for items 


Fleischmann, M., J.A.E.E. van Nunen, B. Gräve. 2003. Integrating closed-loop supply chains and

Francis, A. 2012. Personal communication. Former President, ATC Logistics & Electronics.


Gventer, B. 2012. Personal communication. Former Director of Operations, ATC Logistics & Electronics.


King, J. 2013. Are you maximizing your return on returns? *Consumer Returns Conference 2013, Dallas, Texas*. Presentation by Vice President of ModusLink Global Solutions.


A. Online Appendix

A.1 Technical Appendix: Proofs

Concavity of the Single-Period Profit Function. We begin by showing that the single-period profit function given in (1) is jointly concave in $(D_n, D_r, Q_r)$. First observe that the single-period profit function can be rewritten in a more compact form as follows: $\Pi(D_n, D_r, Q_r) = \tilde{\pi}(D_n, D_r) - \tilde{C}(D_n, D_r, Q_r)$ where $\tilde{\pi}(D_n, D_r) := ((1-\alpha)p_n - c_n + \alpha s)D_n + ((1-\alpha)p_r - c_r - (1-\alpha)s)D_r$ and $\tilde{C}(D_n, D_r, Q_r) := (c_r + s)Q_r + C(z(D_n, D_r, Q_r))$ with $z(D_n, D_r, Q_r) := Q_r - \gamma(1-\alpha)(D_n + D_r)$,

$$C(x) := hE(x - \xi)^+ + c_n E(\xi - x)^+.$$ The concavity of $\tilde{\pi}(D_n, D_r)$ can be verified by its Hessian matrix. That is, $\partial^2_{D_n} \tilde{\pi}(D_n, D_r) = -2(1 - \alpha) < 0$ and the determinant of the Hessian matrix is equal to $|\nabla^2 \tilde{\pi}(D_n, D_r)| = 4(1 - \alpha)^2 (1 - \delta) > 0$. Thus, Hessian is negative definite and $\tilde{\pi}(D_n, D_r)$ is strictly jointly concave in $(D_n, D_r)$. Next observe that $z(D_n, D_r, Q_r)$ is a linear mapping. Since $C(x)$ is convex and composition with an affine mapping preserves convexity, $C(z(D_n, D_r, Q_r))$ is jointly convex in $(D_n, D_r, Q_r)$. Thus, $\tilde{C}(D_n, D_r, Q_r)$ is jointly convex in $(D_n, D_r, Q_r)$, and the joint concavity of $\Pi(D_n, D_r, Q_r)$ follows.

Proof of Lemma 1. Since $\Pi(D_n, D_r, Q_r) = \tilde{\pi}(D_n, D_r) - \tilde{C}(D_n, D_r, Q_r)$, for a given $(D_n, D_r)$, maximizing $\Pi(D_n, D_r, Q_r)$ is equivalent to minimizing $\tilde{C}(D_n, D_r, Q_r)$. Thus, we are interested in finding the optimal $Q_r$ minimizing $\tilde{C}(D_n, D_r, Q_r)$ over the constraint set $0 \leq Q_r \leq \alpha D_n - D_r$, when $(D_n, D_r)$ is given and satisfy $\alpha D_n - D_r \geq 0$. Since $\tilde{C}(D_n, D_r, Q_r)$ is jointly convex, the optimal $Q_r$ is either at the interior of the constraint set or at its bounds. It can be easily verified that $\partial_{Q_r}C(z(D_n, D_r, Q_r)) = (h + c_n)F(Q_r - \gamma(1 - \alpha)(D_n + D_r)) - c_n$ Thus, by the convexity of $\tilde{C}(D_n, D_r, Q_r)$, the unconstrained optimal solution satisfies the equation $\partial_{Q_r} \tilde{C}(D_n, D_r, Q_r) = c_r + s + (h + c_n)F(Q_r - \gamma(1 - \alpha)(D_n + D_r)) - c_n = 0$, or equivalently $F(Q_r - \gamma(1 - \alpha)(D_n + D_r)) = \frac{c_n - c_r - s}{h + c_n}$. Since $F(\cdot)$ has an inverse, the unconstrained optimal solution is found as $Q_r^* = \gamma(1 - \alpha)(D_n + D_r) + \tilde{z}$. By the assumption $c_n > c_r + s$, the derivative of $\tilde{C}(D_n, D_r, Q_r)$ evaluated at the lower bound of $Q_r$ is negative, that is, $\partial_{Q_r} \tilde{C}(D_n, D_r, Q_r)|_{Q_r = 0} = c_r + s - c_n < 0$. Therefore, by the convexity of $\tilde{C}(D_n, D_r, Q_r)$, it follows that the optimal $Q_r$ cannot be found at its lower bound, and if the unconstrained solution is greater than $\alpha D_n - D_r$, the optimal solution is $Q_r^* = \alpha D_n - D_r$.

Proof of Proposition 1. For a given $(D_n, D_r)$ the interior solution of the optimal earmarking quantity is equal to $Q_r^* = \gamma(1 - \alpha)(D_n + D_r) + \tilde{z}$. By substituting this in $\Pi(D_n, D_r, Q_r)$ and making necessary simplifications, we obtain that $\Pi(D_n, D_r) = \tilde{\pi}(D_n, D_r) - (c_r + s)\gamma(1 - \alpha)(D_n + D_r) - (c_r + s)\tilde{z} - C(\tilde{z})$. Thus, the single-period optimization problem becomes maximizing $\Pi(D_n, D_r)$ subject
to $D_n, D_r \geq 0$. We form the Lagrangian $L(D_n, D_r) = \Pi(D_n, D_r) + \lambda_1 D_n + \lambda_2 D_r$ and write the first-order conditions of the problem as follows: $\partial_{D_n} L(D_n, D_r) = \partial_{D_n} \pi(D_n, D_r) - (c_r + s)\gamma(1 - \alpha) + \lambda_1 = 0$, $\partial_{D_r} L(D_n, D_r) = \partial_{D_r} \pi(D_n, D_r) - (c_r + s)\gamma(1 - \alpha) + \lambda_2 = 0$, $\lambda_1 D_n = 0$, $\lambda_2 D_r = 0$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$.

Since the optimization problem exists only if there is a positive new product sales, we are interested in the solutions with $D^*_n > 0$. Thus, $\lambda_1 = 0$, and depending on the value of $D_r$, we have two cases: (i) If $D_r > 0$ then $\lambda_2 = 0$, and from the first-order conditions above, we have the equations $\partial_{D_n} \pi(D_n, D_r) = (c_r + s)\gamma(1 - \alpha)$ and $\partial_{D_r} \pi(D_n, D_r) = (c_r + s)\gamma(1 - \alpha)$, the solution of which yield $D^*_n$ and $D^*_r$ as given in the first line of the table in Proposition 1. Since the optimal solution must satisfy $D^*_n > 0$ and $D^*_r > 0$, we observe that $D^*_n > 0 \iff M_n - M_r > 0$ and $D^*_r > 0 \iff M_r - \delta M_n > 0$.

Combining the two yields the condition $M_r < M_n < M_r/\delta$ as given in Proposition 1. (ii) If $D_r = 0$ then $\lambda_2 \geq 0$, and the first-order conditions become $\partial_{D_n} \pi(D_n, D_r)|_{D_r = 0} = (c_r + s)\gamma(1 - \alpha)$ and $\lambda_2 = (c_r + s)\gamma(1 - \alpha) - \partial_{D_r} \pi(D_n, D_r)|_{D_r = 0}$. Solving the first equation yields $D^*_n$ as given in the second line of the table in Proposition 1. Substituting $D^*_n$ in the second equation and reorganizing the terms give $\lambda_2 = \delta M_n - M_r$. In the optimal solution, $D^*_n > 0$ and $\lambda_2 \geq 0$ must hold. Hence, for the optimal solution to be in this case, the conditions $M_n > 0$ and $\delta M_n \geq M_r$ must hold. In both cases, substituting $(D^*_n, D^*_r)$ in the definitions of $Q^*_r, p^*_n$, and $p^*_r$ yields the optimal expressions for these variables.

Proof of Corollary 1. We observe from Proposition 1 that in the interior solution, assuming that selling new products is profitable, remarketing is profitable only if the condition $M_n < M_r/\delta$ holds. Thus, equation (3) follows directly from the condition $M_n < M_r/\delta$ and the definitions of $M_n$ and $M_r$.

Proof of Corollary 2. In the benchmark scenario, the OEM does not refurbish consumer returns to cover warranty demand except for remarketing ($Q_r = 0$). Thus, the OEM’s optimization problem boils down to maximizing $\Pi(D_n, D_r) = ((1 - \alpha)p_n - c_n)D_n + ((1 - \alpha)p_r - c_r)D_r - c_n E(R_w(D_n, D_r, \xi)) + s(\alpha D_n - (1 - \alpha)D_r)$ subject to $0 \leq D_r \leq \alpha D_n$. Let $(D^*_n, D^*_r)$ denote the optimal solution to this problem. Similar to the original model, the benchmark model is interesting only if $D^*_r > 0$. Thus, it has three solution cases depending on the value of $D^*_r$, i.e., $D^*_r$ is either in the interior or at the boundaries of the constraint set. The interior solution of the benchmark model is characterized as $D^*_n = \frac{M^*_n - M^*_r}{2(1 - \alpha)}$ and $D^*_r = \frac{M^*_r - \delta M^*_n}{2(1 - \alpha)}$ with $M^*_n := 1 - \alpha - c_n + \alpha s - c_n \gamma(1 - \alpha)$ and $M^*_r := \delta(1 - \alpha) - c_r - (1 - \alpha)s - c_n \gamma(1 - \alpha)$. We are interested in comparing refurbished product sales of the benchmark model and original model, when the interior solution is optimal in both models.
The interior solution of the original model is given by Proposition 1. Thus, the difference between the optimal refurbished product sales of both models is given as $D^* - D^0 = \frac{M_r - \delta M_n - M^o_r - \delta M^o_n}{2(1-\alpha)(1-\delta)\delta}$, and making necessary simplifications yields the result as $\gamma(c_n - c_r - s)/2\delta$.

\[ \square \]

**Proof of Corollary 3.** When remarketing is optimal, from the first line of the table in Proposition 1, we obtain that $\partial_\alpha D^*_n = -\frac{c_n - c_r - s}{2(1-\alpha)^2(1-\delta)}$, and by the assumption $c_n - c_r - s > 0$, it follows that $\partial_\alpha D^*_n < 0$. Similarly, taking the derivative of $D^*_r$ with respect to $\alpha$ gives $\partial_\alpha D^*_r = \frac{\delta c_n - c_r - \delta s}{2(1-\alpha)^2(1-\delta)\delta}$. The sign of $\partial_\alpha D^*_r$ depends on the sign of the term $\delta c_n - c_r - \delta s$. By the optimality condition $M_r > \delta M_n$, we observe that $\delta c_n - c_r > s(1 - \alpha + \alpha \delta) + (c_r + s)\gamma(1 - \alpha)(1 - \delta)$. Since $\delta < 1$, it can be easily shown that $s(1 - \alpha + \alpha \delta) > \delta s$. Also, $(c_r + s)\gamma(1 - \alpha)(1 - \delta) > 0$ since $\alpha < 1$, $\gamma > 0$ and $c_r + s > 0$. Thus, $s(1 - \alpha + \alpha \delta) + (c_r + s)\gamma(1 - \alpha)(1 - \delta) > \delta s$ and therefore, $\delta c_n - c_r > \delta s$ and $\partial_\alpha D^*_r > 0$. The derivative of the optimal earmarking quantity with respect to $\alpha$ is given as $\partial_\alpha Q^*_r = -\frac{\gamma(1-s-(c_r+s)\gamma)}{2\delta}$. Clearly, the sign of $\partial_\alpha Q^*_r$ depends on the sign of the term $\delta - s - (c_r + s)\gamma$. It can be easily shown that the optimality condition $M_r < M_n < M_r/\delta$ implies $M_r > 0$. Thus, by the definition of $M_r$, we have $\delta(1 - \alpha) - (1 - \alpha)s - (c_r + s)\gamma(1 - \alpha) > c_r > 0$, which implies that $(1 - \alpha)(\delta - s - (c_r + s)\gamma) > 0$, and since $\alpha \in (0,1)$, we get $\partial_\alpha Q^*_r < 0$. When remarketing is not optimal, from the second line of the table in Proposition 1, we obtain that $\partial_\alpha D^*_n = -\frac{c_n - c_r - s}{2(1-\alpha)^2}$, which is negative since $c_n - c_r - s > 0$. Similarly, the derivative of the earmarking quantity with respect to $\alpha$ is found as $\partial_\alpha Q^*_r = -\frac{\gamma(1-s-(c_r+s)\gamma)}{2\delta}$, and its sign depends on the term $1 - s - (c_r + s)\gamma$. Note that $M_n > 0$ must hold for the optimal solution to be in this case. Thus, by the definition of $M_n$, we can rewrite this condition as $1 - \alpha - (1 - \alpha)s - (c_r + s)\gamma(1 - \alpha) > c_n - s > 0$, which implies that $(1 - \alpha)(1 - s - (c_r + s)\gamma) > 0$, and therefore $\partial_\alpha Q^*_r < 0$.

\[ \square \]

**Proof of Corollary 4.** When remarketing is optimal, from the first line of the table in Proposition 1, we obtain that $\partial_{c_r} D^*_n = \frac{1}{2(1-\alpha)(1-\delta)} > 0$, and $\partial_{c_r} D^*_r = -\frac{1 + \frac{\gamma(1 - \alpha)(1 - \delta)}{2(1-\alpha)(1-\delta)}}{M_r} < 0$, where the signs of the derivative follow from that $\alpha, \delta, \gamma \in (0,1)$. Similarly, differentiating $Q^*_r$ with respect to $c_r$ yields that $\partial_{c_r} Q^*_r = -\frac{2(1 + \frac{\gamma(1 - \alpha)\gamma}{2\delta})}{M_r} + \partial_{c_r} \tilde{z}$. To decide the sign of $\partial_{c_r} \tilde{z}$, first observe that $\tilde{z}$ solves $F(\tilde{z}) = \frac{c_n - c_r - s}{c_n + \tilde{h}}$ since $F(\cdot)$ has inverse. Thus, by letting $h(\tilde{z}, c_r) := F(\tilde{z}) - \frac{c_n - c_r - s}{c_n + \tilde{h}}$ and applying the implicit function theorem gives that $\partial_{c_r} \tilde{z} = -\frac{\partial_{\tilde{z}} h(\tilde{z}, c_r)}{\partial_{c_r} h(\tilde{z}, c_r)} = -\frac{1}{\gamma(c_n + \tilde{h})} f(\tilde{z})$, where $f(\cdot)$ is the pdf of $\xi$. Hence, $\partial_{c_r} \tilde{z} < 0$ and since the first term in $\partial_{c_r} Q^*_r$ is negative, we conclude that $\partial_{c_r} Q^*_r < 0$. When remarketing is not optimal, from the second line of the table in Proposition 1, we obtain that $\partial_{c_r} D^*_n = -\gamma/2 < 0$ and $\partial_{c_r} Q^*_r = -\frac{(1 - \alpha)\gamma^2}{2} + \partial_{c_r} \tilde{z}$, which is negative since $\alpha, \gamma \in (0,1)$ and $\partial_{c_r} \tilde{z} < 0$.

\[ \square \]
Proof of Proposition 2. (a) For any $t = 0, ..., T - 1$, assume $V_{t+1}(x)$ is concave and observe that the function $h(y, D_n, D_r) := y - R_w(D_n, D_r, \xi) = y - \gamma(1 - \alpha)(D_n + D_r) - \xi$ is an affine mapping. Thus, the composition $V_{t+1}^+(h(y, D_n, D_r)) = V_{t+1}^+(y - R_w(D_n, D_r, \xi))$ is jointly concave in $(y, D_n, D_r)$. Clearly, $E(V_{t+1}^+(y - R_w(D_n, D_r, \xi)))$ is also jointly concave in $(y, D_n, D_r)$. By straightforward calculus, it can be shown that the function $\pi_t(D_n, D_r)$ is also jointly concave in $(D_n, D_r)$. Define the function $C_t(x) := hE(x - \xi)^+ + bE(\xi - x)^+$ and note that $G_t(y, D_n, D_r) = C_t(y - \gamma(1 - \alpha)(D_n + D_r))$. Since $C_t(x)$ is convex in $x$, and $y - \gamma(1 - \alpha)(D_n + D_r)$ is an affine mapping, $G_t(y, D_n, D_r)$ is jointly convex in $(y, D_n, D_r)$. Hence, $-G_t(y, D_n, D_r)$ is jointly concave. Since the second and third terms in the right-hand side of (5) are linear in $D_n, D_r$ and $y$, it thus follows that, if $V_{t+1}(x)$ is concave then $J_t(y, D_n, D_r)$ is jointly concave in $(y, D_n, D_r)$ since it is a summation of concave functions, and the concavity of $V_t(x)$ is immediate from (4). Next, observe that $V_T^+(x)$ is a concave function since $c_n - c_r - s > 0$. Then, the above argument iterates backwards through the periods $t = T - 1, ..., 0$ and completes the proof.

(b) Let $H(y) := (c_r + s)(1 - \beta)y + G_t(t, D_n, D_r)$ and observe that $\lim_{|y| \to \infty} H(y) = \infty$ since $H(y)$ is convex, $\lim_{y \to -\infty} H'(y) = (c_r + s)(1 - \beta) - b < 0$, and $\lim_{y \to \infty} H'(y) = (c_r + s)(1 - \beta) + h > 0$. Assume that $J_t(y, D_n, D_r)$ has a finite maximizer $(\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r)$ for $t = 0, ..., T - 1$. Then, by the concavity of $V_t^+(x)$, it follows that $E(V_t^+(y - R_w(D_n, D_r, \xi))) \leq V_t^+(\hat{y}_t)$ for any $(y, D_n, D_r)$. Thus, from $\lim_{|y| \to \infty} H(y) = \infty$ and (5) we have, for all $(D_n, D_r) \in \Omega$, that $\lim_{|y| \to \infty} J_{t-1}(y, D_n, D_r) = -\infty$. Together with the joint concavity of $J_{t-1}(y, D_n, D_r)$, this implies that $J_{t-1}(y, D_n, D_r)$ has a finite maximizer. Next, observe that $\lim_{|x| \to \infty} V_T^+(x) = -\infty$. It follows from (5) and $\lim_{|y| \to \infty} H(y) = \infty$ that $\lim_{|y| \to \infty} J_{T-1}(y, D_n, D_r) = -\infty$, for all $(D_n, D_r) \in \Omega$. Then, by the concavity of $J_{T-1}(y, D_n, D_r)$ it follows that $J_{T-1}(y, D_n, D_r)$ has a finite maximizer. The argument iterates backwards through the periods $t = T - 1, ..., 0$ and completes the proof.

(c) For any $t = 0, ..., T - 1$: (c.1) if $x > \hat{y}_t$, by the joint concavity of $J_t(y, D_n, D_r)$, the optimal earmark-up-to level is $\hat{y}_t$. To see this more clearly, take a decision tuple $(y_t^n, D_t^n, D_t^r)$ such that $x < y_t^n$, and define $x := \theta \hat{y}_t + (1 - \theta)y_t^n, D_t^n := \theta \hat{D}_t^n + (1 - \theta)D_t^n, D_t^r := \theta \hat{D}_t^r + (1 - \theta)D_t^r$, where $\theta \in [0, 1]$. Then, by the concavity of $J_t(y, D_n, D_r)$ and the global optimality of $(\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r)$, we have that $J_t(x, D_t^n, D_t^r) \geq J_t(\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r) = (1 - \theta)J_t(y_t^n, D_t^n, D_t^r) \geq J_t(y_t^n, D_t^n, D_t^r)$. Thus, if $x > \hat{y}_t$, the optimal earmark-up-to level $y_t^n(x)$ is at its lower bound $x$. (c.2) if $\hat{y}_t - \hat{K}_t \leq x \leq \hat{y}_t$ (or equivalently $x \leq \hat{K}_t \leq x + \hat{K}_t$), by the concavity of $J_t(y, D_n, D_r)$ and global optimality of $(\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r)$, the optimal earmark-up-to level is $(\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r)$. (c.3) If $x < \hat{y}_t - \hat{K}_t$ then $x < \hat{y}_t$. Assume that there exists $(D_t^n(x), D_t^r(x))$ such that $\hat{y}_t - x < \alpha D_t^n(x) - D_t^r(x)$. Then, by the global optimality of $(\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r)$ it follows that $(y_t^n(x), D_t^n(x), D_t^r(x)) = (\hat{y}_t, \hat{D}_t^n, \hat{D}_t^r)$. By the earmarking capacity
constraint, this implies that \( x < \hat{y}_t \leq x + \hat{K}_t \), which is a contradiction with \( x < \hat{y}_t - \hat{K}_t \). Thus, if \( x < \hat{y}_t - \hat{K}_t \), the optimal sales quantities should be such that \( x + \alpha D^*_n(x) - D^*_r(x) < \hat{y}_t \), which implies \( y^*_t(x) < \hat{y}_t \).

A.2 Inter-temporal Behavior of the Optimal New Product Sales

In this section we present and discuss the inter-temporal behavior of the optimal new product sales \( (D^*_n) \) for the three scenarios considered in Figures 2–4 under Section 4.1. Figure 7 shows the changes in the optimal new product sales during the life-cycle with respect to the changes in the consumer return and failure rates.

Figure 7: Dynamics of the Optimal New Product Sales \( (D^*_n) \)

(a) Decreasing Consumer Return Rate   (b) Increasing Consumer Return Rate   (c) Decreasing Failure Rate

A decreasing consumer return rate implies an increasing warranty demand rate during the life-cycle, and it becomes optimal to allocate most of the consumer returns to the earmarking option rather than the remarketing option at the early stages in the life-cycle. Hence, we observe from Figure 7a that the new products are sold at a higher rate at the early stages in the life-cycle. Over time, the optimal refurbished product sales increase due to the decrease in the optimal earmarking quantity, and to partially offset the increase in the total sales and warranty demand rate, the optimal new product sales decrease. On the other hand, for increasing consumer return rate, we observe that the optimal new products sales have a concave behavior. This is because an increasing consumer return rate implies a relatively high warranty demand rate along with a scarce refurbishing capacity at the early stages in the life-cycle. Thus, to increase the refurbishing capacity and speed up the earmarked inventory buildup, the new products are sold at a higher rate at the early stages in the life-cycle. Once the consumer return rate is sufficiently high or enough earmarked inventory

\(^{10}\)The behavior of the optimal refurbished product sales and the optimal earmarking quantity in face of the changes in the consumer return rate and failure rate are discussed in detail in Section 4.1.
is built up, the refurbished products are introduced to the market, and therefore the optimal new product sales decrease.

Similarly, the behavior of the optimal new product sales in the failure rate is primarily driven by the interactions between the refurbished product sales and earmarking quantity. High failure rates imply that the majority of the consumer returns are allocated to the earmarking option, and as the failure rate drops, the remarketed products are introduced to the market more aggressively. Thus, the optimal new product sales are higher at the beginning of the life-cycle and decrease gradually throughout the life-cycle as the remarketed products become more dominant.

A.3 Estimation of the Manufacturing Cost

In this section we outline the method we use to estimate the manufacturing cost and calibrate our model for the numerical experiments. In our model, the construction of the demand functions relies, without loss of generality, on willingness-to-pay parameters normalized to the \([0, 1]\) range. To calibrate the model using existing data, we relax this assumption and allow the underlying willingness-to-pay parameter to be uniformly distributed on \([0, b]\), estimate the maximum willingness to pay \(b\) and then normalize the manufacturing cost to the range \([0, 1]\) by dividing it by \(b\).

To obtain a first-cut estimation of the maximum willingness-to-pay, we use a base model which yields an analytically convenient expression for \(b\). Since some OEMs do not sell refurbished products, our base model is where an OEM sells only new products, receives consumer returns and uses only new products to cover warranty demand. As such the model is a special case of our original model. The profit function for this special case can be given as follows:

\[
\Pi(\bar{D}_n) = (\bar{p}_n - c_n)\bar{D}_n - \bar{p}_n \alpha \bar{D}_n - c_n E(\gamma(1-\alpha)\bar{D}_n + \xi),
\]

which can be rewritten in more compact form as

\[
\Pi(\bar{D}_n) = ((1-\alpha)\bar{p}_n - (1 + \gamma(1-\alpha))\bar{c}_n)\bar{D}_n - \bar{c}_n E(\xi).
\]

Because the consumer willingness-to-pay for a new product is uniformly distributed in the interval \([0, b]\) with \(b > 0\), we obtain \(\bar{p}_n = b(1 - \bar{D}_n)\), where \(\bar{p}_n\) and \(\bar{c}_n\) denote the actual price and manufacturing cost, respectively.

Assuming that the OEM prices the product so as to sell optimally in the market, maximizing \(\Pi(\bar{D}_n)\) yields that 

\[
\bar{D}_n^* = \frac{(1-\alpha)b - \bar{c}_n(1+\gamma(1-\alpha))}{2(1-\alpha)b},
\]

and by the relation 

\[
\bar{p}_n^* = b(1 - \bar{D}_n^*),
\]

we obtain that 

\[
b = 2\bar{p}_n^* - \frac{\bar{c}_n(1+\gamma(1-\alpha))}{1-\alpha}.
\]

Thus, for a given \(\alpha, \bar{c}_n, \gamma\) and \(\bar{p}_n^*\), the maximum willingness-to-pay \(b\) can be inferred from this simple expression. Note that dividing \(\Pi(\bar{D}_n)\) with \(b\) gives the normalized profit function with the normalized manufacturing cost of \(c_n = \bar{c}_n/b\).

To estimate the manufacturing cost that are used in the numerical study, we choose tablet computers as a representative example of a typical consumer electronics product. The material
costs of the products chosen, which are the most sensitive data that few firms are willing to share, are based on publicly available reports by a market intelligence firm (www.isuppli.com/Teardowns). The retail prices are obtained from the OEMs’ or the dedicated sellers’ online shopping websites (e.g., www.store.apple.com/us). To simplify the calibration, we take the material cost as a proxy for the manufacturing cost and also consider the unsubsidized price without a contract for services such as data plans or cellular service. From these public resources, we find that, for example, the manufacturing cost of Ipad 32GB without cellular functionality is estimated to be about $250, and its retail price is reported as $599. If we take realistic values of 10% consumer return rate ($\alpha$) and 3% base warranty demand rate ($\gamma$), the above formula yields a maximum willingness-to-pay of $913. Dividing the reported manufacturing cost of $250 by $913 gives the normalized manufacturing cost of 0.27. We conduct this exercise for different tablet computer models (reported in Table 4) as well as different $\alpha$ and $\gamma$ values. For each model we estimate the maximum willingness-to-pay by averaging the observed values for different $\alpha$ and $\gamma$ combinations. As can be seen from Table 4, the estimated manufacturing (total material) costs are within the interval [0.25, 0.30]. Hence, in our numerical study we set the manufacturing cost to 0.25 and 0.30 to capture these upper and lower bounds.

Table 4: Material Costs and Retail Prices for Tablet Computers

<table>
<thead>
<tr>
<th>Part Description</th>
<th>iPad 2 32GB GSM/HSPA w/o 3G</th>
<th>iPad 16GB 3G w/o 3G</th>
<th>iPad 16GB 3G</th>
<th>iPad 32GB 3G w/o 3G</th>
<th>iPad 64GB 3G w/o 3G</th>
<th>iPad 64GB 3G</th>
<th>Motorola Xoom 3G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Display/Touchscreen</td>
<td>$127.00</td>
<td>$80.00</td>
<td>$80.00</td>
<td>$80.00</td>
<td>$80.00</td>
<td>$80.00</td>
<td>$140.00</td>
</tr>
<tr>
<td>Memory</td>
<td>$65.70</td>
<td>$29.50</td>
<td>$29.50</td>
<td>$59.00</td>
<td>$118.00</td>
<td>$118.00</td>
<td>$80.40</td>
</tr>
<tr>
<td>App. Procs./Baseband</td>
<td>$82.70</td>
<td>$17.00</td>
<td>$41.50</td>
<td>$17.00</td>
<td>$17.00</td>
<td>$41.50</td>
<td>$35.43</td>
</tr>
<tr>
<td>Camera</td>
<td>$4.30</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$14.00</td>
</tr>
<tr>
<td>WiFi/Bluetooth</td>
<td>$9.00</td>
<td>$8.05</td>
<td>$8.05</td>
<td>$8.05</td>
<td>$8.05</td>
<td>$8.05</td>
<td>$7.61</td>
</tr>
<tr>
<td>Misc.</td>
<td>$87.90</td>
<td>$84.80</td>
<td>$87.40</td>
<td>$84.80</td>
<td>$87.40</td>
<td>$87.40</td>
<td>$82.48</td>
</tr>
<tr>
<td>Total Mat. Cost</td>
<td>$326.60</td>
<td>$219.35</td>
<td>$246.45</td>
<td>$248.85</td>
<td>$307.85</td>
<td>$334.95</td>
<td>$359.92</td>
</tr>
</tbody>
</table>

Estimated Max WTP | $1073.81 | $739.97 | $968.09 | $905.27 | $1035.86 | $1263.98 | $1174.61 |

Estimated Total Mat. Cost | 0.304 | 0.296 | 0.255 | 0.275 | 0.297 | 0.265 | 0.306 |