

# The Value of Collaborative Forecasting in Supply Chains

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## Abstract

Motivated by the mixed evidence concerning the adoption level and value of collaborative forecasting (CF) implementations in retail supply chains, in this paper, we explore the conditions under which CF offers the highest potential. We consider a two-stage supply chain with a single supplier selling its product to consumers through a single retailer. We assume that both the supplier and the retailer can improve the quality of their demand forecasts by making costly forecasting investments to gather and analyze information. First, we consider a *non-collaborative* (NC) model where the supplier and the retailer can invest in forecasting but do not share forecast information. Next, we examine a *collaborative forecasting* (CF) model where the supplier and the retailer combine their information to form a single shared demand forecast. We investigate the value of CF by comparing each party's profits in these scenarios under three contractual forms that are widely used in practice (two variations of the simple wholesale price contract as well as the buyback contract). We show that for a given set of parameters, CF may be Pareto improving for none to all three of the contractual structures, and that the Pareto regions under all three contractual structures can be expressed with a unifying expression that admits an intuitive interpretation. We observe that these regions are limited and explain how they are shaped by the contractual structure, power balance and relative forecasting capability of the parties. In order to determine the specific value of collaborative forecasting as a function of different factors, we carry out a numerical analysis and observe the following: First, under non-coordinating contracts, improved information due to CF has the added benefit of countering the adverse effects of double marginalization in addition to reducing the cost of supply-demand mismatch. Second, one may expect the value of CF to increase with bargaining power, however this does not hold in general: The value of CF for the newsvendor first increases and then

decreases in his bargaining power. Finally, while one may expect CF to be more valuable under coordinating contracts, rather than a simple wholesale price contract that is prone to double marginalization, the *magnitude of the gain* from CF is in many cases higher in the absence of quantity coordination.

**Key words:** supply chain management, collaborative forecasting, information exchange, forecast quality.

## 1 Introduction

During the last two decades, retail supply chains have experimented with various information sharing arrangements in an effort to reduce the operational inefficiencies arising from demand uncertainty. One such initiative is *Collaborative Planning, Forecasting, and Replenishment* (CPFR), which aims to improve production and replenishment decisions through the use of a single shared demand forecast that is agreed on by the retailer and the supplier. The CPFR collaboration template (Seifert, 2003) prescribes that the retailer and the supplier each create a demand forecast and enter it into the collaboration platform. If there are any differences between the two forecasts, they are reconciled through discussion and information sharing among managers, and a single shared demand forecast is generated, which serves as the basis for quantity decisions.

The existing academic research (Aviv, 2001, 2002, 2007) on the value of collaborative forecasting suggests that collaborative forecasting can result in significant benefits. Indeed, there are many success stories. Successful implementations include Procter & Gamble's experiences with retail partners Metro and Tesco, and Wal-Mart's experiences with many of its suppliers such as Sara Lee (<http://www.vics.org/committees/cpfr>). At the same time, it was also reported that many collaboration initiatives did not succeed (GMA, 2002; Supply Chain Digest, 2008). A study (GMA, 2002) conducted for Grocery Manufacturers of America reveals that "67% of GMA member companies are engaged in some form of collaborative planning, forecasting and replenishment activity, with only 19% moving beyond pilot studies."

The high start-up costs of the collaboration platform and the complexity of the process have often been cited as culprits in the lack of adoption of collaborative forecasting (Seifert, 2003; White and Roster, 2004; Supply Chain Digest, 2008), but there has been no exploration of the factors that explain whether sustained benefits will accrue to both supply chain partners once the collaborative forecasting platform and framework are adopted. Yet, the mixed results observed in practice from those collaborations that do go forward lead us to believe that the benefits from collaborative

forecasting must be context specific. Consequently, this paper explores the conditions under which collaborative forecasting offers the highest potential in retail supply chains. In particular, we focus on the influence of costly internal forecast process improvement and the nature of the supply chain relationship: The parties must internally make costly investments in obtaining relevant data, improving data quality and integrity, and generating forecasts. Since forecast improvements by one party typically benefit the other, implementing collaborative forecasting can lead to a change in each party’s forecasting investment. Moreover, Collaborative Forecasting is typically superimposed on an existing supply chain relationship. The nature of this relationship (contractual structure and power balance) may have an important effect on its success. Our goal is to understand how these effects interact to determine the value of collaborative forecasting.

To this end, we consider a two-stage supply chain with a single supplier selling its product to consumers through a single retailer. We assume that the supplier and the retailer have heterogeneous forecasting capabilities and that both can improve the quality of their demand forecasts by making costly forecasting investments. First, we consider a *non-collaborative* (NC) model where the supplier and the retailer can invest in forecasting but do not share forecast information. Next, we examine a *collaborative forecasting* (CF) model where the supplier and the retailer combine their information to form a single shared demand forecast. We investigate the value of collaborative forecasting by comparing each party’s profits in these scenarios under the following three contractual forms that are widely used in practice: Two variations of the simple wholesale price contract, *Retailer Managed Inventory with wholesale pricing (RMI)* and *Supplier Managed Inventory with wholesale pricing (SMI)*, where the retailer and the supplier, respectively, make the quantity decision, and *Retailer Managed Inventory with buyback (BB)* where the retailer makes the quantity decision but returns unsold inventory to the supplier for a refund. We proxy the power of a given party by the share of expected operating profits he appropriates, which is determined by the existing contract parameters (also called “terms of trade”).

Our paper explores the following questions: What are the conditions that are conducive to the adoption of collaborative forecasting? What are the factors that yield a high value from CF for the retailer and the supplier, respectively? How do these factors depend on the underlying contract structure or power balance in the supply chain? Is CF more valuable in settings where a simple wholesale price contract is used or where a coordinating buyback contract is used?

To answer these questions, we characterize the equilibrium forecast investments and identify the Pareto-improving region where both parties benefit from collaborative forecasting. Our main

analytical contribution is to show that the Pareto region under all three contractual structures we study can be expressed with a unifying expression that admits an intuitive interpretation. We observe that these regions are limited and explain how they are shaped by the contractual structure, power balance and relative forecasting capability of the parties. We then investigate the value of collaborative forecasting in a numerical study and delineate its drivers.

Our work contributes to the recent body of literature on collaborative forecasting. Aviv (2001) proposes a supply chain model where the supplier and the retailer dynamically update their forecasts and that can be used to quantify the inventory and service performance of supply chain. The paper studies the benefit of collaborative forecasting and finds that it may provide substantial benefits for the supply chain, especially when the correlation between trading partners' forecasts is low. Aviv (2002) extends this model to the case of autocorrelated demand. Aviv (2007) investigates a similar model but explicitly models the manufacturer's production environment as well as the internal service performance. The paper investigates the value of collaborative forecasting and identifies elements such as forecast correlation and supply side agility as being important for the success of collaborative practices. A more detailed review on research done on the value of CPFR can be found in a recent book chapter by Aviv (2004).

Aviv's papers are rich in modeling the planning and replenishment elements and of the forecast evolution process. They capture the essence of forecast collaboration by combining two forecasts of exogenous accuracy. Hence the accuracies of the individual forecasts are independent of whether collaborative forecasting is implemented or not. Our research, on the other hand, captures the essence of the quantity decisions by using the newsvendor model. It brings a new dimension to the forecasting element by assuming that supply chain partners can invest in improving their forecasts such that forecast accuracies are endogenously determined. Moreover, we capture the competitive interaction whereby once the supply chain partners agree to share their forecasts, they may strategically change their forecast investment levels. This allows us to examine the implications of the competitive aspects of collaborative forecasting in depth. However, issues that arise in multi-period settings (such as the implications of order fluctuations for operational efficiency discussed in depth by Aviv 2001 and 2007) remain beyond the scope of this model.

There is a vast literature on supply chain contracts (Cachon, 2003) that investigates the impact of various contracts on supply chain performance and the value that can be obtained from eliminating double marginalization in supply chains. Our paper makes a contribution to the intersection of this literature with the collaborative forecasting literature. We do this by considering

the forecasting investments under different contractual structures and show that the contractual form and the power balance in the supply chain are important drivers of the value of collaborative forecasting. For example, we find that the value of collaborative forecasting under the wholesale price contract can be higher or lower than that in the coordinating buyback contract depending on the share of the operating profit appropriated by the supplier.

Economists have studied investment in forecasting and information sharing in the context of horizontal competition where oligopolists have private information about uncertain market characteristics. These models incorporate either an information-sharing stage, where forecast accuracy is exogenous (e.g., Novshek and Sonnenschein, 1982; Vives, 1984; Gal-Or, 1985), or an investment stage, where forecasts are not shared (e.g., Li et al., 1987; Vives, 1988). Main findings include that firms often wish to distort or fully withhold information. The supply chain literature has mainly studied one-sided information exchange in vertical competition, where it is typically the retailer that has a forecast of demand. Because of the strategic nature of the supply chain interaction, there can again be an incentive to distort information. A stream of papers focuses on designing mechanisms that induce truthful information sharing through signaling (Cachon and Lariviere, 2001; Özer and Wei, 2006) and screening (Özer and Wei, 2006; Lariviere, 2002).

Interestingly, it has also been shown that truthful information sharing can occur under the simple contracts we study. In a setting with a simple wholesale price contract, Özer et al. (2010) show that truthful information sharing may take place (in a controlled laboratory experiment) even in the absence of reputation-building mechanisms and complex contracts, and argue that the underlying reason for cooperation is trust. Li (2002) says truthful information sharing can be ensured via the participation of an external agent. Miyaoka (2003) finds that a coordinating buy-back contract provides incentives for two-sided truthful information sharing. Taylor and Xiao (2009) show that buyback contracts can provide incentives to influence the retailer's forecasting decision and elicit forecast information.

Consequently, in this paper, we do not focus on mechanism design, instead, we characterize the value of collaborative forecasting assuming truthful information sharing under contracts that are prevalent in practice; in its absence the parties would default to non-collaborative forecasting since the information would be uninformative. In this way, we isolate the impact of the strategic interaction at the investment stage. We show that the competitive aspects of collaborative forecasting alone can limit its adoption in practice.

The rest of the paper is organized as follows: §2 describes the model and discusses the assump-

tions. §3 derives the equilibrium forecasting investments and profits in the noncollaborative and collaborative settings and characterizes the Pareto region where both parties would prefer collaborative forecasting. A detailed numerical study concerning the value of collaborative forecasting is presented in §4. Finally, §5 concludes with a summary of the insights from the analysis.

## 2 The Model

The goal of collaborative forecasting initiatives is to improve the supply chain’s ability to match supply and demand. It has been extensively reported in trade literature that such initiatives have the highest potential when they are implemented in the context of promotion planning and new product introductions in retail supply chains with long lead-times (Accenture, 2002; Småros et al., 2004), while little value is expected otherwise: “Products with inherently short lead times, that are not highly promoted, or those with future demand patterns that can be easily extrapolated from past demand provide less of a financial payback using CPFR” (Andrews, 2008). Promotions and new product introduction are “irregular” one-time events with relatively large underage and overage costs even for “regular” products that are sold on a continuous basis such as toothpaste and diapers, a point that was confirmed during our discussions with practitioners. Since the single-period newsvendor model has been extensively used for capturing the supply-demand mismatch cost in such settings, it is the framework we adopt in this paper. Our results can be interpreted as evaluating the value of collaborative forecasting where it inherently has high potential.

To answer the question of how the contractual structure affects the value of collaborative forecasting, we focus on the simple wholesale price contract ( $w$ ), which is one of the most prevalent contract types in practice (Cachon, 2004; Perakis and Roels, 2007) and the buyback contract ( $w, b$ ), which is commonly used for items such as music, books, software, and clothing (Pasternack, 1985; Kandel, 1996; Padmanabhan and Png, 1997). We further refine the wholesale price contract to consider Retailer-Managed Inventory (RMI) and Supplier-Managed Inventory (SMI). Under the RMI contract, the retailer is the “newsvendor”: He determines the stocking quantity, purchases it from the supplier before demand is realized and incurs the supply-demand mismatch cost. We note that the buyback contract is an RMI contract where the cost of supply-demand mismatch is shared. Under the SMI contract, the supplier is the “newsvendor.” He determines the production quantity before the selling season and incurs the supply-demand mismatch cost.

Collaborative forecasting is usually implemented in the context of a broader commercial frame-

work including pricing and assortment that is negotiated between the retailer and the supplier, typically on a yearly basis. Therefore, we take terms of trade under each contractual structure as given and we focus on the sequence of events starting with the investments for forecasting and ending with the realization of demand. The sequence of events is as follows: (1) Forecast investment; (2) Demand forecast updating; (3) Quantity decision; (4) Demand realization. We assume that unfulfilled demand is lost. Under the *non-collaborative* (NC) forecasting model, the supplier or the retailer make their quantity decision (if any) based on their own demand forecasts, whereas under *collaborative forecasting* (CF), they use a joint demand forecast which takes into account both the supplier’s and retailer’s demand forecasts. In the remainder of this section, we discuss how we model forecasting and information sharing.

The essence of collaborative forecasting is that the two parties independently create a demand forecast based on their own information. They then combine these to create a joint demand forecast. In creating these forecasts, the supplier and the retailer make use of information sets (expert estimates, market research reports, or retail test results (Fisher and Rajaram, 2000)) which may be distinct. For instance, the supplier may have information about the sales of a product in other stores, while the retailer may have the ability to generate better information about local demand through daily interactions with customers.

To model forecast collaboration, we use a forecasting model based on Winkler (1981) and Clemen and Winkler (1985). Both the supplier and the retailer have common prior information and believe that demand  $D$  is distributed normally with mean  $\mu$  and variance  $\sigma_0^2$ . The information acquisition process is as follows: The supplier and the retailer can each draw (without replacement) costly signals from a pool of signals that are informative about the demand. When  $D = d$ , signal  $i$  is a realization of the random variable  $d + \xi_i$ , where  $\xi_i$  is a random variable that represents the error of signal  $i$ . Suppose  $n$  signals are drawn. Then the error vector  $\Xi_n = (\xi_1, \dots, \xi_n)$  is assumed to be a multivariate normal random variable with zero mean and positive definite covariance matrix. The covariance matrix is intraclass (Press, 1972), i.e.,  $\text{VAR}[\xi_i] = \sigma^2$ ,  $\forall i = 1, \dots, n$ , and  $\text{COV}[\xi_i, \xi_j] = \rho\sigma^2$ ,  $\forall i \neq j$ . To ensure that the covariance matrix is positive definite, we assume the correlations between error terms are nonnegative ( $\rho \geq 0$ ), which is appropriate to capture reality as  $\rho > 0$  models overlapping information. The variances and correlations reflect the quality of the additional information obtained from a signal; a higher variance means the signal is more noisy, whereas a higher correlation means there is more information overlap between the signals. Then it follows that a vector of  $n$  signals is also multivariate normal with mean  $\tilde{\mu}_n = (\mu, \dots, \mu)$  and covariance

matrix  $\tilde{\Sigma}_n$  such that  $\tilde{\Sigma}_{ii} = \sigma_0^2 + \sigma^2$ ,  $\forall i$  and  $\tilde{\Sigma}_{ij} = \rho\sigma^2$ ,  $\forall i \neq j$ .

Let  $D|\Omega_n$  be the conditional distribution of demand after observing  $n$  signal realizations  $\Omega_n = (\psi_1, \dots, \psi_n)$ . Then  $D|\Omega_n \sim N(\mu_n(\Omega_n), \sigma_n^2)$ , where  $\mu_n(\Omega_n) = \frac{(1+(n-1)\rho)\sigma^2\mu + \sigma_0^2\sum_{i=1}^n\psi_i}{(1+(n-1)\rho)\sigma^2 + n\sigma_0^2}$  and  $\sigma_n^2 = \left[\frac{1}{\sigma_0^2} + \frac{n}{[1+(n-1)\rho]\sigma^2}\right]^{-1}$ . Notice that  $\mu_n(\Omega_n)$  is a weighted average of the prior mean  $\mu$  and the sum of the observed signals  $\psi_1, \dots, \psi_n$ .  $\sigma_n^2$  is independent of the signals and is decreasing and convex in  $n$  (Clemen and Winkler, 1985). In order to simplify the exposition of our analytical results, we assume a diffuse prior (i.e.,  $\sigma_0^2 \rightarrow \infty$ ) and uncorrelated signals (i.e.,  $\rho = 0$ ). Under these assumptions, the updated mean and variance after observing  $n$  signals are given by  $\mu_n(\Omega_n) = \frac{\sum_{i=1}^n\psi_i}{n}$  and  $\sigma_n^2 = \frac{\sigma^2}{n}$ . In our numerical study, we relax the assumption of uncorrelated signals and investigate the value of collaborative forecasting as a function of the correlation as well.

Let  $s$  and  $r$  denote the number of signals drawn by the supplier and the retailer, respectively, and let  $\Omega_s$  and  $\Omega_r$  denote their  $s$ - and  $r$ -dimensional vectors of observed signals, respectively. In the non-collaborative scenario, the supplier and the retailer individually update their demand forecasts  $D|\Omega_r$  and  $D|\Omega_s$  and act on their own demand forecasts. In the collaborative forecasting scenario, we model the updated forecast as  $D|(\Omega_r \cup \Omega_s)$ , which reflects truthful information sharing. This is assured via the contract structure in the buyback contract. With the wholesale price contract, it can occur in practice thanks to factors such as trust or reputation. In the absence of truthful information sharing, the parties would disregard the information provided by the other and effectively default to the non-collaborative scenario. Collaborative forecasting would not have any value in this case.

To capture the costly nature of forecasting investment, we assume that the cost of observing  $n$  signals is given by  $C_i(n) = k_i n^q$ ,  $i \in \{R, S\}$ . Firms have inherently different forecasting capabilities. To capture this, we allow the forecasting cost parameters  $k_R$  and  $k_S$  to be different. A more capable party has a lower value of  $k$ . We refer to the parameter  $q$  as the *forecasting technology* parameter and assume that it is the same for both parties. We limit our analysis to cases where  $q \geq 1$  as it is more reasonable to assume that each additional signal comes at an equal or higher cost. The magnitude of  $q$  captures the diseconomies in the forecast investment. Finally, we assume that the investments into forecasting are non-contractible, due to the difficulty of proving in court whether a poor forecast is due to a low forecasting investment or to the inherent variability in the demand.

### 3 Analysis

We first characterize the forecast investments for the retailer and the supplier under noncollaborative and collaborative forecasting in Sections 3.1 and 3.2, respectively. In Section 3.3, we compare

the forecast investments and profits under these two scenarios.

### 3.1 Noncollaborative Forecasting

In the noncollaborative (NC) forecasting scenario, forecast investments by the retailer and the supplier only improve their own forecast accuracies. In the RMI and SMI settings, respectively, the retailer and the supplier face a trade-off between the investment cost of improving their own forecast and the uncertainty cost arising from making the stocking decision based on an inaccurate forecast. We characterize the expected profit functions of the retailer and the supplier for given investment levels  $r$  and  $s$  under the three contractual settings under study and show that they can be expressed in the same form that admits an easy interpretation. Lemma 1 then presents the equilibrium investment levels and profits for the three settings. Throughout the paper we use the convention that  $\pi$  and  $\Pi$  refer to the supplier and retailer profits, respectively.  $\phi$  and  $\Phi$  denote the p.d.f and c.d.f of the standard normal distribution. Profit derivations under the three contracts can be found in Appendix A, while the proofs are presented in Appendix B.

**Retailer-Managed Inventory with Wholesale Pricing (RMI).** Under RMI, the supply chain operates under a simple wholesale price contract,  $w$ , and the retailer determines the stocking quantity and bears all inventory risk. Suppose that the retailer makes a forecast investment  $r$  and updates his demand forecast based on observed signals  $\Omega_r$ . The retailer then places an order  $Q$  that balances the expected cost of lost sales with that of leftover inventory; unmet demand is lost. It is straightforward to show that the retailer's order quantity given  $r$  and  $\Omega_r$  is given by  $Q_{NC}^{RMI}(r, \Omega_r) = \mu_r(\Omega_r) + z_R \sigma_r$ , where  $\mu_r(\Omega_r)$  and  $\sigma_r$  are the updated mean and standard deviation of the retailer's forecast given  $r$  and  $\Omega_r$ , and  $z_R \doteq \Phi^{-1}(1 - w/p)$ . The supplier does not invest in forecasting as he gains no benefit from doing so under the non-collaborative scenario. Substituting the retailer's optimal order quantity into the retailer's and supplier's profit functions, and taking expectation over all signal realizations (Appendix A), the retailer's and the supplier's expected profits as a function of  $r$  can be written as

$$\pi_{NC}^{RMI}(r) = (w - c)\mu - H_S^{RMI}\sigma_r \quad (1)$$

$$\Pi_{NC}^{RMI}(r) = (p - w)\mu - H_R^{RMI}\sigma_r - k_R r^q, \quad (2)$$

where  $\sigma_r = \sigma/\sqrt{r}$ ,  $H_R^{RMI} \doteq p\phi(z_R)$  and  $H_S^{RMI} \doteq -(w - c)z_R$ . We henceforth call  $H$  the unit cost of uncertainty as it translates an increase in demand uncertainty as measured by  $\sigma$  to a change in profits. Note that  $H_R^{RMI} > 0$ , hence the retailer's profit decreases in  $\sigma_r$ , or equivalently, increases in his forecast accuracy. However, the sign of  $H_S^{RMI}$  depends on  $z_R$ . Consequently, the supplier's

profit increases (decreases) in the retailer's accuracy when  $z_R < 0$  ( $z_R > 0$ ). This follows from the fact that when  $z_R$  is negative (positive), a more accurate forecast leads to a larger (smaller) order.

**Supplier-Managed Inventory with Wholesale Pricing (SMI).**

SMI is also based on a simple wholesale price, however it differs from the RMI contract in one aspect: The supplier makes the production decision in the face of demand uncertainty, while the retailer places an order after demand realization (hence the supplier bears all inventory risk). Suppose that the supplier makes a forecast investment of  $s$ , and updates his demand forecast based on observed signals  $\Omega_s$ . It is straightforward to show that the supplier's production quantity given his investment  $s$  and the observed signals  $\Omega_s$  is given by  $Q_{NC}^{SMI}(s, \Omega_s) = \mu_s(\Omega_s) + z_S \sigma_s$ , where  $\mu_s(\Omega_s)$  and  $\sigma_s$  are the updated mean and standard deviation of the supplier's demand forecast given  $s$  and  $\Omega_s$ , and  $z_S \doteq \Phi^{-1}(1 - c/w)$ . The retailer does not invest in forecasting as he gains no benefit from doing so under the noncollaborative scenario. Substituting the optimal production quantity into the retailer's and the supplier's profit functions, and taking expectation over all signal realizations, the retailer's and the supplier's expected profits as a function of  $s$  can be written as

$$\pi_{NC}^{SMI}(s) = (w - c)\mu - H_S^{SMI} \sigma_s - k_S s^q \quad (3)$$

$$\Pi_{NC}^{SMI}(s) = (p - w)\mu - H_R^{SMI} \sigma_s, \quad (4)$$

where  $\sigma_s = \sigma/\sqrt{s}$ ,  $H_S^{SMI} \doteq w\phi(z_S)$  and  $H_R^{SMI} \doteq (p - w) \left[-\frac{z_S c}{w} + \phi(z_S)\right]$ . Note that  $H_R^{SMI} > 0$  (because the standard loss function  $L(z_S) \doteq \left[-\frac{z_S c}{w} + \phi(z_S)\right]$  is always positive (Tong, 1990)) and  $H_S^{SMI} > 0$ . Hence, both the supplier's and the retailer's profit increase in the supplier's forecast accuracy. Comparing RMI and SMI, we note that because of the vertical nature of the relationship, how one party's forecast accuracy affects the other is not symmetric: The supplier's profit under RMI is determined by the retailer's order  $Q_{NC}^{RMI}$  such that a lower retailer accuracy benefits the supplier when this translates into a larger order. In contrast, the retailer's profit under SMI is determined by  $\min(D, Q_{NC}^{SMI})$ , and his expected loss from his maximum attainable expected profit  $(p - w)\mu$  increases as the supplier's ability to anticipate demand and set the production quantity appropriately diminishes.

**Retailer Managed Inventory with a Buyback Contract (BB).**

With a buyback contract, the supplier charges the retailer  $w$  per unit purchased, but pays the retailer  $0 < b < w$  per unit for all the returned units remaining at the end of the season (RMI with a buyback contract with  $b = 0$  is equivalent to RMI with a wholesale price contract). As such, buyback contracts involve risk sharing. Furthermore, buyback contracts also allow the parties to achieve quantity coordination

by eliminating double marginalization. It can be shown that the retailer's order quantity given signals  $\Omega_r$  is given by  $Q_{NC}^{BB}(\Omega_r) = \mu_r(\Omega_r) + z_{Rb}\sigma_r$  where  $z_{Rb} \doteq \Phi^{-1}\left(\frac{p-w}{p-b}\right)$ . The supplier does not invest in improving his forecast accuracy: he obtains no benefit from doing so, because the retailer determines the stocking quantity. In a similar manner as above, the retailer's and the supplier's expected profits as a function of  $r$  are given by

$$\pi_{NC}^{BB}(r) = (w - c)\mu - H_S^{BB}\sigma_r \quad (5)$$

$$\Pi_{NC}^{BB}(r) = (p - w)\mu - H_R^{BB}\sigma_r - k_R r^q, \quad (6)$$

where  $H_S^{BB} \doteq b\phi(z_{Rb}) - \left(w - c - \frac{b(p-w)}{p-b}\right)z_{Rb}$  and  $H_R^{BB} \doteq (p-b)\phi(z_{Rb})$ . As with RMI with a simple wholesale price contract, the retailer's unit cost of uncertainty  $H_R^{BB}$  is positive, so he benefits from improving his forecast accuracy. When  $z_R > 0$ ,  $H_S^{BB}$  is negative at  $b = 0$  and strictly increases in  $b$ . Consequently, below a threshold value  $\bar{b}$ , the supplier makes less profit when the retailer's accuracy improves, but stands to gain from the retailer's accuracy improvement if his exposure to demand risk is high enough ( $b > \bar{b}$ ). Given a wholesale price  $w$ , suppose that the parties set  $b$  such that the supply chain is coordinated, i.e.,  $\frac{p-w}{p-b} = 1 - \frac{c}{p}$ , or  $b(w) = p\frac{w-c}{p-c}$ . Let  $\lambda \doteq \frac{w-c}{p-c}$ ; this is the proportion of the total supply chain profit appropriated by the supplier. It is easy to show that for the coordinating buyback contract  $H_S^{BB} = \lambda p\phi(z_{Rb}) > 0$  and  $H_R^{BB} = (1 - \lambda)p\phi(z_{Rb}) > 0$ , such that both parties benefit from the retailer's increased accuracy. In the rest of the paper, we only focus on coordinating buyback contracts.

A few observations are in order. First, while the profit functions for the retailer and the supplier under the three contractual forms are distinct, they all have the same structure. All profit functions are composed of three parts: profit in the absence of uncertainty, the cost of uncertainty and the cost of forecast improvement (if any). Second, the cost of uncertainty can be broken down further into the standard deviation ( $\sigma_r$  or  $\sigma_s$ ) and the cost of uncertainty for one unit of standard deviation ( $H_R$  and  $H_S$  for the retailer and the supplier, respectively). Under each contractual form,  $H_R$  and  $H_S$  embody the impact of economic parameters,  $c$ ,  $w$  and  $p$ ; however, the expressions for  $H_R$  and  $H_S$  differ across contractual forms, and their sensitivity with respect to a specific parameter can go in opposite directions. To give an example, both  $H_R^{SMI}$  and  $H_R^{BB}$  decrease in  $w$ , while  $H_S^{SMI}$  and  $H_S^{BB}$  increase in  $w$ ; and  $H_R^{RMI}$  first increases and then decreases in  $w$ , while  $H_S^{RMI}$  increases in  $w$ .

Lemma 1 summarizes the retailer's and the supplier's equilibrium forecast investments,  $r_{NC}$  and  $s_{NC}$ , in the non-collaborative forecasting scenario:

**Lemma 1** *For any  $q \geq 1$ , the supplier's and retailer's forecasting investments in the non-collaborative*

forecasting scenario can be characterized as follows:

- (i) Under RMI with a wholesale price contract,  $s_{NC}^{RMI} = 0$  and  $r_{NC}^{RMI} = \left(\frac{H_R^{RMI}\sigma}{2qk_R}\right)^{2/(2q+1)}$ .
- (ii) Under SMI with a wholesale price contract,  $s_{NC}^{SMI} = \left(\frac{H_S^{SMI}\sigma}{2qk_S}\right)^{2/(2q+1)}$  and  $r_{NC}^{SMI} = 0$ .
- (iii) Under RMI with a buyback contract,  $s_{NC}^{BB} = 0$  and  $r_{NC}^{BB} = \left(\frac{H_R^{BB}\sigma}{2qk_R}\right)^{2/(2q+1)}$ .

We can make several observations based on this result. First, in the absence of forecast collaboration, only one party invests into forecast improvement: The party making the quantity decision in the face of uncertainty (“the newsvendor”) benefits from improving its forecast by gathering more information, but the other party gains nothing from its own forecast investment since its profits are solely determined by the newsvendor’s decision. Second, the forecast improvement effort increases in  $H_R$  under the RMI contracts and in  $H_S$  under the SMI contract. This is because as  $H_R$  ( $H_S$ ) increases, the retailer (supplier) has more to lose for a unit increase in standard deviation (and hence has more to gain from improving forecast accuracy). Third, the equilibrium investment level for the newsvendor is high when signals are more noisy. Fourth, if the forecasting capability is low ( $k$  is high), then a lower investment is made at optimality. Finally, if the forecasting technology parameter ( $q$ ) increases, the forecasting investment decreases.

### 3.2 Collaborative Forecasting

The timeline in the collaborative forecasting scenario is identical to the noncollaborative scenario with the exception that the parties develop a single shared demand forecast based on their pooled information set  $\Omega_j = \Omega_r \cup \Omega_s$  before making any quantity decisions. The profit functions of the retailer and the supplier under collaborative forecasting with contract  $x \in \{RMI, SMI, BB\}$  can be written using a similar representation as under non-collaborative forecasting,

$$\pi_{CF}^x(s, r) = (w - c)\mu - H_S^x\sigma_j - k_S s^q \quad (7)$$

$$\Pi_{CF}^x(s, r) = (p - w)\mu - H_R^x\sigma_j - k_R r^q, \quad (8)$$

with  $H_S^x$  and  $H_R^x$  defined as in the previous section for each contractual structure. Let  $s_{CF}^x$  and  $r_{CF}^x$  denote the supplier’s and the retailer’s equilibrium investment levels in the collaborative forecasting scenario. As seen from Lemma 2, there exists a unique pure-strategy Nash equilibrium in the investment levels for all contracts except when  $q = 1$  and  $\frac{H_S^x}{k_S} = \frac{H_R^x}{k_R}$  jointly hold.

**Lemma 2** *The supplier’s and the retailer’s equilibrium forecasting investments under each contractual structure are as follows:*

(i) Let  $q = 1$ . If  $\frac{H_S^x}{k_S} > \frac{H_R^x}{k_R}$ , then  $s_{CF}^x = \left(\frac{H_S^x \sigma}{2k_S}\right)^{\frac{2}{3}}$  and  $r_{CF}^x = 0$ . If  $\frac{H_S^x}{k_S} < \frac{H_R^x}{k_R}$ , then  $s_{CF}^x = 0$  and  $r_{CF}^x = \left(\frac{H_R^x \sigma}{2k_R}\right)^{\frac{2}{3}}$ . If  $\frac{H_S^x}{k_S} = \frac{H_R^x}{k_R}$ , there exist multiple equilibria such that  $s_{CF}^x + r_{CF}^x = \left(\frac{H_R^x \sigma}{2k_R}\right)^{2/3} \left(= \left(\frac{H_S^x \sigma}{2k_S}\right)^{2/3}\right)$ .

(ii) Let  $q > 1$ . If  $H_S^x \leq 0$ ,  $s_{CF}^x = 0$  and  $r_{CF}^x = \left(\frac{H_R^x \sigma}{2qk_R}\right)^{\frac{2}{2q+1}}$ ; otherwise  $s_{CF}^x = \left(\frac{H_S^x \sigma}{2k_S q}\right)^{\frac{2}{2q+1}} \left(1 + \left(\frac{H_R^x/k_R}{H_S^x/k_S}\right)^{\frac{1}{q-1}}\right)^{\frac{-3}{2q+1}}$  and  $r_{CF}^x = \left(\frac{H_R^x \sigma}{2k_R q}\right)^{\frac{2}{2q+1}} \left(1 + \left(\frac{H_S^x/k_S}{H_R^x/k_R}\right)^{\frac{1}{q-1}}\right)^{\frac{-3}{2q+1}}$ .

The most revealing observation from Lemma 2 is that the ratios  $H_R/k_R$  and  $H_S/k_S$  and their relative magnitude play a crucial role in the collaborative forecasting equilibrium. Recall that  $H$  is the unit cost of uncertainty, while  $k$  determines the cost of forecasting (a party with a low value of  $k$  is said to have a high forecasting capability since it achieves the same accuracy at a lower cost). We call the ratio  $H/k$  the “normalized cost of uncertainty.” All else being equal, a higher  $H$  or a lower  $k$  would result in a larger normalized cost of uncertainty and a larger forecasting investment.

With the linear forecasting technology ( $q = 1$ ), only one of the parties invests into forecasting if the parties’ normalized costs of uncertainty are unequal. The party making the investment is the one with the larger normalized cost of uncertainty. With the convex forecasting technology ( $q > 1$ ), both the retailer and the supplier invest into forecasting under CF as long as  $H_S^x > 0$ . The investment level for either party depends on both  $H_R^x/k_R$  and  $H_S^x/k_S$  values, and increases in his own normalized cost of uncertainty while it decreases in the other’s.

To translate this discussion into specific contract parameters, let us take RMI with a wholesale price contract and consider  $w$ . We note that there is a threshold value  $\bar{w}$  above which  $\frac{H_S^x}{k_S} > \frac{H_R^x}{k_R}$ . This follows from the fact that  $H_S^x$  increases in  $w$  while  $H_R^x$  decreases in  $w$  under RMI when  $H_S^x > 0$ . For  $H_S^x < 0$ , the supplier does not invest. For  $q = 1$ , only the supplier invests when  $w > \bar{w}$ ; otherwise only the retailer invests. For  $q > 1$ , both parties invest in equilibrium, but as  $w$  increases, the investment by the supplier increases while the investment by the retailer decreases monotonically: As the wholesale price increases, the effect of double marginalization becomes more pronounced and the retailer becomes more conservative in its order quantity. In this case, the supplier invests in equilibrium in order to reduce the uncertainty experienced by the retailer, thereby increasing the retailer’s order quantity and his own profit. Given the forecast support from the supplier, the retailer reduces his forecast investment. As such, forecast collaboration helps the supplier alleviate the negative effect of double marginalization through uncertainty reduction.

### 3.3 Conditions that Favor the Adoption of Collaborative Forecasting

The main goal of this paper is to understand the conditions that are conducive to the adoption of collaborative forecasting in practice. For this analysis, we adopt the notion of Pareto improvement and compare the profits in the NC and CF scenarios. To this end, we first compare the investment levels and the final forecast accuracies in Proposition 1 and then compare the supplier's and retailer's profits in the NC and CF scenarios under each contractual structure in Proposition 2.

**Proposition 1** *Implementing CF results in*

1. *a weakly lower investment by the retailer ( $r_{CF} \leq r_{NC}$ ) and a weakly higher investment by the supplier ( $s_{CF} \geq s_{NC}$ ) under the two types of RMI contracts.*
2. *a weakly lower investment for the supplier ( $s_{CF} \leq s_{NC}$ ) and a weakly higher investment for the retailer ( $r_{CF} \geq r_{NC}$ ) under the SMI contract.*
3. *a forecast accuracy improvement (equivalently,  $r_{CF} + s_{CF} \geq r_{NC} + s_{NC}$ ) under all three contracts.*

Under all contracts, the “newsvendor” is the only one who invests in forecasting under NC. When CF is implemented, the other party contributes to the final demand forecast if his own (equilibrium) benefit from reducing the uncertainty faced by the newsvendor dominates the cost of his forecast investment. The newsvendor exploits this increase in forecast investment by reducing his own investment level. Nevertheless, the accuracy of the final demand forecast is always higher under the CF scenario. This is because the total investment into forecasting under CF is (weakly) higher than the newsvendor's investment in the NC scenario.

Next, we identify the set of parameters that are Pareto-improving, that is, for which switching to CF is (weakly) beneficial for both supply chain partners. Let  $\mathbf{\Gamma} \doteq (c, w, p, k_S, k_R)$ . We denote the Pareto region for contractual structure  $x \in \{RMI, SMI, BB\}$  with  $\mathbf{P}_x$ . Then  $\mathbf{P}_x = \{\mathbf{\Gamma} | (\pi_{CF}^x(\mathbf{\Gamma}) \geq \pi_{NC}^x(\mathbf{\Gamma})) \cap (\Pi_{CF}^x(\mathbf{\Gamma}) \geq \Pi_{NC}^x(\mathbf{\Gamma}))\}$ . The next proposition simplifies the characterization of the Pareto-improving region.

**Proposition 2** *The retailer (supplier) is never worse off from switching to CF under the RMI contracts (the SMI contract), while the supplier (retailer) can be worse off.*

This result can be interpreted as follows: The supplier does not make any investment into forecasting in the NC scenario but invests into forecasting in the CF scenario under the RMI contracts. Therefore, despite the gain in forecast accuracy, switching to CF can result in a profit

loss for the supplier. On the other hand, the retailer reduces his own investment into forecasting and incurs a lower forecast investment cost, while the final forecast accuracy (weakly) increases. As a result, the retailer is never worse off from implementing CF under the RMI contracts. A parallel argument applies under the SMI contract, with the conclusion that the supplier always benefits from switching to CF, while the retailer can be worse off.

Proposition 2 implies that the Pareto region is defined by the parameters where the supplier benefits from switching to CF in the RMI contracts, and where the retailer benefits in the SMI contract, i.e.  $\mathbf{P}_x = \{\Gamma | \Pi_{CF}^x(\Gamma) \geq \Pi_{NC}^x(\Gamma)\}$  for  $x = RMI, BB$ , and  $\mathbf{P}_{SMI} = \{\Gamma | \pi_{CF}^{SMI}(\Gamma) \geq \pi_{NC}^{SMI}(\Gamma)\}$ . This property is exploited to derive Proposition 3, where  $R_x \doteq \frac{H_S^x/k_S}{H_R^x/k_R}$  under contract  $x$ .

**Proposition 3** *Let  $q = 1$ . Then  $\mathbf{P}_x = \{\Gamma \mid R_x \geq 27/8\}$  for  $x \in \{RMI, BB\}$  and  $\mathbf{P}_x = \{\Gamma \mid R_x \leq 8/27\}$  for  $x = SMI$ . Let  $q > 1$ . Then*

$$\mathbf{P}_x = \left\{ \Gamma \mid \left(1 + R_x^{\frac{1}{q-1}}\right)^{\frac{1-q}{2q+1}} + \frac{R_x^{\frac{1}{q-1}}}{2q} \left(1 + R_x^{\frac{1}{q-1}}\right)^{\frac{-3q}{2q+1}} \leq 1 \right\} \text{ for } x \in \{RMI, BB\} \quad (9)$$

$$\mathbf{P}_x = \left\{ \Gamma \mid \left(1 + (1/R_x)^{\frac{1}{q-1}}\right)^{\frac{1-q}{2q+1}} + \frac{(1/R_x)^{\frac{1}{q-1}}}{2q} \left(1 + (1/R_x)^{\frac{1}{q-1}}\right)^{\frac{-3q}{2q+1}} \leq 1 \right\} \text{ for } x = SMI. \quad (10)$$

Proposition 3 is our central analytical result. It establishes that the Pareto region under any contractual structure can be written as a function of solely  $R$  and  $q$ . The former construct captures the economic parameters in the form of the relative normalized cost of uncertainty of the two parties, and the latter reflects the diseconomies in the forecast investment.

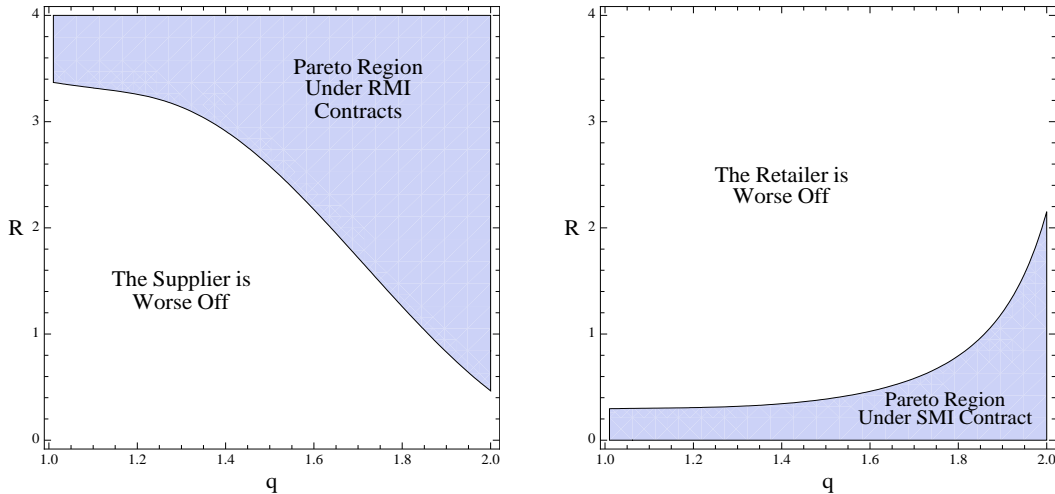


Figure 1: The Pareto regions for all contract types can be characterized as a function of  $q$  and  $R$ . Panel (a) illustrates the Pareto region for the two types of RMI contracts, while panel (b) illustrates it for the SMI contract.

Figure 1 illustrates the Pareto region for all three contractual structures. This figure reveals that collaborative forecasting is Pareto improving only in a limited region. As the retailer always weakly prefers CF, the Pareto region under the RMI contracts (Figure 1a) is the region where the supplier is better off; it occurs at high values of  $R$  (where the supplier's normalized cost of uncertainty is sufficiently high relative to the retailer's) and at high values of  $q$  (where the forecasting technology favors splitting the forecasting investment between the two parties). Similarly, the Pareto region for SMI (Figure 1b) is the region where the retailer is better off under CF; it emerges when the retailer's normalized cost of uncertainty is larger. The role of the forecasting technology is the same regardless of contractual structure and economic parameters: a higher  $q$  enhances the value of CF.

The Pareto regions plotted in Figure 1 can easily be translated to Pareto regions in any parameter (keeping all others constant). To see how, let us denote the indifference curves in Figure 1 by  $\bar{R}_x(q)$ . Let the parameter of interest be denoted by  $\gamma$ , and the indifference curve in this parameter by  $\bar{\gamma}(q)$ . Then  $\bar{\gamma}(q)$  is the value of  $\gamma$  such that  $R_x(\gamma; \Gamma \setminus \{\gamma\}) = \bar{R}_x(q)$ , i.e. the new indifference curve is constructed by finding the value of the parameter  $\gamma$  that yields  $\bar{R}_x(q)$ . This is illustrated in Figure 2, where the Pareto region is plotted in terms of  $\gamma = w$  and  $q$  (assuming all other parameters are fixed). The structure of the Pareto regions in Figure 2 parallels those in Figure 1 because  $R_{RMI}$ ,  $R_{BB}$ , and  $R_{SMI}$  increase in  $w$  in this parameter range.

We observe that CF is Pareto improving at high wholesale prices for the RMI contract. As the wholesale price increases, the retailer's order quantity decreases. Under collaborative forecasting, the supplier has the opportunity to counteract this by making a forecast investment that reduces the uncertainty experienced by the retailer, thereby increasing the retailer's order quantity and his own profit. Above a wholesale price threshold, the benefit this provides the supplier outweighs his forecast investment cost and collaborative forecasting becomes Pareto-improving. Following a similar logic, CF is Pareto improving at low wholesale prices under the SMI contract.

Although the Pareto regions for the two RMI contracts look identical in Figure 1, these regions are different in Figure 2 because for a given wholesale price  $w$ ,  $R_{RMI} \neq R_{BB}$ . As seen in the next corollary, the Pareto region under RMI with a buyback contract always includes the Pareto region under RMI with a wholesale price contract in the  $(q, w)$  space.

**Corollary 1**  $\bar{w}_{BB}(q) \leq \bar{w}_{RMI}(q)$ .

The intuition behind this result is as follows: In contrast to RMI with a wholesale price, the supplier shares the risk for unsold inventory under RMI with buyback. While the supplier is

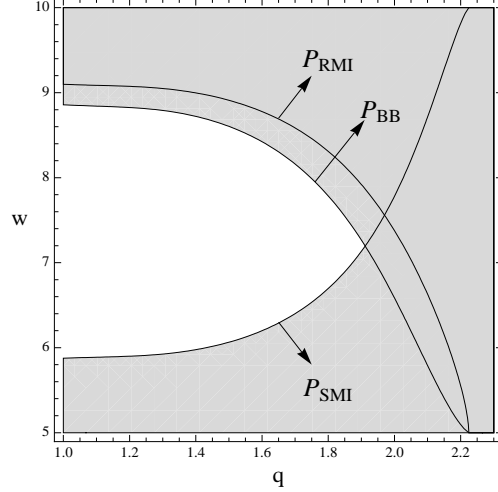


Figure 2: Pareto regions for RMI, SMI, and BB Contracts when  $c = 5$ ,  $p = 10$ ,  $k_S = k_R = 2$ .

exposed to some risk and has no way to counter it in the NC scenario with a buyback contract, he has the opportunity to do so in the CF scenario. Consequently, the Pareto region under the buyback contract is larger.

One important implication of these findings is that collaborative forecasting will not emerge in situations where the “newsvendor” appropriates a large share of the supply chain margin. For example, in RMI with a wholesale price contract, the supplier would be willing to implement collaborative forecasting only when he appropriates a larger share of the supply chain margin (i.e.,  $w$  is high). Similarly, for the SMI contract, CF would be desirable for both supply chain partners only when the retailer appropriates a large share of the margin (i.e.,  $w$  is low). Therefore, we conclude that CF will emerge where the quantity decision making and the power to appropriate a larger share of the operating profits reside at the opposite ends of the supply chain. The magnitude of benefits due to CF under each type of contract as a function of the underlying parameters, and how they differ qualitatively is explored numerically in the next section.

## 4 Numerical Results

In this section, we conduct a numerical analysis to evaluate the value of collaborative forecasting. After analyzing the drivers of CF under the three contractual structures in §4.1, in §4.2 we investigate whether the value of CF is enhanced when it is implemented in conjunction with a coordinating contract.

#### 4.1 Determinants of the Value of Collaborative Forecasting

This section examines the directional effect of the model parameters on the value of CF for the two parties. We calculate the equilibrium forecast investments and the resulting profits for the two parties under the three contract structures over an experimental grid with  $\mu \in \{100, 200\}$ ,  $\sigma \in \{25, 100\}$ ,  $\rho \in \{0, 0.3, 0.5\}$ ,  $c = 5$ ,  $p \in \{7, 13\}$ ,  $k_R, k_S \in \{3, 6\}$ ,  $q \in \{1, 1.5, 2\}$ , and  $w = c + i(p - c)/5$  for  $i = 1, 2, 3, 4$  which result in a total of 3456 parameter combinations. In order to capture the first-order effects, we run linear regressions where the profit difference between the CF and NC scenarios is the dependent variable and the model parameters are the independent variables, using an approach similar to Souza et al. (2004). Table 1 summarizes the  $t$  values and the signs of the effects for the retailer, supplier and supply chain profits for each contract type. Several observations about directional effects can be made from this table.

Contract Type	Parameters	$V_R^{CF}$		$V_S^{CF}$		$V_{SC}^{CF}$	
		$t$	Sign	$t$	Sign	$t$	Sign
RMI	$\rho$	18.6	-	4.0	-	15.7	-
	$\sigma$	11.8	+	2.7	+	10	+
	$p$	20.9	-	4.5	-	17.7	-
	$w$	39.0	+	7.5	+	32.3	+
	$k_R$	13.8	+	8.9	+	15.3	+
	$k_S$	10.7	-	8.6	-	12.9	-
	$q$	4.4	+	12.7	+	11.0	+
SMI	$\rho$	3.45	-	19.5	-	16.9	-
	$\sigma$	11.8	+	12.4	+	10.9	+
	$p$	3.9	+	34.0	+	28.2	+
	$w$	5.2	-	34.3	-	29.3	-
	$k_R$	8.7	-	12.1	-	14.7	-
	$k_S$	8.9	+	15.4	+	17.3	+
	$q$	13.7	+	6.6	+	13.6	+
BB	$\rho$	21.4	-	6.1	-	20.7	-
	$\sigma$	13.6	+	4.3	+	13.4	+
	$p$	5.8	-	11.4	-	13.0	-
	$w$	21.7	+	17.3	+	29.3	+
	$k_R$	18.8	+	8.7	+	20.6	+
	$k_S$	15.0	-	8.2	-	17.4	-
	$q$	3.6	+	11.4	+	11.3	+

Table 1: The sensitivity of the value of CF for the retailer and supplier under the RMI, SMI and BB contracts to model parameters.

**Observation 1** *The effect of  $q$ ,  $\sigma$ , and  $\rho$  is independent of the contract structure or the identity of the newsvendor.*

As  $q$  increases, the cost of gathering information by one party increases disproportionately to the gain from information. Since the two parties can achieve the same forecast accuracy at a lower cost by sharing the investment, the value of CF for both parties increases as  $q$  increases. The value of CF also increases with the standard deviation of the signal noise: Since achieving a given level of accuracy requires more forecasting investment, the benefit of sharing the investment increases in signal noise for both parties. Finally, the value of CF decreases for both parties as signal correlation increases, as the value of each additional signal shared is lower due to information redundancy.

**Observation 2** *The effect of  $p$  depends solely on the contract type.*

An increase in the retail price  $p$  decreases the value of CF for both parties under the RMI contracts, and increases it under the SMI contract. All else being equal, an increase in  $p$  makes the retailer more willing to invest into forecast improvement due to a higher margin. In RMI, the retailer already makes a forecast investment in the noncollaborative scenario, so the gain from forecast collaboration decreases in  $p$ . In SMI, it is the supplier who makes the forecast investment in the non-collaborative scenario, so the gain from forecast collaboration comes from the retailer's contribution. As  $p$  increases, the retailer increases his forecast investment, and both parties' benefit from CF increases.

**Observation 3** *The effect of  $k_R$  and  $k_S$  depends solely on each party's identity.*

The value of CF for both parties decreases in the capability of the “newsvendor,” and increases in the capability of the “non-newsvendor.” Consider either RMI contract, where the retailer is the newsvendor. If the retailer has a low forecasting capability ( $k_R$  is high), then the value of CF is high because the forecast accuracy in the absence of collaboration is low. Moreover, the value of CF is commensurate with the incremental quality of information that the supplier can bring: If he has a low forecasting capability ( $k_S$  is high), then the value of CF is lower. The reverse is true under SMI - the value of CF increases with  $k_S$  and decreases with  $k_R$ , but the logic is the same: The value of CF decreases in the capability of the party that makes the forecast investment under NC (the newsvendor), and increases in the capability of the party that contributes additional forecasting resources under CF (the non-newsvendor).

**Observation 4** *The effect of  $w$  depends on the contract type - newsvendor identity pairing, and is monotone for the non-newsvendor, but not for the newsvendor.*

The value of CF for the supplier under the RMI contracts increases with the wholesale price, while it decreases for the retailer under the SMI contract. Consider the RMI contracts. As the wholesale price increases, the retailer stocks a lower quantity due to his small share of operating profits, and invests little into forecast improvement as his gains are quite limited. Consequently, the supplier’s benefit from investing into forecasting, and sharing it with the retailer with the intention to increase the retailer’s order quantity becomes larger. The same is true for the retailer under the SMI contract: The retailer can increase the production quantity at the supplier by helping improve its forecast accuracy.

While the first-order relationship observed between  $w$  and the value of CF for the retailer is also positive under RMI, when the quadratic term  $w^2$  was included in the regression together with  $w$ , the coefficients for both  $w$  and  $w^2$  were significant, suggesting a nonmonotonic relationship. As illustrated in Figure 3, the value of CF for the retailer first increases and then decreases with the wholesale price. When the retailer captures practically the entire supply chain profit, the supplier does not invest in forecasting, hence, there is no value to be gained from CF for the retailer. At the other extreme where the supplier captures a large fraction of the operating profits, the value of CF for the retailer is again low: Since the retailer’s forecasting investment is already very small, the scope for reducing his own forecasting cost is very low. In addition, he gets negligible benefit from the increased forecast accuracy CF brings since his share of operating profits is low. Consequently, his gain from forecast collaboration is maximized at intermediate values of his share in operating profits. The same pattern is observed for the supplier under SMI. We examine the effect of  $w$  on the value of CF in more detail in Section 4.2.

## 4.2 Comparing the Value of CF under Coordinating and Non-Coordinating Contracts

In this section, we answer the following question: Is the value of CF higher under a coordinating contract (e.g. RMI with buyback) or a non-coordinating contract (e.g. RMI with wholesale pricing)? It is not immediately clear how the value of CF is moderated by the presence of coordination. On the one hand, the parties may better leverage collaborative forecasting investments when the quantity decision is coordinated. On the other hand, the opportunity for collaborative forecasting to make a more significant contribution may be higher when the quantity decision is not coordinated. In order to develop insight, a numerical study was conducted, comparing the value of CF in the RMI and BB scenarios for 1296 parameter combinations. The parameter values were

$\mu = 300$ ,  $\rho = 0$ ,  $\sigma \in \{25, 50, 100\}$ ,  $c = 5$ ,  $p \in \{7, 10, 13\}$ ,  $k_R, k_S \in \{1, 5, 10, 20\}$ ,  $q \in \{1, 1.5, 2\}$ . For consistency of comparison along the wholesale price dimension, for each parameter combination, we generated three values of the wholesale price ( $w_1, w_2, w_3$ ) that are equally spaced between the  $c$  and  $p$  values in that parameter combination. To control for supply chain power when comparing RMI with BB, the buyback contract parameters corresponding to the wholesale price contract with parameter  $w$  are set such that each party's share of the expected operating profit is the same under the two contracts. Our observations from this numerical study can be summarized as follows:

**Observation 5** *The retailer's forecast investment is higher under the non-coordinating contract. The supplier's forecast investment is higher under the non-coordinating (coordinating) contract if his share of the profits is sufficiently high (low).*

**Observation 6** *When the supplier appropriates a large share of the profits, the value of CF is higher under the non-coordinating contract for both parties. When the supplier appropriates a small share of the profits, there is little, if any, value from CF to the supplier under either contract, while the retailer benefits more from CF under the coordinating contract.*

To understand the drivers for these results, consider the main difference between RMI and BB: The former is subject to double marginalization while the latter is not. From the supplier's perspective, the value of collaborative forecasting under BB derives from decreasing the newsvendor loss of the supply chain, and it increases in his share of the expected operating profit. The value of CF for the supplier under RMI derives from countering double marginalization, which is increasingly detrimental at high values of  $w$  (where the retailer decreases his ordering quantity). Consequently, the supplier's forecast investment increases rapidly in  $w$  under RMI and eventually surpasses his forecast investment under BB. At high values of  $w$ , the value of CF for the supplier is higher under RMI as well since this contract is highly inefficient in this range. These phenomena are illustrated in Figure 3 for a representative parameter combination.

Next, consider the value of CF for the retailer. While the supplier makes little or no investment into forecasting at low values of  $w$  under RMI, he has an additional incentive to improve forecast accuracy under buyback contracts due to shared newsvendor costs. Hence, at low values of  $w$ , the value of CF to the retailer is higher under the BB contract relative to RMI. At high values of  $w$ , the supplier makes a larger forecasting investment in order to counter the strong double marginalization effect, which has the effect of reducing the retailer's forecasting costs and increasing his revenues. Therefore, when  $w$  is high, the value of CF for the retailer is higher under the RMI contract.

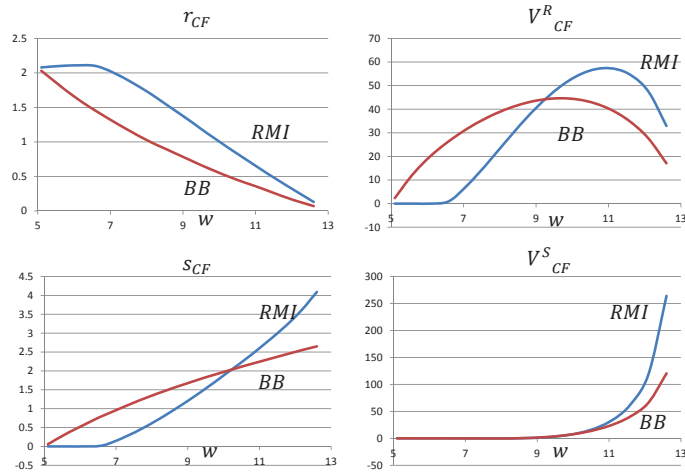


Figure 3: Representative forecasting investments and value from forecast collaboration for the parties under RMI and BB as a function of  $w$ .

Our findings underline that under the non-coordinating contract, improved information due to CF has the added benefit of countering the adverse effects of double marginalization in addition to reducing the cost of supply-demand mismatch. Hence, when the inefficiency arising from double marginalization is high, collaborative forecasting can be highly effective in countering it and delivering value for both parties.

## 5 Conclusions

Motivated by the mixed evidence concerning the adoption level and value of CPFR implementations in retail supply chains, this paper explores the conditions under which collaborative forecasting (CF) offers the highest potential. The key features of our model are (1) the endogenous nature of the forecasting accuracy of the supply chain parties; (2) the presence of strategic interaction in forecasting investments; and (3) the inclusion of different contractual structures that are widely used in retail supply chains. We derive closed-form expressions for the equilibrium forecast investments and profits for each contractual structure in both the non-collaborative and collaborative forecasting scenarios. Moreover, we succinctly express whether collaborative forecasting is Pareto improving as a function of two factors: the parties' relative normalized cost of uncertainty and the diseconomies in the forecast investment cost. Finally, to determine the specific value of collaborative forecasting as a function of different factors, we carry out a numerical analysis based on the closed-form profit expressions we derive. Below, we highlight some of our findings and their implications.

**What settings are conducive to the adoption of forecast collaboration?** There is no one industry where CF should or should not be implemented. Indeed, our analysis shows how product- and relationship-specific the adoption potential of CF is: Pareto improvement requires either that the non-newsvendor's relative normalized cost of uncertainty is sufficiently large relative to the newsvendor's, or that the diseconomies in the forecast investment cost is sufficiently large. Some implications of the first condition are the following: Regardless of the contractual structure, the adoption of collaborative forecasting requires the newsvendor to neither appropriate a large share of the operational profits nor have a high forecasting capability. All else being equal, CF has a larger adoption potential under the buyback contract than the wholesale price contract.

**In what settings are collaborative forecasting initiatives that go forward expected to generate the most value?** We find that some factors' effects are relationship-specific while others hold more generally. For instance, the contractual structure is the main determinant of the impact of retail price: An increase in the retail price decreases the value of CF for both parties under the RMI contracts, and increases it under the SMI contract. The identity of the newsvendor is the main determinant of the impact of forecast capability and is symmetric across contracts: The value of CF always decreases in the forecasting capability of the newsvendor, while it increases in the capability of the other party. In these cases, it is not the industry characteristic per se, but how the specific relationship is structured that determines the value of CF. In contrast, some factors hold promise in any supplier-retailer relationship: The value of CF always increases for both parties when the information available to the supplier and the retailer becomes more distinct, when uncertainty increases, and when there are larger diseconomies in forecast investment.

**What is the impact of power on the value obtained from collaborative forecasting?** We use the relative share of operating profits appropriated by each party as a proxy for supply chain power. For a given party, while one may expect the value of collaborative forecasting to increase with his power, we find that this does not hold in general: The value of CF for the newsvendor is non-monotonic in his power. For example, consider the retailer's gain from collaborative forecasting under the RMI contract with buyback. If the retailer is powerful and appropriates a large share of the operating profit, the supplier's forecast investment is low, limiting the value of collaborative forecasting for both parties. As the supplier's share of the operating profit increases, his investment level increases; the retailer's benefit from CF initially does increase but starts decreasing when the supplier is too powerful.

**Does supply chain coordination make collaborative forecasting more effective?** One may

expect CF to be more valuable under coordinating contracts, rather than a simple wholesale price contract that is prone to double marginalization. Indeed, the *adoption* of collaborative forecasting is always enhanced by quantity coordination (the Pareto region for RMI with a buyback contract subsumes that for RMI with a wholesale price contract). However, the *magnitude of the gain* from collaborative forecasting is in many cases higher in the absence of quantity coordination: When the supplier appropriates a larger share of the operating profit, he gains more from collaborative forecasting under the wholesale price contract. Since a larger forecasting investment by the supplier translates into a higher positive spillover effect on the retailer (in the form of higher accuracy achieved at lower cost), the retailer’s gain from CF is higher under the non-coordinating contract as well. This finding reveals that the improved information due to CF has the added benefit of countering the adverse effects of double marginalization inherent in the wholesale price contract, in addition to reducing the cost of the supply-demand mismatch.

We close with a discussion of the robustness of our results to some of our assumptions. For ease of exposition, we assumed that the forecasting technology parameter  $q$  is identical for both parties. This assumption can be relaxed to allow either party to have a more efficient forecasting technology (that is, a lower  $q$ ). All else being equal, as the efficiency of the non-newsvendor’s forecast technology increases, his forecast accuracy, and consequently, his gain from CF increases. Given the non-newsvendor’s higher contribution to the joint forecast, CF becomes more valuable for the newsvendor as well. As a result, the Pareto region expands. Conversely, as the newsvendor’s forecasting technology becomes more efficient, his forecast accuracy under the NC scenario increases, and the gain to be obtained from CF decreases. Consequently, the Pareto region shrinks.

We assumed that the forecast investment cost is (weakly) convex ( $q \geq 1$ ), but there could be economies of scale in forecasting ( $q < 1$ ). When the analysis of the NC and CF scenarios is extended to this case, the Pareto region continues to shrink as  $q$  decreases. The primary difference between the CF equilibria in the Pareto improving region is that for  $q > 1$ , both parties invest, while for  $q \leq 1$ , only the non-newsvendor invests. In both regions, as  $q$  decreases, the forecast investment level under NC increases, which reduces the gain from adopting CF. There is a second driver when  $q > 1$ : Collaborative forecasting allows the parties to achieve the same accuracy level at a lower total forecast investment, and as  $q$  decreases, this benefit decreases. Consequently, as  $q$  decreases, the Pareto region shrinks over the whole range of  $q$  values.

In Section 3, the Pareto region was characterized for the full range of wholesale prices, capturing different bargaining power distributions in the supply chain. We can further pose the following

question: Under the terms of trade set by a powerful retailer or supplier so as to maximize his own profits under the non-collaborative forecasting scenario, would collaborative forecasting emerge? As discussed at the end of Section 3, when the newsvendor is the powerful party and dictates the contract terms, collaborative forecasting is not expected to emerge. In order to investigate the same question when the non-newsvendor is the powerful party, a numerical study was conducted, paralleling the study in Section 4 (see Appendix C for details). This study shows that the optimal wholesale price set by the supplier in the RMI scenario falls into the Pareto improving range for 75.6% of the parameter combinations. Furthermore, the factors that favor the emergence of CF are found to be a higher retail price, a more efficient forecasting technology, a higher forecasting capability at the supplier, a lower forecasting capability at the retailer, and less signal noise. Since the SMI contract is symmetric, parallel results hold under the SMI contract when the wholesale price is set by the retailer.

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## Appendix A: Derivation of Profits

**Non-collaborative Forecasting.** Let us consider the RMI contract. After making its forecasting investment  $r$  and observing signals  $\Omega_r$ , the retailer sets its order quantity to maximize its expected profit by solving  $\max_Q \mathbb{E}_{D|\Omega_r}[p \min\{Q, D\} - wQ]$ . The retailer’s profit can be rewritten as

$\mathbb{E}_{D|\Omega_r}[pQ - p(Q - D)^+ - wQ]$ . The first-order condition is given by  $P(Q > D|\Omega_r) = 1 - \frac{w}{p}$ . Solving the first-order condition, the retailer's optimal order quantity is given by  $Q_{NC}^{RMI}(\Omega_r) = \mu_r(\Omega_r) + z_R\sigma_r$  where  $\mu_r(\Omega_r) = \frac{\sum_{i=1}^r \psi_i}{r}$  is the mean and  $\sigma_r^2 = \frac{\sigma^2}{r}$  is the variance of the updated demand forecast and  $z_R = \Phi^{-1}(1 - w/p)$ .

Substituting the optimal order quantity back into the retailer's profit function, the retailer who draws  $r$  signals and observes the vector  $\Omega_r$  expects to make the following operating profit:  $\mathbb{E}_{D|\Omega_r}[p \min\{Q_{NC}^{RMI}(\Omega_r, r), D\} - wQ_{NC}^{RMI}(\Omega_r, r)]$  which simplifies to  $(p-w)\mu_r(\Omega_r) - p\phi(z_R)\sigma_r$ . Taking expectation over  $\Omega_r$ , the retailer who is to draw  $r$  signals calculates its expected profit to be  $\Pi_{NC}^{RMI}(r) = \mathbb{E}_{\Omega_r}[(p-w)\mu_r(\Omega_r) - p\phi(z_R)\sigma_r] - k_R r^q = (p-w)\mu - H_R^{RMI}\sigma_r - k_R r^q$  where  $H_R^{RMI} \doteq p\phi(z_R)$  is the cost of uncertainty per unit of standard deviation for the retailer under RMI contract.

The supplier's profit given that the retailer orders  $Q_{NC}^{RMI}$  is given by  $(w-c)Q_{NC}^{RMI} = (w-c)\mu_r(\Omega_r) + (w-c)z_R\sigma_r$ . The supplier's expected profit if the retailer and the supplier are to draw  $r$  and  $s$  signals, respectively, is given by  $\pi_{NC}^{RMI}(r, s) = \mathbb{E}_{\Omega_r}[(w-c)(\mu_r(\Omega_r) + z_R\sigma_r)] - k_S s^q = (w-c)\mu - H_S^{RMI}\sigma_r - k_S s^q$  where  $H_S^{RMI} = -(w-c)z_R$ .

The analysis of the SMI and BB contracts parallels the above, and is omitted for brevity.

**Collaborative Forecasting.** The derivation of profits in the collaborative forecasting scenario is similar to the non-collaborative scenario with the only difference being that in the collaborative forecasting scenario the decision making party decides on the order/production quantity based on the combined information set  $\Omega_j \doteq \Omega_r \cup \Omega_s$ .

## Appendix B: Proofs

### Proof of Lemma 1: Characterizing the Investment Levels in NC

Both the supplier and the retailer select forecasting investments to maximize expected profits, that is, they solve  $\max_s \pi_{NC}(s, r)$  and  $\max_r \Pi_{NC}(r)$ , respectively. While in reality the parameters  $s$  and  $r$  are integers, we treat the supplier's and retailer's profit functions as being continuous in these decision variables. In the RMI and BB contracts, it is easy to see  $\frac{\partial \pi_{NC}}{\partial s} = -qk_S s^{q-1}$  and  $\frac{\partial \Pi_{NC}}{\partial r} = \frac{\sigma H_R}{2r\sqrt{r}} - qk_R r^{q-1}$ . The supplier makes no investment as its profit strictly decreases in its forecast investment, hence  $s_{NC} = 0$ . Solving the first-order condition for the retailer yields  $r_{NC} = \left(\frac{H_R\sigma}{2qk_R}\right)^{2/(2q+1)}$ . The second derivative of the retailer's profit is strictly negative for  $r > 0$  ensuring uniqueness. A similar analysis finds  $s_{NC} = \left(\frac{H_S\sigma}{2qk_S}\right)^{2/(2q+1)}$  and  $r_{NC} = 0$  under SMI. ■

### Proof of Lemma 2 : Characterizing the Investment Levels in CF

#### Existence and Uniqueness of the Equilibrium:

Both the supplier's and the retailer's expected profits are bounded, but the forecasting costs are unbounded. Thus, for large values of  $s$  and  $r$ , the supplier's and the retailer's expected profit functions can become negative. We define the upper bounds for the retailer's and the supplier's investment levels as  $\bar{r} = \left(\frac{(p-c)\mu}{k_R}\right)^{1/q}$  and  $\bar{s} = \left(\frac{(w-c)\mu}{k_S}\right)^{1/q}$  and limit the action space to  $[0, \bar{r}] \times [0, \bar{s}]$ .

From Theorem 1.2 in Fudenberg and Tirole (1991), a pure strategy Nash equilibrium exists if (1) each player's strategy space is a nonempty, compact convex subset of a Euclidean space, and (2) the supplier's and retailer's profit functions are both continuous in  $s$  and  $r$  and respectively quasiconcave in  $r$  and  $s$ . Condition (1) is satisfied since we restrict the strategy space to  $[0, \bar{r}] \times [0, \bar{s}]$ . Therefore, if the supplier's and retailer's expected profits are quasiconcave, then a pure-strategy Nash equilibrium exists. We have  $\frac{\partial^2 \pi_{CF}}{\partial s^2} = -\frac{3\sigma H_S}{4(s+r)^{5/2}} - q(q-1)k_S s^{q-2} \leq 0$  and  $\frac{\partial^2 \Pi_{CF}}{\partial r^2} = -\frac{3\sigma H_R}{4(s+r)^{5/2}} - q(q-1)k_R r^{q-2} < 0$  for  $H_R > 0$  and  $H_S \geq 0$ . Therefore, we conclude that there exists an equilibrium in the forecasting investments under all three contracts. The uniqueness of the equilibrium follows from the cross derivatives being negative (i.e., both best response functions are decreasing), when  $H_R \geq 0$  and  $H_S \geq 0$ , that is  $\frac{\partial^2 \pi_{CF}}{\partial s \partial r} = -\frac{3\sigma H_S}{4(s+r)^{5/2}} \leq 0$  and  $\frac{\partial^2 \Pi_{CF}}{\partial r \partial s} = -\frac{3\sigma H_R}{4(s+r)^{5/2}} \leq 0$ .

Next, we characterize the equilibrium investment levels under all three contracts. In doing so, we distinguish between the case where the forecasting technology is linear (i.e.,  $q = 1$ ) and the case where the forecasting technology is convex (i.e.,  $q > 1$ ).

### Characterization of the Investments with Linear Forecasting Technology $q = 1$

When  $q = 1$ ,  $\frac{\partial \pi_{CF}}{\partial s} = \frac{\sigma H_S}{2(s+r)^{3/2}} - k_S$  and  $\frac{\partial \Pi_{CF}}{\partial r} = \frac{\sigma H_R}{2(s+r)^{3/2}} - k_R$ . Solving the first order conditions yields the following best response functions

$$r(s) = \left(\frac{\sigma H_R}{2k_R}\right)^{2/3} - s \quad s(r) = \left(\frac{\sigma H_S}{2k_S}\right)^{2/3} - r. \quad (11)$$

Note that both best responses are linear in the opponent's strategy. Therefore, the equilibrium is characterized as follows: If  $\frac{H_R}{k_R} > \frac{H_S}{k_S}$ , then only the retailer invests into forecasting and the equilibrium investment levels are given by  $r_{CF} = \left(\frac{\sigma H_R}{2k_R}\right)^{2/3}$  and  $s_{CF} = 0$ . If, on the other hand,  $\frac{H_S}{k_S} > \frac{H_R}{k_R}$ , then the equilibrium investments are given by  $r_{CF} = 0$  and  $s_{CF} = \left(\frac{H_S \sigma}{2k_S}\right)^{2/3}$ . If  $\frac{H_R}{k_R} = \frac{H_S}{k_S}$ , then there exist multiple equilibria such that  $s + r = \frac{H_R}{k_R} = \frac{H_S}{k_S}$ .

### Characterization of the Investments with Convex Forecasting Technology $q > 1$

In this case,  $\frac{\partial \pi_{CF}}{\partial s} = \frac{\sigma H_S}{2(s+r)^{3/2}} - qk_S s^{q-1}$  and  $\frac{\partial \Pi_{CF}}{\partial r} = \frac{\sigma H_R}{2(s+r)^{3/2}} - qk_R r^{q-1}$ . Solving the first-order

conditions simultaneously, we obtain the equilibrium investments for  $q > 1$  as follows:

$$r_{CF} = \left( \frac{H_R \sigma}{2k_R q} \right)^{\frac{2}{2q+1}} \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{-3}{2q+1}} \quad \text{and} \quad s_{CF} = \left( \frac{H_S \sigma}{2k_S q} \right)^{\frac{2}{2q+1}} \left( 1 + \left( \frac{H_R k_S}{H_S k_R} \right)^{\frac{1}{q-1}} \right)^{\frac{-3}{2q+1}}$$

■

### Proof of Proposition 1: Comparing the Investment Levels

Comparing the supplier's and retailer's investment levels in Lemma 1 and 2 under each contract for  $q = 1$  and  $q > 1$  reveals the following relations:

#### Retailer Managed Inventory with Wholesale Price (RMI)

For  $q = 1$ ,  $s_{NC} \leq s_{CF}$  and  $r_{NC} \geq r_{CF}$ . For  $q > 1$ ,  $s_{NC} < s_{CF}$  and  $r_{NC} > r_{CF}$ .

#### Supplier Managed Inventory with Wholesale Price (SMI)

For  $q = 1$ ,  $s_{NC} \geq s_{CF}$  and  $r_{NC} \leq r_{CF}$ . For  $q > 1$ ,  $s_{NC} > s_{CF}$  and  $r_{NC} < r_{CF}$ .

#### Retailer Managed Inventory with Buyback (BB)

For  $q = 1$ ,  $s_{NC} \leq s_{CF}$  and  $r_{NC} \geq r_{CF}$ . For  $q > 1$ ,  $s_{NC} < s_{CF}$  and  $r_{NC} > r_{CF}$ .

Combining the cases for  $q = 1$  and  $q > 1$  reveals that while the supplier increases his investment ( $s_{CF} \geq s_{NC}$ ), the retailer reduces his investment ( $r_{NC} \geq r_{CF}$ ) in the CF scenario under RMI and BB contracts. On the other hand, the supplier reduces its investment ( $s_{NC} \geq s_{CF}$ ), and the retailer increases its investment ( $r_{CF} \geq r_{NC}$ ) in the CF scenario under the SMI contract.

Forecast accuracy in each scenario is defined as  $1/\sigma^2$ . The forecast accuracy in the NC scenario for the RMI contracts is given by:  $\frac{r_{NC}}{\sigma^2} = \frac{1}{\sigma^2} \left( \frac{H_R \sigma}{2k_R q} \right)^{\frac{2}{2q+1}}$ . The final accuracy of the joint forecast under CF depends on the investment levels of both parties. We consider the following cases:

- (1)  $q = 1$  and  $\frac{H_R}{k_R} > \frac{H_S}{k_S}$  (i.e., only the retailer invests). The forecast accuracy of the joint demand forecast in this case is given by  $\frac{r_{CF} + s_{CF}}{\sigma^2} = \frac{1}{\sigma^2} \left( \frac{H_R \sigma}{2k_R q} \right)^{\frac{2}{2q+1}} = \frac{r_{NC}}{\sigma^2}$ .
- (2)  $q = 1$  and  $\frac{H_R}{k_R} < \frac{H_S}{k_S}$  (i.e., only the supplier invests), the accuracy of the joint demand forecast is given by  $\frac{r_{CF} + s_{CF}}{\sigma^2} = \frac{1}{\sigma^2} \left( \frac{H_S \sigma}{2k_S q} \right)^{\frac{2}{2q+1}} > \frac{r_{NC}}{\sigma^2}$  because  $\frac{H_R}{k_R} < \frac{H_S}{k_S}$ .
- (3)  $q > 1$ . The accuracy of the joint demand forecast is given by

$$\frac{r_{CF} + s_{CF}}{\sigma^2} = \frac{1}{\sigma^2} \left[ \left( \frac{H_R \sigma}{2k_R q} \right)^{\frac{2}{2q+1}} \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{-3}{2q+1}} + \left( \frac{H_S \sigma}{2k_S q} \right)^{\frac{2}{2q+1}} \left( 1 + \left( \frac{H_R k_S}{H_S k_R} \right)^{\frac{1}{q-1}} \right)^{\frac{-3}{2q+1}} \right]$$

and simplifies to  $\frac{1}{\sigma^2} \left( \frac{H_R \sigma}{2k_R q} \right)^{\frac{2}{2q+1}} \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{2q-2}{2q+1}}$  which is greater than  $\frac{r_{NC}}{\sigma^2}$ . Therefore, we can conclude that the forecast accuracy in CF is always better than the forecast accuracy under

NC under RMI contracts. The proof is similar for the forecast accuracy in the SMI contract. ■

**Proof of Proposition 2: Comparing the Profits in NC and CF**

**Case 1: RMI and BB with Linear Forecasting Technology ( $q = 1$ )**

The supplier is better off under CF if  $\pi_{CF}(\mathbf{\Gamma}) > \pi_{NC}(\mathbf{\Gamma})$  which is  $-\frac{H_S\sigma}{\sqrt{r_{CF}+s_{CF}}} - k_S s_{CF} > -\frac{H_S\sigma}{\sqrt{r_{NC}}}$ .

If  $\frac{H_S}{k_S} < \frac{H_R}{k_R}$  so that only the retailer invests in forecasting in CF, the supplier is indifferent between NC and CF because  $r_{CF} = s_{CF}$  and  $s_{CF} = 0$ . If, on the other hand,  $\frac{H_S}{k_S} > \frac{H_R}{k_R}$  so that only the supplier invests in CF, the above inequality can be rewritten as  $\frac{H_S\sigma}{\left(\frac{H_S\sigma}{2k_S}\right)^{1/3}} + k_S \left(\frac{H_S\sigma}{2k_S}\right)^{2/3} < \frac{H_S\sigma}{\left(\frac{H_R\sigma}{2k_R}\right)^{1/3}}$  and simplifies to  $\frac{H_S k_R}{H_R k_S} > \frac{27}{8}$  and is not satisfied when  $\frac{H_S k_R}{H_R k_S} \in (1, 27/8)$ . Therefore, we conclude that the supplier could be worse off under CF for some parameter combinations under RMI contracts.

The retailer is better off under CF if  $\Pi_{CF}(\mathbf{\Gamma}) > \Pi_{NC}(\mathbf{\Gamma})$  which is  $-\frac{H_R\sigma}{\sqrt{r_{CF}+s_{CF}}} - k_R r_{CF} > -\frac{H_R\sigma}{\sqrt{r_{NC}}} - k_R r_{NC}$ . If  $\frac{H_S}{k_S} < \frac{H_R}{k_R}$  so that only the retailer invests in CF, the retailer makes the same investment in the NC and CF scenarios and the supplier does not invest in either scenario. Therefore, the retailer is indifferent between NC and CF scenarios. If, on the other hand,  $\frac{H_S}{k_S} > \frac{H_R}{k_R}$  so that only the supplier invests in CF, the above condition can be written as  $-\frac{H_R\sigma}{\left(\frac{H_S\sigma}{2k_S}\right)^{1/3}} > -\frac{H_R\sigma}{\left(\frac{H_R\sigma}{2k_R}\right)^{1/3}} - k_R \left(\frac{H_R\sigma}{2k_R}\right)^{2/3}$  which is equivalent to  $\frac{H_S k_R}{H_R k_S} > \frac{8}{27}$  and is always satisfied when  $\frac{H_S}{k_S} > \frac{H_R}{k_R}$ . Therefore, we conclude that the retailer is never worse off implementing CF under the RMI and BB contracts.

**Case 2: RMI and BB with Convex Forecasting Technology ( $q > 1$ )**

The supplier is better off under CF if  $\pi_{CF}(\mathbf{\Gamma}) > \pi_{NC}(\mathbf{\Gamma})$  which is satisfied if

$$(w - c)\mu - k_S \left[ 2q \left( 1 + \left( \frac{H_R k_S}{H_S k_R} \right)^{\frac{1}{q-1}} \right) + 1 \right] s_{CF}^q > (w - c)\mu - \frac{H_S\sigma}{\sqrt{r_{NC}}} \quad (12)$$

which simplifies to  $\left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{1-q}{2q+1}} + \frac{\left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}}}{2q} \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{-3q}{2q+1}} < 1$ . This inequality holds for some parameters but is not always satisfied, which implies that the supplier is not always better off implementing CF under the RMI contracts.

The retailer is better off under CF if  $\Pi_{CF}(\mathbf{\Gamma}) > \Pi_{NC}(\mathbf{\Gamma})$  which is satisfied if

$$(p - w)\mu - k_R \left[ 2q \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right) + 1 \right] r_{CF}^q > (p - w)\mu - k_R (2q + 1) r_{NC}^q \quad (13)$$

The above simplifies to  $2q \left[ \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{1-q}{2q+1}} - 1 \right] < 1 - \left( 1 + \left( \frac{H_S k_R}{H_R k_S} \right)^{\frac{1}{q-1}} \right)^{\frac{-3q}{2q+1}}$  and always holds since the term on the right hand side is negative and the term on the left hand side is positive.

**Supplier Managed Inventory (SMI).** This case (omitted) closely parallels the RMI analysis.

■

### Proof of Proposition 3: Pareto Regions

The proof of Proposition 3 follows from the proof of Proposition 2. The Pareto region under each contract  $x \in \{RMI, SMI, BB\}$  is defined as  $\mathbf{P}_x = \{\Gamma | (\Pi_{CF}^x(\Gamma) \geq \Pi_{NC}^x(\Gamma)) \cap (\pi_{CF}^x(\Gamma) \geq \pi_{NC}^x(\Gamma))\}$ . Our result in Proposition 2 states that the retailer is never worse off implementing CF under the RMI and BB contracts. That is  $\Pi_{CF}^x(\Gamma) \geq \Pi_{NC}^x(\Gamma)$  always holds for  $x \in \{RMI, BB\}$ . Therefore, the Pareto region under these two contracts is characterized by the set of parameters under which the supplier is no worse off implementing CF. Therefore, the Pareto region under RMI and BB is characterized as  $\mathbf{P}_x = \{\Gamma | R_x > 27/8\}$  when  $q = 1$  and

$$\mathbf{P}_x = \left\{ \Gamma \mid \left(1 + R^{\frac{1}{q-1}}\right)^{\frac{1-q}{2q+1}} + \frac{R^{\frac{1}{q-1}}}{2q} \left(1 + R^{\frac{1}{q-1}}\right)^{\frac{-3q}{2q+1}} < 1 \right\} \text{ when } q > 1. \quad (14)$$

Under the SMI contract, the supplier is never worse off implementing CF (i.e.,  $\pi_{CF}^x(\Gamma) \geq \pi_{NC}^x(\Gamma)$  always holds) but the retailer could be worse off implementing CF. Therefore, the Pareto region is characterized as the set of parameters for which the retailer is no worse off from implementing CF. The Pareto region under the SMI is characterized as  $\mathbf{P}_x = \{\Gamma | R_x < 8/27\}$  when  $q = 1$  and

$$\mathbf{P}_x = \left\{ \Gamma \mid \left(1 + R^{\frac{-1}{q-1}}\right)^{\frac{1-q}{2q+1}} + \frac{R^{\frac{-1}{q-1}}}{2q} \left(1 + R^{\frac{-1}{q-1}}\right)^{\frac{-3q}{2q+1}} < 1 \right\} \text{ when } q > 1. \quad (15)$$

■

### Proof of Corollary 1: $\mathbf{P}_{RMI}$ versus $\mathbf{P}_{BB}$

Let  $\bar{R}_{BB}$  and  $\bar{R}_{RMI}$  be the indifference curves for the supplier in the BB and RMI scenarios. Let  $\bar{w}_{BB}(q)$  and  $\bar{w}_{RMI}(q)$  be the supplier's indifference curves with respect to the parameter  $w$ . Then  $\bar{w}_{BB}(q)$  and  $\bar{w}_{RMI}(q)$  are given by the solution to  $R_{RMI}(w) = \bar{R}_{RMI}(q)$  and  $R_{BB}(w) = \bar{R}_{BB}(q)$ , respectively. Therefore  $\bar{w}_{BB}(q) < \bar{w}_{RMI}(q)$  implies that the Pareto region under the RMI contract is a subset of the Pareto region under the coordinating buyback contract. Since both  $R_{BB}$  and  $R_{RMI}$  are increasing in  $w$ , the above condition is satisfied if  $R_{BB}(w) > R_{RMI}(w)$  holds. Next, we show that this condition is satisfied for each  $w$ :  $R_{BB}(w) > R_{RMI}(w)$  is equivalent to  $\frac{H_S^{BB}}{H_R^{BB}} > \frac{H_S^{RMI}}{H_R^{RMI}}$  which is equivalent to  $\frac{\lambda p \phi(z_{Rb})}{(1-\lambda)p\phi(z_{Rb})} > \frac{-(w-c)z_R}{p\phi(z_R)}$ . This inequality is always satisfied for  $z_R > 0$ . We need to show that it also holds for  $z_R < 0$ . Substituting  $\lambda = \frac{w-c}{p-c}$ , we get  $\frac{w-c}{p-w} > \frac{-(w-c)z_R}{p\phi(z_R)}$ . Simplifying, we get  $\frac{1}{1-w/p} > \frac{-z_R}{\phi(z_R)}$  Using the optimality condition for the newsvendor in the RMI scenario which

is  $1 - \frac{w}{p} = \Phi(z_R)$ , our condition can be rewritten as  $\frac{1}{\Phi(z_R)} > \frac{-z_R}{\phi(z_R)}$ . Using  $\phi(z) = \phi(-z)$  and  $\Phi(z) = 1 - \Phi(-z)$  for standard normal distribution, we get  $\frac{1}{1 - \Phi(-z_R)} > \frac{-z_R}{\phi(-z_R)}$  which can be further rewritten as  $\phi(-z_R) - z_R\Phi(-z_R) + z_R > 0$  which always holds for  $z_R < 0$  and follows from p.175 in Tong (1990). Therefore, we conclude that the Pareto region under the RMI contract is a subset of the Pareto region under the BB contract. ■

## Appendix C: Numerical Study

To answer the question “Does the optimal wholesale price (under no CF) fall within the range of Pareto optimality of CF?” and to identify the parameters for which this is more likely to be the case, we conducted a numerical study ( $\mu = 200$ ,  $\sigma \in \{25, 50, 75, 100\}$ ,  $p \in \{6, 8, 10, 12, 14\}$ ,  $c = 5$ ,  $k_R \in \{3, 6, 9\}$ ,  $k_S \in \{3, 6, 9\}$ ,  $q \in \{1, 1.5, 2\}$ ) under the RMI contract, with the wholesale price set by the supplier (the non-newsvendor). For each parameter combination, we numerically identified the optimal wholesale price that would be set by a powerful supplier under NC, and checked whether this wholesale price fell in the Pareto region where CF would emerge (defining  $I(w^*) = 1$  if  $w^* \geq w_B$  and  $I(w^*) = 0$  otherwise). We found that the optimal wholesale price fell in the Pareto range for 75.6% of the parameter combinations.

In order to examine the environments in which  $w^*$  would fall in the Pareto region, we ran a logistic regression with  $I(w^*)$  as the dependent variable and the model parameters as the independent variables. All coefficients were significant (see Table 2 below), where the regression coefficients are interpreted as increasing/decreasing the likelihood of  $I(w^*) = 1$ .

	Coefficient	Standard Error	p-value
$\sigma$	-0.082	0.010	0.000
$p$	0.511	0.079	0.000
$k_R$	0.700	0.097	0.000
$k_S$	-1.065	0.124	0.000
$q$	6.145	0.725	0.000

Table 2: Results of the logistic regression

The results of the logistic regression can be interpreted as follows:

First, the likelihood of  $w^*$  falling in the Pareto region ( $w^* \geq w_B$ ) decreases as  $\sigma$  increases. This is intuitive: The Pareto region frontier  $w_B$  is independent of  $\sigma$ . All else being equal, a higher demand uncertainty results in the supplier setting a lower wholesale price  $w^*$ . Consequently, the likelihood of  $w^* \geq w_B$  holding decreases.

Second, the likelihood of  $w^*$  falling in the Pareto region decreases as  $k_S$  increases. A higher  $k_S$

results in a lower  $H_S/k_S$  ratio, and hence a lower  $R$ . As seen in Figure 1, the Pareto region shrinks as  $R$  decreases (a larger  $q$  is needed for CF to be Pareto optimal), which explains this result. The opposite is true for  $k_R$ : As  $k_R$  increases,  $R$  increases, and CF is Pareto optimal in a larger region.

Third, the likelihood of  $w^*$  falling in the Pareto region increases as  $p$  increases. All else being equal, increasing  $p$  reduces  $R$ . However, when  $p$  increases, the supplier's optimal  $w^*$  increases. Table 1 tells us that the second effect is stronger.

Finally, the likelihood of  $w^*$  falling in the Pareto region increases as  $q$  increases. This is consistent with Figure 1 where the region where CF is Pareto improving expands as  $q$  increases.