# How Collection Cost Structure Drives the Manufacturer's Reverse Channel Choice

This paper discusses the impact of collection cost structure on the optimal reverse channel choice in decentralized uncoordinated supply chains. Using collection cost functions that capture collection rate and collection volume dependency, we show that the optimal reverse channel choice is driven by how the cost structure moderates the manufacturer's ability to shape the retailer's sales and collection quantity decisions.

## 1. Introduction

The focus of this paper is the optimal reverse channel choice of manufacturers such as Apple, HP, and Kodak that sell remanufactured versions of their products: whether to implement manufacturer, retailer or third party collection of returns. Savaşkan et al. (2004) develop a model of an uncoordinated decentralized supply chain and conclude that retailer collection is optimal for the manufacturer. They identify the value of the proximity of the retailer to the end-market in avoiding the inefficiency caused by double marginalization as the primary reason for this phenomenon: "The closer an agent is to the market, the more efficient is the collection of used products..." (p.246) "[...] by being closer to the final demand, the retailer can efficiently reflect unit cost savings from remanufacturing to the final price of the product, and jointly optimize the investment in used-product collection. The manufacturer is at a disadvantage in coordinating pricing and used-product return rates, because he faces double marginalization in the forward channel..." (p.240).

In reaching this conclusion, Savaşkan et al. (2004) focus on the investment cost required to achieve a given collection rate. In practice, the operational cost of the collection activity is also an important factor. In this paper, we introduce a generic cost function to capture both types of costs. Our contribution is to show that proximity to the market is only half of the story: It is in fact the structure of the collection cost, and in particular, whether it indeed allows the manufacturer to effectively utilize the retailer's proximity to the market, that determines the value of retailer collection. If the collection cost structure exhibits a "scale effect" strong enough for the manufacturer to profitably incite the retailer to increase its sales volume when it is the collecting agent, retailer collection is preferred by the manufacturer. Otherwise, the manufacturer prefers to collect himself.

Many firms doing reverse logistics are (rightly) focused on reducing collection cost to increase profitability. However, they typically do not analyze how collection cost structure should drive their collection strategy. Revealing the importance of such analysis is the practical significance of our paper.

## 2. Modeling Assumptions

To focus on the effect of cost structure on the collection channel choice, we use the same model as in Savaşkan et al. (2004), but use a more general cost structure. We briefly summarize their model and refer the reader to Savaşkan et al. (2004) for a detailed discussion of the model assumptions.

**The Forward Channel.** Undifferentiated new and remanufactured products are sold through the same retailer in a decentralized uncoordinated two-echelon (manufacturer and retailer) supply chain. The archetypal example for undifferentiated products is the Kodak single-use camera where the customer knows that the company utilizes used parts in the production of some cameras, but does not know whether a specific product contains used parts or not. The manufacturer is the Stackelberg leader and offers a wholesale price contract to the retailer. Since the products are undifferentiated, the manufacturer sells both new and remanufactured products to the retailer at the same wholesale price $w$, who in turn sells both products at the same price $p$ on the market. The demand is given by $q = \phi(1-p)$. The cost of producing a new product is $c_m$ and the cost of producing a remanufactured product is $c_{rm}$. We assume $c_m < 1$ to ensure positive demand at positive margin for new products. Define $\tau$ as the fraction of demand satisfied by remanufactured units. Then the manufacturer's profit from selling $q$ items to the retailer is $wq - c_m(1-\tau)q - c_{rm}\tau q$. Defining $\Delta \doteq c_m - c_{rm} > 0$ as the cost saving per unit from remanufacturing, we can rewrite this profit as $(w - c_m + \Delta\tau)q$. Note that with undifferentiated products, it is optimal to remanufacture and sell as many of the returned units as possible; thus $\tau$ is also the collection rate of products from the previous generation's sales volume $q$ (under the implicit assumption that all returns are remanufacturable). The collection volume, $q_r$, equals $\tau q$.

**The Reverse Channel.** The collecting agent is defined as the one who determines the collection quantity, and incurs collection-related costs such as acquisition, advertising and logistics (regardless of who handles the actual collection operation). The collecting agent can be the manufacturer, the retailer or a third party (Figure 1) and incurs a linear (e.g., constant per unit) acquisition cost $Aq_r$ and collection cost $C(\tau; q)$. If the retailer or the third party is the collecting agent, he transfers all collected units to the manufacturer and receives a transfer price $b$ per unit.

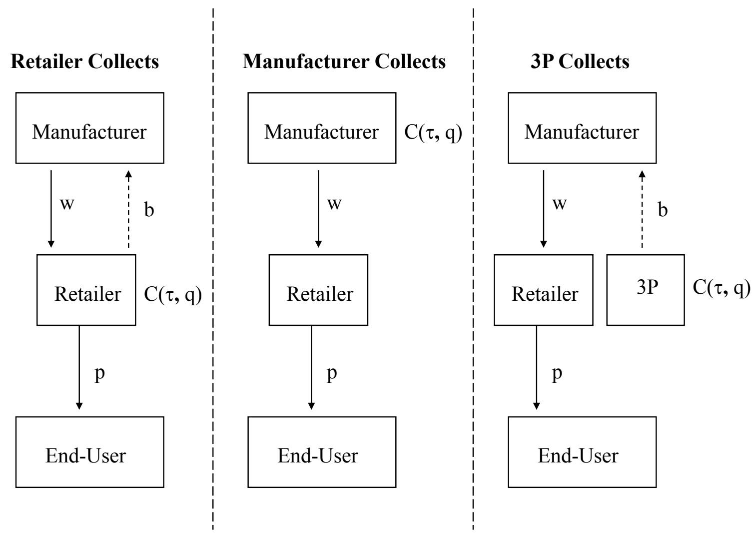


Figure 1: Decentralized Reverse Channel Options. When the collecting agent is the retailer or the third party, he incurs a cost $C(\tau; q)$ for collecting $q_r = \tau q$ units and is compensated at a unit transfer price $b$ by the manufacturer. When the manufacturer collects, he incurs the collection cost $C(\tau; q)$.

**The Collection Cost.** We focus on two components of total collection cost: (i) The cost of inciting the consumers to return their products, when such action is needed. In particular, we define $C_1(\tau) \doteq C_L \tau^l$ as the total cost to achieve a collection rate of $\tau$. (ii) The cost of physically collecting the product, $C_2(q_r) \doteq \eta q_r^k$, defined in terms of the collection volume $q_r$. As such, we model the total cost to collect a fraction $\tau$ of sales volume $q$ as $C(\tau; q) = C_L \tau^l + \eta q_r^k$, with $q_r \doteq \tau q$.

This model is capable of representing a variety of settings. First, it captures the investment cost analyzed by Savaşkan et al. (2004). As argued in their paper, promotional/advertising activities may be needed to encourage consumers to return their products. Such investments would be expected to exhibit diminishing returns ($l > 1$); Savaşkan et al. (2004) let $l = 2$ such that $\tau = \sqrt{C_1/C_L}$. Second, it captures the operational cost of collecting returns, where $k < 1$ means there are economies of scale in collection, while $k > 1$ captures diseconomies of scale. We present two examples corresponding to these cases.

Consider company A, an IT manufacturer that sells print cartridges to end-consumers, who are provided pre-paid return envelopes to return their end-of-use cartridges. The contract with the postal network consists of a quantity discount schedule, leading to *economies of scale* in collection ($k < 1$). Similarly, Cycleon, a European company that provides reverse logistics services, has contracted with several European postal companies, where the contract consists of a quantity discount schedule (Drake et al. 2009).

A number of authors argue that reverse logistics cost can exhibit diseconomies of scale (Guide et al. 2003, Guide and Van Wassenhove 2001, Ferguson and Toktay 2006) when the firm collects from increasingly distant locations to increase its collection volume. The case of company B, a chemical producer that has industrial customers in the automotive industry, confirms the existence of such an effect. Each customer of this company generates a given volume of returns, and the company decides whether to collect from each customer. If yes, a subcontractor collects all the quantity available at that customer. The data we obtained from this company reveals that of their 140 customers, they collect from the 39 closest customers. This is not surprising, as it is optimal to rank order customers according to increasing distance and collect according to that order. In this case, Company B can increase the collection volume by collecting from progressively more distant (and hence expensive) sources. Consequently, increasing the collection volume yields a total cost function that exhibits *diseconomies of scale*.

The rest of the paper is organized as follows: We first focus on the operational cost $C(q_r) = \eta q_r^k$ in Section 3.1. Then, we focus on a generalized version of Savaşkan et al. (2004) in Section 3.2. Section 3.3 presents a general model that combines the two cost structures. We discuss some extensions in Section 4, and conclude in Section 5.

# 3. Analysis

In this section, we follow Savaşkan et al. (2004) and compare the optimal solutions under manufacturer, retailer and third party collection and determine which reverse channel choice is preferred by the manufacturer. We let $\Pi_x^y$ denote the profits of party $x$ when party $y$ is the collecting agent, where $x, y \in \{M, R, 3P\}$. Here $M$ denotes the manufacturer, $R$ denotes the retailer and $3P$ denotes the third party collector.

**Manufacturer Collection:** When the manufacturer is responsible for collection, the retailer determines his sales price $p$ to maximize $\Pi_R^M(p) = (p-w)q(p)$, given the wholesale price $w$ quoted by the manufacturer. Let $p(w)$ denote the retailer's best response and $q(w)$ the corresponding sales quantity. The manufacturer's objective is to maximize $\Pi_M^M(w,\tau) = (w - c_m + \tau(\Delta - A))q(w) - C(\tau; q(w))$.

**Retailer Collection:** When the retailer is responsible for collection, he determines his sales price $p$ and collection rate $\tau$ to maximize $\Pi_R^R(p,\tau) = (p-w)q(p) + (b-A)\tau q(p) - C(\tau; q(p))$, where $b$ is the payment the retailer receives from the manufacturer per collected unit. Let $p(w,b)$ and $\tau(w,b)$ denote the retailer's best response and $q(w,b)$ denote the corresponding sales quantity. The manufacturer's objective is to maximize $\Pi_M^R(w,b) = (w - c_m + \Delta\tau(w,b))q(w,b) - b\tau q(w,b)$ by choosing $w$ and $b$.

**Third Party Collection:** Here, the retailer determines his sales price $p$ to maximize $\Pi_R^{3P}(p) = (p-w)q(p)$ given the wholesale price $w$ quoted by the manufacturer. Let $p(w)$ denote the retailer's best response and $q(w)$ the corresponding sales quantity. For any sales quantity $q$, the third party collector determines his collection rate $\tau$ to maximize $\Pi_{3P}^{3P}(\tau) = (b-A)\tau q - C(\tau; q)$, where $b$ is the payment the collector receives from the manufacturer per collected unit. Let $\tau(b,q)$ denote the collector's best response. The manufacturer anticipates the retailer's and the collector's reactions to his choice of $w$ and $b$. His objective is to maximize $\Pi_M^{3P}(w,b) = (w - c_m + \Delta\tau(b, q(w)))q(w) - b\tau(b, q(w))q(w)$ by choosing $w$ and $b$.

## 3.1 Reverse Logistics Cost

In this section, we focus on the physical cost of collection only. This analysis can be done directly in terms of the collection volume $q_r$ subject to $q_r \leq q$, but for consistency with the next subsection, we use the notation $C(\tau; q) = \eta(\tau q)^k$, where $\tau = \frac{q_r}{q}$ and $\tau \leq 1$.

### 3.1.1 Scale Economies in the Collection Volume

**Proposition 1** *Under economies of scale in collection ($k < 1$), the optimal solution with manufacturer, retailer or third party collection is at one of the two boundaries $\tau^* = 0$ or $\tau^* = 1$.*

**Proof.** All proofs are provided in Appendix A. ■

This result is intuitive: The collection cost exhibits economies of scale in the collection volume, while the revenue from collection increases linearly in the collection volume. Thus, the profit in the reverse channel is convex increasing in the collection volume. As long as the collecting party finds collection profitable for any collected quantity, he is better off by collecting a larger volume, so that $\tau^* = 1$; otherwise he does not undertake collection and $\tau^* = 0$. Unfortunately, a solution to the embedded optimization problems cannot be obtained in closed form for general $k < 1$ values because the collecting party's best response functions cannot be characterized in closed form. However, the numerical analysis presented in Tables 11 - 13 in Appendix C reveals the following:

**Observation 1** *Under economies of scale ($k < 1$), manufacturer profits are maximized under retailer collection. The manufacturer may find retailer collection so profitable that $b$ may be larger than $\Delta$ in equilibrium. The sales price is the lowest with retailer collection, and consequently the sales quantity is maximized when the retailer collects. The collection rate is (weakly) highest with retailer collection.*

By Proposition 1, when the retailer collects, he finds it optimal to collect all possible units as return channel profits are (convex) increasing in the collection volume. An increase in the transfer price $b$ incites the retailer to sell more so as to collect more and increase profits on the reverse channel. For the manufacturer, the resulting increase in the sales volume is so high as to dominate the margin lost from increasing the transfer price. The manufacturer capitalizes on this property, so much so that he may pass the entire remanufacturing savings or more to the retailer ($b^*$ may exceed $\Delta$).

When the manufacturer collects, he benefits from the scale economies in collection, so he would like to encourage a high sales volume in order to collect more. However, his only mechanism to influence the retailer is to decrease the wholesale price, which is not very effective, especially in the absence of scale economies accruing to the retailer. Despite the higher wholesale price observed in the retailer collection scenario, the combined effect of the high wholesale price and the high transfer price is a lower effective wholesale price $w^* - b^*$ and higher demand $q^*$ than with manufacturer collection, yielding higher profits for the manufacturer with retailer collection.

With third party collection, the manufacturer is even worse off than manufacturer collection, because not only does he have only one lever to incite the retailer to increase the collection volume, but he also shares the reverse channel profits with the third party.

**Observation 2** *Under economies of scale ($k < 1$), the retailer or supply chain profits may be highest under manufacturer or retailer collection.*

While the manufacturer always prefers retailer collection under scale economies, the same is not necessarily true from the retailer's perspective. Our numerical analysis (see Table 14 in Appendix C) reveals that the retailer can prefer manufacturer collection under scale economies. We discuss this effect in further detail in Section 3.2.

#### 3.1.2 Scale Diseconomies in the Collection Volume

Under scale diseconomies, there exists a threshold $\bar{\eta} \doteq \frac{\Delta - A}{(\phi(1-c_m)/4)^{k-1}}$ so that $\tau^* < 1$ for $\eta > \bar{\eta}$ regardless of who the collecting agent is. We solve the problem analytically in this case and resort to numerical analysis for $\eta < \bar{\eta}$.

**Proposition 2** *Let $k > 1$ and $\eta > \bar{\eta}$. The manufacturer makes more profit when he collects. With manufacturer collection, the collection rate is higher than if the retailer or the third party were to collect, but the equilibrium wholesale price, sales price and consequently the sales quantity are the same in all cases.*

When $\tau^* < 1$, the forward and reverse channels are decoupled in the sense that increasing the sales volume does not affect the collection volume. This explains why the wholesale price (and hence the retail price and sales volume) are the same in all three cases. It is intuitive that the collection rate $\tau^*$ and manufacturer profits are always higher when the manufacturer collects: While the manufacturer obtains the full benefit $\Delta$ when he is the collecting agent, he shares it with the other party otherwise, leading the other party to collect a lower volume and the manufacturer to make less profit.

**Corollary 1** *Let $k > 1$ and $\eta > \bar{\eta}$. The retailer's profits are highest when the retailer collects ($\Pi_R^R \geq \Pi_R^M = \Pi_R^{3P}$). The total supply chain profit is highest with manufacturer collection ($\Pi_M^M + \Pi_R^M \geq \Pi_M^R + \Pi_R^R \geq \Pi_M^{3P} + \Pi_R^{3P}$).*

Corollary 1 reveals that the retailer's and the manufacturer's preferences for the collection channel are not aligned under diseconomies of scale. This is intuitive as well: As explained above, due to the decoupling of forward and reverse channels, the wholesale price, and hence the retailer's margin and sales volume on the forward channel, are invariant in the collecting agent. With either manufacturer or third party collection, the retailer only makes money on the forward channel, while he makes additional money on the reverse channel when he is the collecting agent. The total supply chain profit (the sum of retailer and manufacturer profits) is highest under manufacturer collection. This is again because forward channel profits are independent of the collecting agent, and the profit on the return channel is higher without double marginalization.

Now consider the cost range where any party would set the collection rate to 1 in equilibrium, i.e., the forward and reverse channels do not decouple. Unfortunately, for general $k > 1$, there is no closed form solution in this case. Nevertheless, a numerical analysis suggests that the manufacturer again prefers to be the collecting agent (see Table 15 in Appendix C). To see why, consider the retailer collection case. The boundary solution $\tau_R^* = 1$ implies that the retailer's profit function on the return channel is locally (concave) increasing in $q_r$ at $q_r = q$. Thus, in response to an increase in $b$, the retailer would increase the sales volume to increase the collection volume. However, unlike the economies of scale case, her profit on the reverse channel increases at a decreasing rate, creating a moderate increase in sales volume in response to an increase in $b$. In other words, the manufacturer's instrument $b$ is not as effective at achieving an increase in sales that is sufficient to compensate the manufacturer's reduced margin from raising $b$. Therefore, the manufacturer prefers to collect himself.

## 3.2 Investment Cost

Now, we focus on the investment cost function $C(\tau; q) = C_L \tau^l$, which captures the effect of increasing the collection rate. Savaşkan et al. (2004) analyze the $l = 2$ case, but impose an upper bound $b \leq \Delta$. The analysis in §3.1.1 showed that the manufacturer may set $b^* > \Delta$, hence, we re-derive their results without imposing this bound.

**Proposition 3** *Let* $C(\tau; q) = C_L \tau^2$ *and* $\frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{8} < C_L < \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$. *The manufacturer makes more profit when the retailer collects. With retailer collection, the equilibrium collection rate is higher, the sales price is lower and consequently the sales quantity is higher.*

The thresholds in Proposition 3 correspond to the region where $\tau_R^* = 1$, but $0 < \tau_M^*, \tau_{3P}^* < 1$[1]. In this region, the main results parallel Savaşkan et al. (2004), i.e., retailer collection is preferred by the manufacturer, and the sales volume and the collection rate are highest with

[1] With no bound on $b$, the retailer's collection rate is either 1 or 0 in equilibrium, just as in the economies of scale case described earlier. In particular, it is 0 when $C_L \geq \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$, so retailer collection is not an option for the manufacturer, and 1 otherwise. When $C_L \leq \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{8}$, $\tau^* = 1$ for the manufacturer and the third party as well.

retailer collection. The effectiveness of retailer collection in this case is due to what Savaşkan et al. (2004) dub the "scale effect" (the average collection cost $C(\tau;q)/q_r = C_L\tau^2/q_r$ decreases in the sales volume for a given collection rate), which parallels the effect of economies of scale discussed in Section 3.1.1: The manufacturer offers the retailer a high transfer price (higher than $\Delta$ for large enough $C_L$). Due to the scale effect, the retailer's response is to raise the sales volume, which has a sufficiently high positive effect to compensate the manufacturer's margin erosion from the high transfer price.

Interestingly, when a broader set of parameters are considered, e.g., when $C_L < \frac{\beta(\Delta-A)^2}{8}$, the manufacturer prefers to collect himself (see Proof of Proposition 3 in Appendix A). Here, the retailer's sales response to an increase in $b$ is weaker, so using the transfer price tool is not as effective for the manufacturer. He prefers to use only the wholesale price tool to affect retail sales, and not incur losses due to double marginalization in the return channel.

Note that this discussion is concentrated on the model of Savaşkan et al. (2004) with $l = 2$ only. Unfortunately, the case $l \neq 2$ does not lend itself to closed-form solutions. However, for any $l > 0$, the so-called "scale effect" holds, i.e., the collecting agent's profitability on the reverse channel increases with volume for a given collection rate, suggesting that the main qualitative insights hold for any $l$. Table 16 in Appendix C provides support for this conjecture.

Another interesting difference from Savaşkan et al. (2004) is regarding retailer and supply chain profits when comparing manufacturer and retailer collection options.

**Corollary 2** *Let $C(\tau;q) = C_L\tau^2$. Then, there exists a $\hat{C}_L$ such that for $C_L \leq \hat{C}_L$, $\Pi_R^R > \Pi_R^M$, and $\Pi_R^R \leq \Pi_R^M$ otherwise. The total supply chain profit is highest with manufacturer collection ($\Pi_M^M + \Pi_R^M \geq \Pi_M^R + \Pi_R^R$) if $C_L > \frac{\phi(\Delta-A)(1-c_m+\Delta-A)}{4}$ and with retailer collection otherwise.*

Corollary 2 parallels Observation 2 in Section 3.1.1 and demonstrates that the retailer's preferences may not be aligned with those of the manufacturer; the retailer may prefer manufacturer collection. There is an intuitive reason behind these results. For higher costs of investment (higher $C_L$) and collection (higher $\eta$), the retailer obtains a lower profit from the reverse channel. Nevertheless, the manufacturer's capability of adjusting the wholesale price and the transfer price simultaneously gives the manufacturer good control over the retailer's behavior. Thus, the manufacturer, as the Stackelberg leader, can still obtain higher profits with retailer collection. When the manufacturer collects however, he cannot use the transfer price instrument. The retailer enjoys the benefits of the manufacturer's wholesale price reduction, as well as not paying for the collection related costs and prefers manufacturer collection. In line with this discussion, the supply chain profits are not necessarily maximized with retailer collection, signaling the difficulty of aligning stakeholder preferences.

## 3.3 Discussion

The central result emerging from the preceding analysis is that the scale effect is key in determining the optimal collecting agent: With economies of scale in the reverse logistics cost, or a strong enough scale effect in the investment cost, the manufacturer prefers the retailer to be the collecting agent, while with diseconomies of scale in the reverse logistics cost, or a weak scale effect in the investment cost, he prefers to be the collecting agent. The third-party option is never preferred by the manufacturer. Furthermore, the retailers do not necessarily prefer to be the collecting party; they may be better off with manufacturer collection when investment costs or reverse logistics costs are high.

Our analysis treated the reverse logistics costs and investment costs separately. Due to its complicated structure, the combined cost formulation $C(\tau; q) = C_L\tau^l + \eta(\tau q)^k$ with general $k$ and $l \neq 2$ does not lend itself to closed-form solutions. Therefore, we carry out a numerical study that compares manufacturer and retailer collection. We omit the third-party collection option as it is always dominated by these alternatives. Not surprisingly, our numerical study demonstrates that the solution to the general case is parameter dependent; the optimal reverse channel choice depends on the values that $\eta$, $C_L$, $k$ and $l$ take. Nevertheless, as may be expected, the optimal channel choice is driven by which of the two previously discussed effects dominates.

Consider the example presented in Figure 2. In this example, the investment cost has parameters $l = 2$ and $0.3 \leq C_L \leq 1$. The reverse logistics cost exhibits scale diseconomies with $k = 3$ and $\eta = 0.3, 0.7, 1$. For these parameters, retailer and manufacturer collection, respectively, would be optimal if the reverse channel cost consisted only of investment cost and reverse logistics cost, respectively. The optimal collection channel with the two costs combined then depends on which effect dominates. As seen in the figure, when the reverse logistics component is small ($\eta$ is low), the scale effect arising from the investment cost dominates and retailer collection is optimal ($\Pi_M^R > \Pi_M^M$). As $\eta$ increases, the reverse logistics cost starts dominating, and manufacturer collection becomes optimal for an increasingly large range of $C_L$ values.

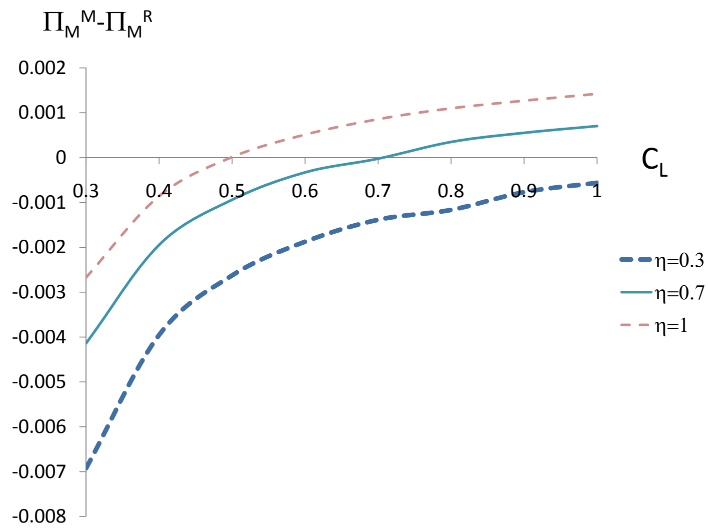


Figure 2: $\Pi_M^M - \Pi_M^R$ with $k = 3$, $l = 2$, $c_m = 0.5$, $A = 0.05$, $\Delta = 0.25$, and $\phi = 1$.

Our analysis identifies three key factors that determine the optimal reverse channel choice of the manufacturer: (i) whether there are scale economies or diseconomies in the reverse logistics cost (whether $k < 1$ or $> 1$); (ii) the magnitude of the reverse logistics cost (how large $\eta$ is); and (iii) the magnitude of the investment cost (how large $C_L$ is). Table 1 summarizes their typical combined impact in a simple informal framework:

- When the reverse logistics cost exhibits scale economies ($k < 1$), the impact of the reverse logistics cost is aligned with the typical impact of the investment cost. The two effects reinforce each other, creating a strong scale effect such that retailer collection is preferred.

  Recall that with a weak enough scale effect in investment cost (low enough $C_L$), manufacturer collection would be optimal if the reverse channel cost consisted only of the investment cost. With the combined cost structure, the reverse logistics cost has to be insignificant ($\eta$ very low) for this effect to persist, so manufacturer collection is excluded from the $k < 1$ column as being an atypical outcome.

- When the reverse logistics cost exhibits scale diseconomies ($k > 1$), the investment cost and the reverse logistics cost have contrasting impacts on the manufacturer's decision, so their respective levels play a more important role. For any $C_L$ value, when the reverse logistics cost component $\eta$ is small enough, the scale diseconomies in the reverse logistics cost is dominated by the scale effect inherent in the investment cost, and retailer collection is optimal. As $\eta$ increases, the scale diseconomies in the reverse logistics cost starts dominating and manufacturer collection becomes optimal above a threshold.

As seen in Figure 2, the Retailer-to-Manufacturer transition happens at lower $\eta$ values when $C_L$ is higher since an increase in the investment cost decreases the profitability of the reverse channel and makes it more difficult for the manufacturer to profitably incite the retailer to collect. We informally express this directional result in Table 1 by including the Manufacturer outcome in the lower middle cell to contrast it to the Retailer only upper middle cell.

| | $k < 1$ | $k > 1$, Low $\eta$ | $k > 1$, High $\eta$ |
|---|---|---|---|
| Low $C_L$ | Retailer | Retailer | Manufacturer |
| High $C_L$ | Retailer | Retailer/Manufacturer | Manufacturer |

Table 1: The Manufacturer's Typical Collection Channel Preference under the General Model

# 4. Cost and Product Differentiation

In this section, we discuss the impact of two effects that have the potential to change our results. We first explore the impact of a potential cost differential between the manufacturer and retailer collection channels. Next, we consider the possibility of differentiation between new and remanufactured products. We show that the main driver of reverse channel choice remains the same, i.e., the collection cost structure, reinforcing our findings so far.

## 4.1 Cost Differentials between Alternative Channels

Our analysis assumes that all collection related cost parameters are the same for the manufacturer, the retailer and the third party collector. These may differ in practice. For instance, a retailer with multiple sales locations can use these as collection points to collect product returns at a lower cost than the manufacturer. Corollary 3 analyzes the impact of cost differences on the optimal collection channel choice of the manufacturer under diseconomies of scale; we focus on this case for insight as it is fully tractable. We present the results for a comparison between the manufacturer and retailer collection options.

**Corollary 3** *Assume $C_L = 0$, $k > 1$ and $\eta > \bar{\eta}$. Then, $\Pi_M^R \geq \Pi_M^M$ if and only if $\eta_R k < \eta_M$ when $A_R = A_M$. Similarly, $\Pi_M^R \geq \Pi_M^M$ if and only if $A_R \leq \Delta - (\Delta - A_M)k^{1/k}$ when $\eta_R = \eta_M$.*

Corollary 3 confirms the intuition that the manufacturer would prefer the retailer to collect even under diseconomies of scale, provided that the collection cost of the retailer is sufficiently lower than that of the manufacturer. For equal acquisition costs, as $k$ increases, a bigger advantage in reverse logistics cost is needed for retailer collection to be optimal at higher scale diseconomies. In other words, the level of the scale effect also has an impact on the way the linear acquisition costs affect the channel choice. Note that this discussion considers the scale diseconomies case only. Similarly, one can show that the results under economies of scale are reversed with a sufficient difference in reverse logistics or acquisition costs.

## 4.2 Differentiated Products

Our model assumes that new and remanufactured products are perfectly substitutable. In this section, we discuss how the manufacturer's optimal reverse channel choice may be affected by product differentiation by making a number of modifications to the base model. Based on our findings from the previous section, we focus on the comparison of manufacturer and retailer collection channels only. Below, we describe the features of our product differentiation model and the underlying assumptions.

**Market Structure:** To model the market for differentiated new and remanufactured products, we borrow a market structure that is widely used in the remanufacturing literature (e.g., Ferguson and Toktay 2006, Ferrer and Swaminathan 2006, Atasu et al. 2008). We assume that consumer valuations for new products are uniformly distributed between 0 and 1. Furthermore, an individual's valuation for the remanufactured product is lower than for the new product: If a consumer values the new product at $\theta$, her valuation for the remanufactured product is $\delta\theta$. With this representation of the market, a consumer with new product valuation $\theta$ obtains a net utility $u_n(\theta) = \theta - p_n$ from the new product, where $p_n$ is the price of the new product. The same consumer obtains net utility $U_r(\theta) = \delta\theta - p_r$ from the remanufactured product, where $p_r$ is the price of the remanufactured product. Given the retailer's prices $p_n$ and $p_r$, the set of consumers buying new and remanufactured products is $\{\theta | U_n(\theta) \geq 0, U_n(\theta) > U_r(\theta)\}$ and $\{\theta | U_r(\theta) \geq 0, U_r(\theta) \geq U_n(\theta)\}$, respectively, yielding inverse demand functions $p_n = 1 - q_n - \delta q_r$ and $p_r = \delta(1 - q_n - q_r)$.

We consider a single-period model as in Savaşkan et al. (2004). To ensure a certain level of tractability, while capturing the fact that products can only be remanufactured a finite number of times, we assume that used products can only be remanufactured once. This assumption is common in the remanufacturing literature and guarantees tractability for a variety of cases (e.g. Debo et al. 2005, Ferguson and Toktay 2006, Atasu et al 2008, Ferrer and Swaminathan 2006, Majumder and Groenevelt 2001). Multiple uses of remanufactured products can tremendously complicate the analysis and requires additional assumptions on the durability of products, product life cycles and product usage durations (see Geyer et al. 2006 for further discussion).

It is also true that the availability of remanufacturable products depends not only on the efficiency of the collection channel, but also on other factors such as the physical condition of collected items or on product usage durations (Debo et al. 2005, Geyer et al. 2006). To capture this, we assume that only a fraction ($\rho \leq 1$) of used products are remanufacturable. With this assumption, if $q_n$ new products are sold, $\rho q_n$ would be the maximum number of products that can be remanufactured.

**Cost Structure:** The manufacturer charges different wholesale prices, $w_n$ and $w_r$, to the retailer for new and remanufactured products, which cost $c_n$ and $c_r$ respectively. When the retailer collects, with differentiated wholesale prices, there is no need for an additional transfer price instrument (such as the transfer price $b$ in the base model); the wholesale price internalizes the transfer price. Figure 3 displays the two reverse channel models under product differentiation.

The collecting party incurs cost $C(\tau, q_n)$ for collecting a fraction $\tau$ of available remanufacturable products $q_n$. The structure of the collection cost $C(\tau, q_n)$ depends on the scale effect as before. Unfortunately, even with the above simplifying assumptions, the problem is less tractable than before. Therefore, we limit our analysis to two special cases of our model affording some level of tractability: The case $k = 1$ and $l = 2$, with $C(\tau, q_n) = C_L\tau^2$, to capture the positive effect of scale, and the case $k = 2$ and $l = 0$, with $C(\tau, q_n) = \eta(\tau q_n)^2$, to capture diseconomies of scale. In Appendix B, we prove that manufacturer collection is optimal for the $k = 2$ and $l = 0$ case. For the $k = 1$ and $l = 2$ case, we resort to numerical analysis due to the complexity of the analytical expressions, and observe that retailer collection is optimal (Table 17 and 18). These findings suggest that the results and intuition from our basic model appear to hold under our model of new and remanufactured product differentiation, reinforcing the finding that the collection cost structure is the key element that determines the optimal reverse channel choice.

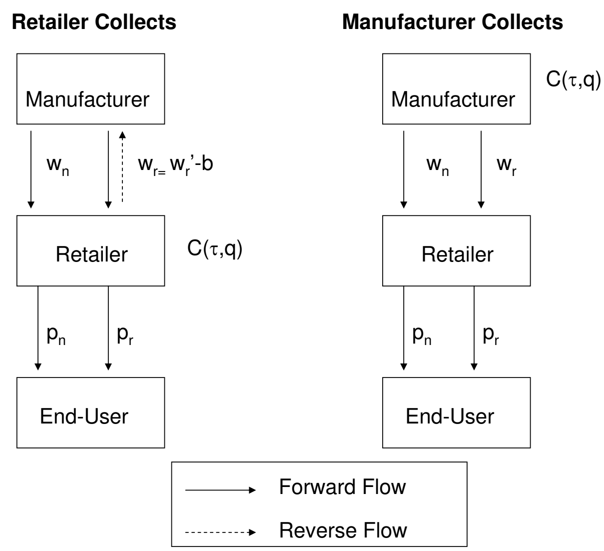


Figure 3: Reverse Channel Models under Product Differentiation

# 5. Conclusion

The theoretical and practical contribution of our research is to show that the structure of the collection cost should play a crucial role in the manufacturer's reverse channel strategy in decentralized supply chains. In particular, if the cost structure is such that the manufacturer can incite the retailer to sufficiently increase new product sales, then the manufacturer may profit from having the retailer collect despite sharing reverse channel profits with him. Thus, with our volume-dependent reverse logistics cost model, retailer collection is optimal when there are economies of scale, while manufacturer collection is optimal when there are diseconomies of scale. With the rate-dependent investment cost model (as modeled by Savaşkan et al. 2004), if the scale effect is strong enough, retailer collection is optimal, while manufacturer collection is optimal otherwise.

Since our model and analysis generalize those in Savaşkan et al. (2004), we highlight how we add to the understanding developed therein. Savaşkan et al. (2004) model the cost of investing to increase the collection rate, and show that retailer collection is optimal. They postulate that the retailer is the best collecting agent because he is closer to the market and can most directly affect demand with his pricing decisions. In contrast, the manufacturer can only affect demand through the wholesale price in an uncoordinated supply chain. Due to double marginalization, the manufacturer‘s ability to affect demand is limited. He therefore prefers to have the retailer collect.

Our contribution is to show that the optimal reverse channel choice should be driven by how the collection cost structure *moderates the manufacturer's ability to shape the retailer's decisions*. Consider our volume-dependent model first. With economies of scale, our result is the same as in Savaşkan et al. (2004) - retailer collection is optimal. With diseconomies of scale, however, despite being closer to the market, the retailer does not reflect the cost savings from remanufacturing in the final price of the product and collects less than the manufacturer would. The manufacturer makes more profit when he collects himself! Why does this happen? It is the efficiency of the transfer price instrument that is key, and this is determined by the collection cost structure. With economies of scale, an increase in the transfer price strongly incites the retailer to stimulate demand and increase the collection volume; the transfer price is a powerful instrument. The manufacturer capitalizes on this fully, even passing all remanufacturing savings or more to the retailer in some cases. The net increase in demand dominates the manufacturer's loss from compensating the retailer, and the manufacturer prefers the retailer to collect. In contrast, with diseconomies of scale, the transfer price is unable to induce the retailer to sell sufficiently more than under manufacturer collection. Consequently, the manufacturer has no incentive to share the reverse channel profits with the retailer and chooses to collect himself.

These results are in fact consistent with Savaşkan et al's: The investment cost they model exhibits a strong scale effect in the range that they analyze, such that the transfer price is a powerful instrument in inciting the retailer to increase its sales volume with its attendant benefits for the manufacturer. At the same time, as shown by our analysis, when the range of parameters is extended to where the scale effect is weak, the manufacturer prefers to collect himself.

This research has important managerial implications. It shows that manufacturers can indeed pass the reverse channel decision rights to retailers in decentralized reverse supply chains to improve the profitability of their businesses. The profitability of this option depends on the structure of the collection cost, which is driven by the characteristics of their operating environment and their products. For instance, consider a consumer electronics manufacturer that sells small electronic devices. Due to their size, these products can be collected through postal networks as in the Cycleon example. Where the postal contract specifies quantity discounts, our results suggest that this manufacturer should have the retailers deal with the collection channel. On the other hand, an industrial copier producer, due to the size or volume of its products, would have to employ a different collection strategy, e.g., the collecting agent would have to collect products directly from end-users. In this case, it is likely that reverse logistics costs exhibit scale diseconomies, since collecting a higher volume of used products may require reaching more remote locations. If so, our results suggest that the manufacturer would be better off collecting himself (unless of course the collection cost of the retailer is sufficiently lower than that of the manufacturer). In sum, while remanufacturing by itself can be an attractive business proposition, carefully managing the reverse channel (by having retailers take responsibility and using the transfer price effectively) can increase sales volume and manufacturer profitability.

Our results have research implications as well. Modeling reverse supply chains is an increasingly popular research topic in the operations management community. By showing how different collection cost structures impact collection channel choice, this paper highlights the importance of ensuring that the model appropriately captures the characteristics of the remanufacturing environment in question.

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# Appendix A: Proofs

## Proof of Proposition 1

### Manufacturer Collection

When the manufacturer is responsible for collection, the retailer determines his sales price $p$ to maximize

$$\Pi_R^M(p) = (p - w)q(p)$$

given the wholesale price $w$ quoted by the manufacturer. Let $p(w)$ denote the retailer's best response and $q(w)$ the corresponding sales quantity. It is straightforward to show that $p(w) = (1 + w)/2$ and $q(w) = (1 - w)/2$.

The manufacturer's objective is to maximize

$$\Pi_M^M(w, \tau) = (w - c_m + \tau\Delta)q(w) - \eta(\tau q(w))^k - A\tau q(w).$$

The unique solution to the first-order conditions of the manufacturer's profit function is $w = (1 + c_m)/2$ and $\tau = \frac{4(\frac{\Delta - A}{\eta k})^{1/(k-1)}}{\phi(1-c_m)}$. However, one can show that when $k < 1$, the determinant of the Hessian is negative at this solution. Thus, this stationary point cannot be a maximizer of the manufacturer's problem, and there are only two possible solutions to the manufacturer's problem: either collect nothing or collect everything.

### Retailer Collection

When the retailer is responsible for collection, he determines his sales price $p$ and collection rate $\tau$ to maximize

$$\Pi_R^R(p, \tau) = (p - w)q(p) - (A - b)\tau q(p) - \eta(\tau q(p))^k,$$

where $b$ is the payment the retailer receives from the manufacturer per collected unit. Let $p(w, b)$ and $\tau(w, b)$ denote the retailer's best response and $q(w, b)$ the corresponding sales quantity.

Similar to the manufacturer collection scenario, there exists a unique solution to the first-order conditions at $p(w, b) = (1 + w)/2$ (with $q(w, b) = (1 - w)/2$)and $\tau(w, b) = \frac{2(\frac{b-A}{\eta k})^{1/(k-1)}}{\phi(1-w)}$. One can show that the determinant of the Hessian is negative at this solution. Thus, the two candidates for optimality are at the boundaries $\tau = 0$ or 1 as in the manufacturer collection case.

### 3P Collection

When the third party is responsible for collection, he determines the collection rate $\tau$ to maximize

$$\Pi_{3P}^{3P}(\tau) = (b - A)q_r(\tau) - \eta(q_r)^k,$$

where $b$ is the payment the third party collector receives from the manufacturer per collected unit. Similar to the manufacturer collection scenario, there exists a unique solution to the first-order condition to the third party's objective at $\tau(b) = \frac{(\frac{b-A}{\eta k})^{1/(k-1)}}{\phi(1-p)}$. One can show that the local determinant of the Hessian is negative at this solution. Thus, the two candidates for optimality are at the boundaries $\tau = 0$ or 1 as in the manufacturer collection case.

## Proof of Proposition 2

### Manufacturer Collection

As in Proposition 1, it is straightforward to show that $p(w) = (1+w)/2$ and $q(w) = (1-w)/2$. The manufacturer's objective is to maximize

$$\Pi_M^M(w, \tau) = (w - c_m + \tau\Delta)q(w) - \eta(\tau q(w))^k - A\tau q(w).$$

Solving the first-order conditions of the manufacturer's objective results in the unique solution $w^* = (1 + c_m)/2$ and $\tau^* = \frac{4(\frac{\Delta-A}{\eta k})^{1/(k-1)}}{\phi(1-c_m)}$. With the assumption $\eta > \bar{\eta}$, $\tau^* < 1$. At this point, the Hessian is negative definite, meaning that this point is the only interior maximizer. The optimal profit is $\Pi_M^M = \frac{1}{8}\phi(c_m - 1)^2 + \eta(k-1)\left(\frac{\Delta-A}{\eta k}\right)^{\frac{k}{k-1}}$.

### Retailer Collection:

Let $\eta > \bar{\eta}_R(w, b) \doteq \frac{b-A}{k(\phi(1-w)/2)^{k-1}}$. In this case, the unique optimal solution to the retailer's problem is obtained at $p(w, b) = (1 + w)/2$ (with $q(w, b) = (1 - w)/2$) and $\tau(w, b) = \frac{2(\frac{b-A}{\eta k})^{1/(k-1)}}{\phi(1-w)}$, where $\tau(w, b) < 1$. The manufacturer anticipates the retailer's reaction to his choice of $w$ and $b$ and his objective is to maximize

$$\Pi_M^R(w, b) = (w - c_m + \Delta\tau(w, b))q(w, b) - b\tau(w, b)q(w, b)$$

by choosing $w$ and $b$. It is straightforward to show that the manufacturer profit is maximized at $w^* = (1 + c_m)/2$ and $b^* = \frac{A(k-1)+\Delta}{k}$. Substituting this solution into $\bar{\eta}_R(w, b)$, we require $\eta > \bar{\eta}_R \doteq \frac{\Delta-A}{k^2(\phi(1-c_m)/2)^{k-1}}$ to guarantee that $\tau^* < 1$. This condition is satisfied when $\eta > \bar{\eta}$ since $k > 1$. At this solution, the optimal profit is $\Pi_M^R = \frac{1}{8}\phi(1 - c_m)^2 + \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k^2}\right)^{\frac{1}{k-1}}}{k}$.

**3P Collection:**

Let $\eta > \bar{\eta}_{3P}(b) \doteq \frac{b-A}{k((\phi(1-p))^{k-1}}$. Then the unique optimal solution to the third party's problem is $\tau(b) = \frac{(\frac{b-A}{\eta k})^{1/(k-1)}}{\phi(1-p)}$, where $\tau(b) < 1$. It is straightforward to show that the retailer chooses $p(w) = (1+w)/2$ and $q(w) = (1-w)/2$. The manufacturer anticipates the retailer's and the collector's reactions to his choice of $w$ and $b$ and his objective is to maximize

$$\Pi_M^{3P}(w,b) = (w - c_m + \Delta\tau(w,b))q(w,b) - b\tau(w,b)q(w,b)$$

by choosing $w$ and $b$. It is again straightforward to show that the manufacturer profit is maximized at $w^* = (1+c_m)/2$ and $b^* = \frac{A(k-1)+\Delta}{k}$. Substituting this solution into $\bar{\eta}_{3P}(b)$, we require $\eta > \bar{\eta}_{3P} \doteq \frac{\Delta - A}{k^2(\phi(1-c_m)/2)^{k-1}}$ to guarantee that $\tau^* < 1$. This condition is satisfied when $\eta > \bar{\eta}$ since $k > 1$. At this solution, the optimal profit is $\Pi_M^{3P} = \frac{1}{8}\phi(1-c_m)^2 + \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k^2}\right)^{\frac{1}{k-1}}}{k}$.

**Comparison**

A summary of results for the three scenarios is provided in Tables 2 and 3, from which we see that the optimal prices are the same in all collection channels and that $\tau_M^* \geq \tau_R^* = \tau_{3P}^*$.

| Manufacturer Collection | Retailer Collection |
|---|---|
| $p^* = \frac{3+c_m}{4}$ | $p^* = \frac{3+c_m}{4}$ |
| $w^* = \frac{1+c_m}{2}$ | $w^* = \frac{1+c_m}{2}$ |
| $\tau^* = \frac{4(\frac{\Delta-A}{\eta k})^{1/(k-1)}}{\phi(1-c_m)}$ | $\tau^* = \frac{4(\frac{b^*-A}{\eta k})^{1/(k-1)}}{\phi(1-c_m)}$ |
| - | $b^* = \frac{A(k-1)+\Delta}{k}$ |
| $\Pi_M^{M^*} = \frac{1}{8}\phi(1-c_m)^2 + \eta(k-1)\left(\frac{\Delta-A}{\eta k}\right)^{\frac{k}{k-1}}$ | $\Pi_M^{R^*} = \frac{1}{8}\phi(1-c_m)^2 + \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k^2}\right)^{\frac{1}{k-1}}}{k}$ |
| $\Pi_R^{M^*} = \frac{1}{16}\phi(1-c_m)^2$ | $\Pi_R^{R^*} = \frac{1}{16}\phi(1-c_m)^2 + \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k^2}\right)^{\frac{1}{k-1}}}{k^2}$ |

Table 2: Manufacturer and Retailer Collection with Diseconomies of Scale in Reverse Logistics Cost

We note that the manufacturer profit at the third party collection solution is equal to the retailer collection solution. It is straightforward to show that the ratio of the second term of $\Pi_M^M$ over the second term of $\Pi_M^R$ is equal to $k^{1/(k-1)}$. The natural log of this term is nonnegative for $k > 1$, thus this term is always large than one. Thus, manufacturer collection dominates.

| Third Party Collection |
|---|
| $p^* = \frac{3+c_m}{4}$ |
| $w^* = \frac{1+c_m}{2}$ |
| $\tau^* = \frac{4(\frac{b^*-A}{\eta k})^{1/(k-1)}}{\phi(1-c_m)}$ |
| $b^* = \frac{A(k-1)+\Delta}{k}$ |
| $\Pi_M^{3P^*} = \frac{1}{8}\phi(1-c_m)^2 + \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k^2}\right)^{\frac{1}{k-1}}}{k}$ |
| $\Pi_R^{3P^*} = \frac{1}{16}\phi(1-c_m)^2$ |

Table 3: 3P Collection with Diseconomies of Scale in Reverse Logistics Cost

## Proof of Corollary 1

The proofs follow from the results in Tables 2 and 3.

**Retailer Profits:** First, we note that the retailer's equilibrium profit at the third party collector solution is equal to the manufacturer collection solution. It is straightforward to show that the second term of $\Pi_R^R$ is always positive since $k > 1$, thus the retailer's equilibrium profit is maximized under retailer collection.

**Channel Profits:** We define the channel profit as the sum of manufacturer and retailer profits at a given collection channel choice.

First, we compare the retailer and third party collection options and demonstrate that the third party option is dominated by the retailer collection option. Note that $\Pi_M^R + \Pi_R^R - \Pi_M^{3P} + \Pi_R^{3P} = \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k^2}\right)^{\frac{1}{k-1}}}{k^2} > 0$. Thus, the third party option is dominated by the retailer collection option.

Next, we compare the channel profits under manufacturer and retailer collection. Note that $\Pi_M^M + \Pi_R^M - \Pi_M^R + \Pi_R^R = \left[k^{1/(k-1)-1-\frac{1}{k}}\right] \frac{(k-1)(\Delta-A)\left(\frac{\Delta-A}{\eta k}\right)^{\frac{1}{k-1}}}{k^2}$.

Thus, $\Pi_M^M + \Pi_R^M - \Pi_M^R + \Pi_R^R \geq 0 \Leftrightarrow \left[k^{1/(k-1)} - 1 - \frac{1}{k}\right] \geq 0$. By simplifying and rearranging terms, this condition is equivalent to $\frac{k}{k-1} \geq \frac{\ln(k+1)}{\ln(k)}$. This condition can be further simplified to $k > \ln(k+1)$, which is always true. Thus, channel profits are maximized with manufacturer collection.

## Proof of Proposition 3

The proofs in this section follow Savaşkan et al. (2004), with the exception that we do not impose the condition $b \leq \Delta$ and we relax the lower bound on $C_L$.

### Manufacturer Collection

When the manufacturer collects, it is straightforward to show that the retailer's best response is $p(w) = \frac{1+w}{2}$. Given the retailer's best response, the manufacturer's objective is to maximize $\Pi_M^M = \phi\left(\frac{1}{2}(-w+1)\right)(\tau(\Delta - A) - c_m + w) - C_L\tau^2$. Substituting and solving the first-order conditions gives the unique stationary point $\tau^* = -\frac{(c_m-1)\phi(A-\Delta)}{\phi(A-\Delta)^2-8C_L}$ and $w^* = \frac{\phi(A-\Delta)^2-4C_L(c_m+1)}{\phi(A-\Delta)^2-8C_L}$.

Note that $\tau^*$ is decreasing in $C_L$ and the condition $8C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$ guarantees that $\tau^* < 1$. Also, $\frac{\partial^2\Pi_M^M}{\partial w^2} < 0$, and the determinant of the Hessian is $2C_L\phi - \frac{1}{4}\phi^2(A-\Delta)^2$, which is always positive when $8C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$. Thus, the unique stationary point maximizes manufacturer profits when $8C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$. The equilibrium profits of the manufacturer and the retailer are $\Pi_M^{M*} = \frac{C_L(c_m-1)^2\phi}{8C_L-\phi(A-\Delta)^2}$ and $\Pi_R^{M*} = \frac{4C_L^2(c_m-1)^2\phi}{(\phi(A-\Delta)^2-8C_L)^2}$.

When $8C_L \leq \phi(A-\Delta)(A+c_m-\Delta-1)$, the interior solution obtained from the manufacturer's first order conditions is not a maximizer. In this case, the manufacturer's objective will be obtained at the boundaries. It is straightforward to show that the manufacturer's profit is maximized at $\tau^* = 1$. When $\tau = 1$, the manufacturer sets $w = \frac{1+c_m+A-\Delta}{2}$. At this solution, the manufacturer obtains $\Pi_M^{M*} = \frac{1}{8}\phi(A+c_m-\Delta-1)^2 - C_L$ and the retailer obtains $\Pi_R^{M*} = \frac{1}{16}\phi(A+c_m-\Delta-1)^2$. These results are summarized in Table 4.

| $C_L \leq \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$ | $C_L > \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$ |
|---|---|
| $\Pi_M^M = \frac{1}{8}\phi(A+c_m-\Delta-1)^2 - C_L$ | $\Pi_M^M = \frac{C_L(c_m-1)^2\phi}{8C_L-\phi(A-\Delta)^2}$ |
| $\Pi_R^M = \frac{1}{16}\phi(A+c_m-\Delta-1)^2$ | $\Pi_R^M = \frac{4C_L^2(c_m-1)^2\phi}{(\phi(A-\Delta)^2-8C_L)^2}$ |

Table 4: Manufacturer Collection with Investment Cost

### Third Party Collection

When the third party collects, the retailer's best response to the manufacturer's wholesale price $w$ is $p(w) = \frac{1+w}{2}$. The third party's objective is to maximize $\phi\tau\left(\frac{1}{2}(-w-1)+1\right)(b - A) - C_L\tau^2$ with respect to $\tau$. This function is concave in $\tau$ and is maximized at $\tau = \frac{\phi(w-1)(A-b)}{4C_L}$ for given $w$ and $b$.

Given the retailer's and the third party's best responses, the manufacturer's objective is to maximize $\Pi_M^M = \phi\left(\frac{1}{2}(-w+1)\right)(\tau(\Delta - A) - c_m + w)$. Substituting and solving the first-order conditions gives the unique stationary point $b = \frac{A+\Delta}{2}$ and $w = \frac{\phi(A-\Delta)^2-8C_L(c_m+1)}{\phi(A-\Delta)^2-16C_L}$.

When $16C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$, $\tau^* < 1$. Also, $\frac{\partial^2 \Pi^M_{3P}}{\partial w^2} < 0$, and the determinant of the Hessian, $\frac{\phi^3(w-1)^2\left(4C_L-\phi\left(A^2+A(\Delta-3b)+3b^2-3b\Delta+\Delta^2\right)\right)}{16C_L^2}$, is positive at this stationary point. Thus the only interior maximizer of manufacturer profit is obtained at $b = \frac{A+\Delta}{2}$ and $w = \frac{\phi(A-\Delta)^2-8C_L(c_m+1)}{\phi(A-\Delta)^2-16C_L}$ when $16C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$. The equilibrium profits of the manufacturer and the retailer at this point are $\Pi^{3P}_M = \frac{2C_L(c_m-1)^2\phi}{16C_L-\phi(A-\Delta)^2}$ and $\Pi^{3P}_R = \frac{16C_L^2(c_m-1)^2\phi}{(\phi(A-\Delta)^2-16C_L)^2}$.

Although the stationary solution obtained above is a local maximizer, the joint concavity of the manufacturer's objective with respect to $b$ and $w$ cannot be guaranteed, thus this is not necessarily a global maximizer. For the sake of completeness, we need to check the manufacturer's objective at the boundaries as well. Based on the third party's best response, it is easy to see that:

- $\tau = 0$ when $b = A$. In this case $\Pi^{3P}_M = \frac{1}{8}\phi(1-c_m)^2$ and $\Pi^{3P}_R = \frac{1}{16}\phi(1-c_m)^2$
- $\tau = 1$ when $b = A + \frac{8C_L}{\phi(1-c_m+\Delta-A)}$. In this case, $\Pi^{3P}_M = \frac{1}{8}\phi(A+c_m-\Delta-1)^2 - 2C_L$ and $\Pi^{3P}_R = \frac{1}{16}\phi(A+c_m-\Delta-1)^2$

By comparing the manufacturer's interior maximizer with his objective at the boundary solutions, we observe that the interior solution dominates the boundary solutions when $16C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$. Otherwise, the boundary solution with $\tau = 1$ dominates. These results are summarized in Table 5.

| $C_L \leq \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{16}$ | $C_L > \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{16}$ |
|---|---|
| $\Pi^{3P}_M = \frac{1}{8}\phi(A+c_m-\Delta-1)^2 - 2C_L$ | $\Pi^{3P}_M = \frac{2C_L(c_m-1)^2\phi}{16C_L-\phi(A-\Delta)^2}$ |
| $\Pi^{3P}_R = \frac{1}{16}\phi(A+c_m-\Delta-1)^2$ | $\Pi^{3P}_R = \frac{16C_L^2(c_m-1)^2\phi}{(\phi(A-\Delta)^2-16C_L)^2}$ |

Table 5: Third Party Collection with Investment Cost

**Retailer Collection**

When the retailer collects, given the wholesale price $w$ and the transfer price $b$, the retailer maximizes $\Pi^R_R = (1-p)\phi(\tau(b-A)+p-w) - C_L\tau^2$. Solving the first-order conditions yields the stationary best response $\tau(w,b) = -\frac{\phi(w-1)(A-b)}{\phi(A-b)^2-4C_L}$ and $p(w,b) = \frac{\phi(A-b)^2-2C_L(w+1)}{\phi(A-b)^2-4C_L}$. Also $\frac{\partial^2 \Pi^R_R}{\partial p^2} < 0$ and the determinant of retailer's Hessian is $\phi\left(4C_L - \phi(A-b)^2\right)$. Therefore, the objective is not necessarily jointly concave in $p$ and $\tau$. In what follows, we prove that the retailer's stationary best response will be forced to the boundary, i.e., $\tau = 1$, by the manufacturer.

Let us assume that the retailer's unique stationary solution is a local maximizer and investigate the manufacturer's choice of $w$ and $b$. Replacing the retailer's best response function in the manufacturer's objective, we obtain:

$$\Pi_M^R = -\frac{2C_L\phi(w-1)(\phi(A-b)(A(c_m-w)+b(-c_m)+b+\Delta(w-1))+4C_L(w-c_m))}{(\phi(A-b)^2-4C_L)^2}$$

In a similar spirit to Savaşkan et al. (2004), we start by investigating the behavior of this function with respect to $w$. Assume that there is a local interior maximizer pair $(w_1, b_1)$. This pair would have to satisfy the first-order condition $\frac{\partial \Pi_M^R}{\partial w} = -\frac{2C_L\phi(\phi(A-b)(A(c_m-2w+1)+b(-c_m)+b+2\Delta(w-1))-4C_L(c_m-2w+1))}{(\phi(A-b)^2-4C_L)^2} = 0$. Thus, a maximizer should satisfy $w(b) = \frac{4C_L(c_m+1)-\phi(A-b)(Ac_m+A-bc_m+b-2\Delta)}{8C_L-2\phi(A-b)(A-\Delta)}$. Replacing this in the manufacturer's objective, we obtain $\Pi_M^R(b) = \frac{C_L(c_m-1)^2\phi}{8C_L-2\phi(A-b)(A-\Delta)}$. It is easy to see that this function is increasing in $b$. Thus, at the retailer's stationary best response, the manufacturer chooses the highest $b$ possible. Now note that $\tau(w,b) = -\frac{\phi(w-1)(A-b)}{\phi(A-b)^2-4C_L}$ is also increasing in $b$. Therefore, at the equilibrium, the value of $b$ will be such that $\tau^* = 1$. This is achieved when $b^* = \frac{8C_L}{\phi(2(\Delta-A)+1-c_m)} + A$. This point is a maximizer of the manufacturer's objective, because $\frac{\partial^2\Pi_M^R}{\partial w^2} = -\frac{(c_m-1)\phi^3(2A+c_m-2\Delta-1)^3}{(\phi(2A+c_m-2\Delta-1)^2-16C_L)^2} < 0$. In this case, $w(b^*) = \frac{1+c_m}{2} + b^* - \Delta$. Similarly, this is a maximizer of the retailer's objective since $\frac{\partial^2\Pi_R^R}{\partial p^2} < 0$ on the $\tau = 1$ boundary. In this case, the manufacturer's profit will be $\Pi_M^R = \frac{1}{8}(c_m-1)\phi(2A+c_m-2\Delta-1)$ and the retailer's profit will be $\Pi_R^R = \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2 - C_L$.

Note also that in order for the equilibrium characterized above to take place, the retailer needs to have non-negative profits. In other words, it has to be true that $\Pi_R^R = \frac{1}{16}\phi(2A + c_m - 2\Delta - 1)^2 - C_L > 0$. This means that when $C_L > \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2$ the retailer is better off not selling nor collecting anything. Thus, when $C_L > \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2$ the retailer will not participate and both parties will obtain zero profits.

Since there is no equilibrium with interior $\tau$, we need to check the retailer's potential equilibria at the boundaries for $\tau$. The above analysis characterizes the case of $\tau = 1$. Therefore, we consider an equilibrium at $\tau = 0$ next. When the manufacturer chooses $b = A$ and $w = (1+c_m)/2$ (i.e., the forward channel only equilibrium), the retailer will choose not to collect but will be selling for profit from the forward channel. In this case $\Pi_M^R = \frac{1}{8}\phi(1-c_m)^2$ and $\Pi_R^R = \frac{1}{16}\phi(1-c_m)^2$.

Comparing the manufacturer profits at the two boundaries, we can fully characterize the manufacturer's decisions and the equilibrium as follows. When $4C_L > \phi(A-\Delta)(A+c_m-\Delta-1)$, $\Pi_M^R(\tau^*=0) = \frac{1}{8}\phi(1-c_m)^2 > \Pi_M^R(\tau^*=1) = \frac{1}{8}(c_m-1)\phi(2A+c_m-2\Delta-1)$, and the equilibrium will take place at $\tau = 0$. (Note that this condition also guarantees that the retailer will have positive profit at the equilibrium.) Otherwise, the equilibrium will take place at $\tau = 1$. These results are summarized Table 6.

| $C_L \leq \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{4}$ | $C_L > \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{4}$ |
|---|---|
| $\Pi_M^R = \frac{1}{8}(c_m-1)\phi(2A+c_m-2\Delta-1)$ | $\Pi_M^R = \frac{1}{8}\phi(1-c_m)^2$ |
| $\Pi_R^R = \frac{1}{16}\phi(2A+c_m-2\Delta-1)^2 - C_L$ | $\Pi_R^R = \frac{1}{16}\phi(1-c_m)^2$ |

Table 6: Retailer Collection with Investment Cost

**Comparison of Manufacturer Profits**

Let us define $C_L^M \doteq \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$, $C_L^R \doteq \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$ and $C_L^{3P} \doteq \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{16}$. With some algebraic manipulation of the data in Tables 4, 5 and 6, Table 7 provides the ranking of manufacturer profits under manufacturer and retailer collection for different parameter levels. The third party collection channel is always dominated and therefore is not presented in this table.

| $C_L \leq \frac{\phi(\Delta-A)^2}{8}$ | $\frac{\phi(\Delta-A)^2}{8} < C_L \leq C_L^{3P}$ | $C_L^{3P} < C_L \leq C_L^M$ | $C_L^M < C_L \leq C_L^R$ | $C_L > C_L^R$ |
|---|---|---|---|---|
| $\Pi_M^M > \Pi_M^R$ | $\Pi_M^R > \Pi_M^M$ | $\Pi_M^R > \Pi_M^M$ | $\Pi_M^R > \Pi_M^M$ | $\Pi_M^M > \Pi_M^R$ |

Table 7: Comparison of Manufacturer Profits

Note that this table provides a broader picture than Proposition 3 for the collection channel choice when an investment cost is considered. Proposition 3 identifies when retailer collection is preferred as in Savaşkan et al. (2004). According to this table however, when the investment cost is sufficiently low ($C_L \leq \frac{\phi(\Delta-A)^2}{8}$) or high ($C_L > C_L^R$), the manufacturer prefers to collect himself.

## Proof of Corollary 2

**Comparison of Retailer Profits**

Defining $C_L^M = \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$, $C_L^R = \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$ and $C_L^{3P} = \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{16}$, with some algebraic manipulation of the data in Tables 4 and 6, Table 8 provides the ranking of retailer profits under manufacturer and retailer collection for different parameter levels. We omit the third party collection option because it is dominated with respect to manufacturer profits. $\hat{C}_L$ is the solution to the equation $\frac{1}{16}\phi(2A + c_m - 2\Delta - 1)^2 - C_L = \frac{4C_L^2(c_m-1)^2\phi}{(\phi(A-\Delta)^2-8C_L)^2}$. The closed form solution to this equation is tedious, but exists, meaning that above this level of investment cost coefficient $C_L$, the retailer will prefer manufacturer collection.

| | | | |
|---|---|---|---|
| $C_L \leq C_L^M$ | $C_L^M < C_L \leq \hat{C}_L$ | $\hat{C}_L < C_L \leq C_L^R$ | $C_L > C_L^R$ |
| $\Pi_R^R > \Pi_R^M$ | $\Pi_R^R > \Pi_R^M$ | $\Pi_R^M > \Pi_R^R$ | $\Pi_R^M > \Pi_R^R$ |

Table 8: Comparison of Retailer Profits

### Comparison of Supply Chain Profits

Defining $C_L^M = \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$, and $C_L^R = \frac{\phi(A-\Delta)(A+c_m-\Delta-1)}{8}$, with some algebraic manipulation of the data in Tables 4 and 6, Table 9 provides the ranking of supply chain profits (i.e., $\Pi_S = \Pi_M + \Pi_R$) under manufacturer and retailer collection for different parameter levels. We omit the third party collection option because it is always dominated.

| | | |
|---|---|---|
| $C_L \leq C_L^M$ | $C_L^M < C_L \leq C_L^R$ | $C_L > C_L^R$ |
| $\Pi_S^R > \Pi_S^M$ | $\Pi_S^R > \Pi_S^M$ | $\Pi_S^M > \Pi_S^R$ |

Table 9: Comparison of Supply Chain Profits

## Proof of Corollary 3

The proof directly follows from comparing the equilibrium profits of the manufacturer under manufacturer and retailer collection options in Proposition 2.

# Appendix B: Differentiated Products

We assume that $c_n < 1$ and $A + c_r < \delta c_n$, so that positive margins can be obtained from selling both new and remanufactured products.

## Diseconomies of Scale

The cost function $C(\tau, q_n) = \eta(\tau q_n)^2$ used in this analysis can be rewritten as $C(q_r) = \eta q_r^2$, so the retailer and manufacturer optimization problems can be written in terms of $q_r$ and $q_n$. We assume that the collection cost coefficient $\eta$ is sufficiently high such that the unconstrained optimizer satisfies $q_r \leq q_n$.

### Manufacturer Collects

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer's sales quantities for any given $(w_n, w_r)$ pair that maximize $\Pi_R^M = (p_n - w_n)q_n + (p_r - w_r)q_r$.

The problem is jointly concave in $q_n$ and $q_r$. The first-order conditions can be written as $\frac{\partial \Pi_R^M}{\partial q_n} = 1 - 2\,q_n - 2\,\delta\,q_r - w_n = 0$, and $\frac{\partial \Pi_R^M}{\partial q_r} = -2\,\delta\,q_n - \delta\,(-1 + q_r) - \delta\,q_r - w_r = 0$. Solving the first-order conditions gives $q_n(w_n, w_r) = \frac{-(1-\delta-w_n+w_r)}{2\,(-1+\delta)}$ and $q_r(w_n, w_r) = \frac{-(\delta\,w_n - w_r)}{2\,(-1+\delta)\,\delta}$.

Given the retailer's anticipated reaction to $w_n$ and $w_r$, the manufacturer's constrained Lagrangian objective, which includes the constraint on remanufacturable product availability, can be written as

$$L(w_n, w_r, \lambda) = \Pi_M^M + \lambda(\rho q_n - q_r) = q_n\,(-c_n + \lambda\,\rho + w_n) - q_r\,(A + c_r + \lambda + \eta\,q_r - w_r)\,.$$

Replacing the anticipated retailer sales quantities in the Lagrangian we obtain $L(w_n, w_r, \lambda) = \frac{(-c_n+\lambda\,\rho+w_n)\,(-1+\delta+w_n-w_r)+\frac{(\delta\,w_n-w_r)\left(A+c_r+\lambda-w_r+\frac{\eta\,(-(\delta\,w_n)+w_r)}{2\,(-1+\delta)\,\delta}\right)}{\delta}}{2\,(-1+\delta)}$.

The problem is jointly concave in $w_n$ and $w_r$. The first-order conditions can be written as $\frac{\partial \Pi_M^M}{\partial w_n} = \frac{-1+A-c_n+c_r+\delta+\lambda+\lambda\,\rho+2\,w_n-\frac{\eta\,(\delta\,w_n-w_r)}{2\,(-1+\delta)\,\delta}-2\,w_r+\frac{\eta\,(-(\delta\,w_n)+w_r)}{2\,(-1+\delta)\,\delta}}{2\,(-1+\delta)} = 0$,
$\frac{\partial \Pi_M^M}{\partial w_r} = \frac{c_n-\lambda\,\rho-w_n+\frac{\left(-1+\frac{\eta}{2\,(-1+\delta)\,\delta}\right)(\delta\,w_n-w_r)}{\delta}-\frac{A+c_r+\lambda-w_r+\frac{\eta\,(-(\delta\,w_n)+w_r)}{2\,(-1+\delta)\,\delta}}{\delta}}{2\,(-1+\delta)} = 0$, and
$\frac{\partial \Pi_M^M}{\partial \lambda} = \frac{\rho\,(-1+\delta+w_n-w_r)+\frac{\delta\,w_n-w_r}{\delta}}{2\,(-1+\delta)} = 0$.

**<u>Case1: $\lambda > 0$</u>**

Solving the first-order conditions gives $w_n = \frac{1+c_n+(A+c_r+(2+c_n)\,\delta)\,\rho+((2+A+c_r-\delta)\,\delta+\eta)\,\rho^2}{2+\eta\,\rho^2+2\,\delta\,\rho\,(2+\rho)}$, $w_r = -\left(\frac{\delta\left(1+c_n+(-1+A+c_n+c_r+3\,\delta)\,\rho+(A+c_r+\delta+\eta)\,\rho^2\right)}{2+\eta\,\rho^2+2\,\delta\,\rho\,(2+\rho)}\right)$ and $\lambda = \frac{-2\,(A+c_r-c_n\,\delta)+(2\,\delta\,(-1-A+c_n-c_r+\delta)+(-1+c_n)\,\eta)\,\rho}{2+\eta\,\rho^2+2\,\delta\,\rho\,(2+\rho)}$. The resulting sales quantities are $q_n = -\left(\frac{-1+c_n+(A+c_r-\delta)\,\rho}{4+2\,\eta\,\rho^2+4\,\delta\,\rho\,(2+\rho)}\right)$ and $q_r = -\left(\frac{\rho\,(-1+c_n+(A+c_r-\delta)\,\rho)}{4+2\,\eta\,\rho^2+4\,\delta\,\rho\,(2+\rho)}\right)$. It is easy to see that both $q_n$ and $q_r$ are positive when both margins are positive, i.e., $\delta > A+c_r$ and $c_n < 1$. The problem is unconstrained if $\lambda > 0$ or $\rho > \rho^M = \frac{-2\,(A+c_r-c_n\,\delta)}{2\,(1+A-c_n+c_r-\delta)\,\delta+\eta-c_n\,\eta}$. The manufacturer's profit when the manufacturer collects is $\Pi_M^M = \frac{(-1+c_n+(A+c_r-\delta)\,\rho)^2}{8+4\,\eta\,\rho^2+8\,\delta\,\rho\,(2+\rho)}$. The retailer's profit in this case is given by
$\Pi_R^M = \frac{(-1+c_n+(A+c_r-\delta)\,\rho)^2\,(1+\delta\,\rho\,(2+\rho))}{4\,(2+\eta\,\rho^2+2\,\delta\,\rho\,(2+\rho))^2}$.

**<u>Case 2: $\lambda = 0$</u>**

Solving the first-order conditions at $\lambda = 0$ gives $w_n = \frac{1+c_n}{2}$ and
$w_r = \frac{\delta\left(-2\,A-2\,c_r-2\,\delta+2\,A\,\delta+2\,c_r\,\delta+2\,\delta^2-\eta-c_n\,\eta\right)}{2\,(-2\,\delta+2\,\delta^2-\eta)}$.

The resulting sales quantities are $q_n = \frac{2\,\delta\,(-1-A+c_n-c_r+\delta)+(-1+c_n)\,\eta}{8\,(-1+\delta)\,\delta-4\,\eta}$ and $q_r = \frac{A+c_r-c_n\,\delta}{4\,(-1+\delta)\,\delta-2\,\eta}$. It is easy to see that both $q_n$ and $q_r$ are positive when $1 - c_n > \delta - (c_r + A)$, i.e. when the margin from manufacturing is higher than the margin from remanufacturing, and when both margins are positive, i.e., $\delta > A+c_r$ and $c_n < 1$. The manufacturer's optimal profit when the manufacturer collects is $\Pi_M^M = \frac{(-1+c_n)^2+\frac{2\,(A+c_r-c_n\,\delta)^2}{-2\,(-1+\delta)\,\delta+\eta}}{8}$. The retailer's profit in this case is given by $\Pi_R^M = \frac{-4\,(-1+\delta)\,\delta\,\left((A+c_r)^2+\left(1+{c_n}^2-2\,c_n\,(1+A+c_r)\right)\delta+(-1+2\,c_n)\,\delta^2\right)-4\,(-1+c_n)^2\,(-1+\delta)\,\delta\,\eta+(-1+c_n)^2\,\eta^2}{16\,(-2\,(-1+\delta)\,\delta+\eta)^2}$.

**Retailer Collects**

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer's sales quantities for any given $(w_n, w_r)$ pair that maximize $\Pi_R^R = (p_n - w_n)q_n + (p_r - w_r)q_r - C(q_r) = -{q_n}^2 - q_n\ (-1 + 2\,\delta\,q_r + w_n) - q_r\ (A - \delta + (\delta + \eta)\ q_r + w_r)$. The problem is jointly concave in $q_n$ and $q_r$. The first-order conditions can be written as $\frac{\partial \Pi_R^R}{\partial q_n} = 1 - 2\,q_n - 2\,\delta\,q_r - w_n = 0$, and $\frac{\partial \Pi_R^R}{\partial q_r} = -A + \delta - 2\,\delta\,q_n - 2\ (\delta + \eta)\ q_r - w_r = 0$. Solving the first-order conditions gives $q_n(w_n, w_r) = \frac{\delta^2+\eta\,(-1+w_n)-\delta\,(1+A-w_n+w_r)}{2\,(-1+\delta)\,\delta-2\,\eta}$ and $q_r(w_n, w_r) = \frac{A-\delta\,w_n+w_r}{2\,(-1+\delta)\,\delta-2\,\eta}$.

Given the retailer's anticipated reaction to $w_n$ and $w_r$, the manufacturer's constrained Lagrangian objective can be written as $L(w_n, w_r \lambda) = \Pi_M^R + \lambda(\rho q_n - q_r) = q_n\ (-c_n + \lambda\,\rho + w_n) - q_r\ (c_r + \lambda - w_r)$.

The problem is jointly concave in $w_n$ and $w_r$. The first-order conditions can be written as $\frac{\partial \Pi_M^R}{\partial w_n} = \frac{-\left(-\delta^2+\eta-\eta\,w_n+(-\delta-\eta)\,(-c_n+\lambda\,\rho+w_n)-\delta\,(c_r+\lambda-w_r)+\delta\,(1+A-w_n+w_r)\right)}{2\,((-1+\delta)\,\delta-\eta)} = 0$,
$\frac{\partial \Pi_M^R}{\partial w_r} = \frac{-(-A+c_r+\lambda+\delta\,w_n+\delta\,(-c_n+\lambda\,\rho+w_n)-2\,w_r)}{2\,((-1+\delta)\,\delta-\eta)} = 0$, and
$\frac{\partial \Pi_M^R}{\partial \lambda} = \frac{-\left(A-\delta\,w_n+w_r+\rho\left(-\delta^2+\eta-\eta\,w_n+\delta\,(1+A-w_n+w_r)\right)\right)}{2\,((-1+\delta)\,\delta-\eta)} = 0$.

**Case 1: $\lambda > 0$**

Solving the first-order conditions gives $w_n = \frac{1+c_n+c_n\,\delta\,\rho+\rho\,(A+c_r+((A+c_r-\delta)\,\delta+2\,\eta)\,\rho+2\,\delta\,(1+\rho))}{2+4\,\delta\,\rho+2\,(\delta+\eta)\,\rho^2}$, and $w_r = \frac{-2\,A+\delta+c_n\,\delta+(\delta\,(-1-3\,A+c_n+c_r+3\,\delta)+(-1+c_n)\,\eta)\,\rho-(A-c_r-\delta)\,(\delta+\eta)\,\rho^2}{2+4\,\delta\,\rho+2\,(\delta+\eta)\,\rho^2}$. The resulting sales quantities are $q_n = -\left(\frac{-1+c_n+(A+c_r-\delta)\,\rho}{4+8\,\delta\,\rho+4\,(\delta+\eta)\,\rho^2}\right)$, and $q_r = -\left(\frac{\rho\,(-1+c_n+(A+c_r-\delta)\,\rho)}{4+8\,\delta\,\rho+4\,(\delta+\eta)\,\rho^2}\right)$. It is easy to see that both $q_n$ and $q_r$ are positive when both margins are positive, i.e., $\delta > A + c_r$ and $c_n < 1$. The problem is unconstrained if $\lambda > 0$ or $\rho > \rho^R = \frac{A+c_r-c_n\,\delta}{(-1-A+c_n-c_r)\,\delta+\delta^2+(-1+c_n)\,\eta}$. The manufacturer profit is $\Pi_M^R = \frac{(-1+c_n+(A+c_r-\delta)\,\rho)^2}{8+8\,\rho\,(\eta\,\rho+\delta\,(2+\rho))}$. The retailer's profit in this case is given by $\Pi_R^R = \frac{(-1+c_n+(A+c_r-\delta)\,\rho)^2}{16\,(1+\eta\,\rho^2+\delta\,\rho\,(2+\rho))}$.

**Case 2: $\lambda = 0$**

Solving the first-order conditions at $\lambda = 0$ gives $w_n = \frac{1+c_n}{2}$ and $w_r = \frac{-A+c_r+\delta}{2}$. The resulting sales quantities are $q_n = \frac{(-1-A+c_n-c_r)\,\delta+\delta^2+(-1+c_n)\,\eta}{4\,(-1+\delta)\,\delta-4\,\eta}$ and $q_r = \frac{A+c_r-c_n\,\delta}{4\,(-1+\delta)\,\delta-4\,\eta}$. It is easy to see that both $q_n$ and $q_r$ are positive when $c_n < 1$ and $A + c_r < \delta c_n$. The manufacturer profit is $\Pi_R^M = \frac{(-1+c_n)^2+\frac{(A+c_r-c_n\,\delta)^2}{\delta-\delta^2+\eta}}{8}$. The retailer's profit in this case is given by $\Pi_R^M = \frac{-\left((-1+\delta)\,\delta\left(-3\,A^2+2\,A\,c_r+c_r{}^2+\left(1+c_n{}^2-2\,c_n\,(1+A+c_r)\right)\delta+(-1+2\,c_n)\,\delta^2\right)+(A-c_r+c_n\,\delta)^2\,\eta\right)}{16\,(-1+\delta)^2\,\delta^2}$.

### Comparison of Alternatives

We would like to know when the manufacturer makes more profit by collecting himself. To compare the profits we need to compare $\rho^R$ and $\rho^M$ first. It is easy to see that $\frac{\rho^M}{\rho^R} = \frac{2\,((1+A-c_n+c_r-\delta)\,\delta+\eta-c_n\,\eta)}{2\,(1+A-c_n+c_r-\delta)\,\delta+\eta-c_n\,\eta} > 1$. Thus, we can compare the profits depending on the value of $\rho$ as follows:

When $\rho \geq \rho^M$, we should compare the unconstrained profits. In this case, $\Pi_M^M - \Pi_M^R = \frac{(A+c_r-c_n\,\delta)^2\,\eta}{8\left(2\,(-1+\delta)^2\,\delta^2-3\,(-1+\delta)\,\delta\,\eta+\eta^2\right)}$. This term is always positive. Thus the manufacturer is always better off when he is collecting.

When $\rho < \rho^R$, we should compare the constrained profits. In this case, $\Pi_M^M - \Pi_M^R = (-1 + c_n + (A + c_r - \delta)\ \rho)^2 \left(\frac{1}{8+4\,\eta\,\rho^2+8\,\delta\,\rho\,(2+\rho)} - \frac{1}{8+8\,\rho\,(\eta\,\rho+\delta\,(2+\rho))}\right)$. Note that the term on the left hand side is always positive. The term on the right hand side within parentheses is also nonnegative since $(8 + 8\,\rho\ (\eta\,\rho + \delta\ (2 + \rho))) - (8 + 4\,\eta\,\rho^2 + 8\,\delta\,\rho\ (2 + \rho)) = 4\,\eta\,\rho^2 \geq 0$. Thus, the manufacturer is always better off collecting himself when $\rho \leq \rho^R$.

When $\rho^R \leq \rho < \rho^M$, the comparison of profits should be done between the case where the unconstrained manufacturer collects and where the constrained retailer collects. In this case, the manufacturer is always better off when he collects since $\Pi_M^M(unconstrained) > \Pi_M^M(constrained)$ by definition, and $\Pi_M^M(constrained) > \Pi_M^R(constrained)$ from the above result. Thus, $\Pi_M^M(unconstrained) > \Pi_M^R(constrained)$.

## Economies of Scale

The cost function used in this analysis is $C(\tau, q_n) = C_L\tau^2$. We note that neither party would optimally collect a higher fraction than what would be remanufactured since the collection cost increases in the collection rate. Thus, the constraint $q_r \leq \tau\rho q_n$ will always be binding in equilibrium, and we use the equality $q_r = \tau\rho q_n$ in the analysis where appropriate. We assume that the collection cost coefficient $C_L$ is sufficiently high such that the unconstrained optimizer satisfies $\tau \leq 1$.

**Manufacturer Collects**

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer's sales quantities for any given $(w_n, w_r)$ pair that maximize $\Pi_R^M = (p_n - w_n)q_n + (p_r - w_r)q_r$. The problem is jointly concave in $q_n$ and $q_r$. The first-order conditions can be written as $\frac{\partial \Pi_R^M}{\partial q_n} = 1 - 2\, q_n - 2\, \delta\, q_r - w_n = 0$, and $\frac{\partial \Pi_R^M}{\partial q_r} = -2\, \delta\, q_n - \delta\, (-1 + q_r) - \delta\, q_r - w_r = 0$. Solving the first-order conditions gives $q_n(w_n, w_r) = \frac{-(1-\delta-w_n+w_r)}{2\,(-1+\delta)}$ and $q_r(w_n, w_r) = \frac{-(\delta\, w_n - w_r)}{2\,(-1+\delta)\,\delta}$. Replacing the retailer's anticipated reaction in the manufacturer's objective, we obtain

$$\Pi_M^M = \frac{\delta\, \left(-2\, (-1+\delta)\, C_L\, \tau^2 + w_n\, (-1 + A + c_r + \delta + w_n)\right) - c_n\, \delta\, (-1 + \delta + w_n - w_r) - (A + c_r + 2\, \delta\, w_n)\, w_r + {w_r}^2}{2\, (-1+\delta)\, \delta}.$$

Thus, the Langrangean is written as $L(w_n, w_r, \tau, \lambda) = \Pi_M^M - \lambda(q_r(w_n, w_r) - q_n(w_n, w_r)\rho\tau)$. The first-order conditions are $\frac{\partial \Pi_M^M}{\partial w_n} = \frac{\delta(\delta+c_r+A-1)+\lambda+\lambda\rho\tau+2w_n-1)-2\delta w_r}{2(\delta-1)\delta} = 0$, $\frac{\partial \Pi_M^M}{\partial w_r} = \frac{-c_r-A-\delta-\lambda\rho\tau\delta-2w_n\delta-\lambda+2w_r}{2(\delta-1)\delta} = 0$, $\frac{\partial \Pi_M^M}{\partial \lambda} = \frac{\delta((\delta-1)\rho\tau+(\rho\tau+1)w_n)-(\delta\rho\tau+1)w_r}{2(\delta-1)\delta} = 0$, and $\frac{\partial \Pi_M^M}{\partial \tau} = \frac{\delta((1-\delta)(4C_L\tau-\lambda\rho)+\lambda\rho w_n)-\delta\lambda\rho w_r}{2(\delta-1)\delta} = 0$. The joint solution to these first-order conditions is not feasible in closed form. Multiple stationary points exist and finding the maximizer requires solving a fifth-order polynomial with respect to $\tau$. Nevertheless, a numerical solution can be obtained, which we use later to compare the two scenarios.

**Retailer Collects**

We solve the Stackelberg game sequentially. The manufacturer anticipates the retailer's sales quantities for a $(w_n, w_r)$ pair that maximize $\Pi_R^R = (p_n - w_n)q_n + (p_r - w_r)q_r - qrA - C_L\tau^2$. The first-order conditions on the retailer's objective are $\frac{\partial \Pi_R^R}{\partial q_n} = -2q_n - 2\delta q_r + \lambda\rho\tau - w_n + 1 = 0$, $\frac{\partial \Pi_R^R}{\partial q_r} = -A - \lambda - 2\delta q_n - \delta(q_r - 1) - \delta q_r - w_r = 0$, $\frac{\partial \Pi_R^R}{\partial \lambda} = q_n\rho\tau - q_r = 0$, and $\frac{\partial \Pi_R^R}{\partial \tau} = \lambda q_n \rho - 2C_L\tau = 0$. Unfortunately, the joint solution to these first-order conditions cannot be obtained in closed form and is not unique. Nevertheless, the retailer's best response curves are unique for any $\tau$. Thus, the manufacturer can anticipate the retailer's best response curves for any $\tau$. Solving the first three first-order conditions, we obtain $q_n(w_n, w_r|\tau) = -\frac{w_n+\rho\tau(A-\delta+w_r)-1}{2\delta\rho\tau(\rho\tau+2)+2}$, $q_r(w_n, w_r|\tau) = -\frac{\rho\tau(w_n+\rho\tau(A-\delta+w_r)-1)}{2\delta\rho\tau(\rho\tau+2)+2}$, and $\lambda(w_n, w_r|\tau) = -\frac{\delta\rho\tau A+A-\delta(w_n+\rho\tau(\delta+w_n-w_r-1))+w_r}{\delta\rho\tau(\rho\tau+2)+1}$. Anticipating the retailer's decision for a given $\tau$, the manufacturer's objective is $\Pi_M^M(w_n, w_r|\tau) = \frac{(c_n-w_n+\rho\tau(c_r-w_r))(w_n+\rho\tau(A-\delta+w_r)-1)}{2\delta\rho\tau(\rho\tau+2)+2}$. It is easy to see that the manufacturer's objective $\Pi_M^M(w_n, w_r|\tau)$ can be written as a function of $w_n + w_r\tau\rho$, i.e., $w_n$ and $w_r$ are linearly dependent in the manufacturer's objective. Thus, any $w_n$, $w_r$ pair that satisfies $w_n = \frac{1+c_n}{2} + \rho\tau\frac{(c_r-w_r)+(\delta-w_r)}{2}$ characterizes the manufacturer's best response. Consequently, the manufacturer announces wholesale prices as a function of the collection rate $w_n(\tau) = \frac{1+c_n}{2} + \rho\tau\frac{(c_r-w_r(\tau))+(\delta-w_r(\tau))}{2}$. (Note that this is equivalent to the effective wholesale price ($w - \tau b$) in the perfect substitution scenario.) This induces the retailer to sell $q_n(\tau) = \frac{-c_n+(A-c_r+\delta)\rho\tau+1}{4\delta\rho\tau(\rho\tau+2)+4}$ and $q_r(\tau) = \frac{\rho\tau(-c_n+(A-c_r+\delta)\rho\tau+1)}{4\delta\rho\tau(\rho\tau+2)+4}$. As in the manufacturer collection scenario, a closed-form characterization of the optimal $\tau$ is not possible, since it requires solving a fifth-order polynomial. Nevertheless, numerically this task is not as challenging as it is analytically.

**Comparison of Alternatives**

Due to the complicated structure of the profit functions and the related parameter set restrictions, it is hard to characterize the manufacturer profit difference between the two collection strategies. However, a numerical analysis reveals that collection at the retailer dominates under economies of scale. Tables 17 and 18 in Appendix C illustrate this finding for with a full factorial analysis for $c_n = 0.4, 0.2$, $\delta = 0.2, 0.5, 0.6, 0.8$, $C_L = 0.008, 0.005$, $\rho = 0.8, 0.4$ and $\gamma = 0.8, 0.6$, where $c_r = c_n\delta\gamma$. These parameter settings are chosen to ensure that both the manufacturer and the retailer choose a positive collection rate $\tau$, new product sales $q_n$ and remanufactured product sales $q_r$.

# Appendix C: Numerical Results

## Numerical Analysis for Observation 1

| Manufacturer Collection |
|---|
| $(w^* = \frac{1+c_m}{2}, p^* = \frac{3+c_m}{4}, \tau^* = 0)$ or<br>$(w^* = \arg\max_w (w - c_m + \Delta - A)q(w) - \eta(q(w))^k, p^* = \frac{1+w^*}{2}, \tau^* = 1)$ |
| Retailer Collection |
| $(w^*, b^*)$ obtained numerically where the retailer's best response $(p(w,b), \tau(w,b))$ is<br>$(\frac{1+w}{2}, 0)$ or $(\arg\max_p (p - w + b - A)q(p) - \eta(q(p))^k, 1)$ |
| 3rd Party Collection |
| $(w^*, b^*)$ obtained numerically where the retailer's best response $p(w) = (1+w)/2$<br>and the third party's best response is<br>$\tau(w,b) = 1 \Leftarrow (b - A)q(w) - \eta(q(w))^k \geq 0$ and $\tau(b) = 0$ otherwise. |

Table 10: Manufacturer, Retailer and 3rd Party Collection with Economies of Scale

Following the proof for Proposition 1, the resulting optimization problems for the three scenarios are summarized in Table 10. Unfortunately, a solution to the embedded optimization problems cannot be obtained in closed form for general $k < 1$ values because of the fact that the collecting party's best response functions cannot be characterized in closed form. However, it is possible to do so numerically. Our numerical results reveal that when $k < 1$, retailer collection is optimal from the manufacturer's perspective. The parameter levels chosen are $c_m = \{0.2, 0.5, 0.8\}$, $\Delta = \{0.2c_m, 0.5c_m, 0.8c_m\}$, $A = \{0.2\Delta, 0.5\Delta, 0.8\Delta\}$, $\eta = \{0.01, 0.1, 1\}$, $\phi = \{0.5, 1, 5\}$ and $k = \{0.2, 0.5, 0.8\}$, and cover a broad range of possible outcomes. Results from a representative sample are tabulated in Table 11, which shows that the manufacturer is better off with retailer collection when $k < 1$. The full set of experiments is available from the authors.

Table 11: (Observation 1) Manufacturer profits under different collection channels for $k < 1$. Representative results from numerical experiments carried out with $c_m = \{0.2, 0.5, 0.8\}$, $\Delta = \{0.2c_m, 0.5c_m, 0.8c_m\}$, $A = \{0.2\Delta, 0.5\Delta, 0.8\Delta\}$, $\eta = \{0.01, 0.1, 1\}$, $\phi = \{0.5, 1, 5\}$ and $k = \{0.2, 0.5, 0.8\}$.

| $c_m$ | $\Delta$ | $A$ | $\eta$ | $k$ | $\phi$ | $\Pi_M^M$ | $\Pi_M^R$ | $\Pi_M^{3P}$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 0.5 | 0.015625 | 0.017735 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 1 | 0.03125 | 0.03647 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 5 | 0.179789 | 0.187208 | 0.17889 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 0.5 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 1 | 0.03125 | 0.03125 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 5 | 0.15625 | 0.170526 | 0.15625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 0.5 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 1 | 0.03125 | 0.03125 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 5 | 0.15625 | 0.15625 | 0.15625 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 0.5 | 0.016307 | 0.017603 | 0.016256 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 1 | 0.034127 | 0.035966 | 0.0338 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 5 | 0.180794 | 0.184923 | 0.17889 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 0.5 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 1 | 0.03125 | 0.03125 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 5 | 0.15625 | 0.15625 | 0.15625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 0.5 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 1 | 0.03125 | 0.03125 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 5 | 0.15625 | 0.15625 | 0.15625 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 0.5 | 0.017744 | 0.017974 | 0.017556 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 1 | 0.035786 | 0.036188 | 0.035778 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 5 | 0.181699 | 0.183165 | 0.17889 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 0.5 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 1 | 0.03125 | 0.03125 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 5 | 0.15625 | 0.15625 | 0.15625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 0.5 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 1 | 0.03125 | 0.03125 | 0.03125 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 5 | 0.15625 | 0.15625 | 0.15625 |

Table 12: (Observation 1) Sales prices under different collection channels for $k < 1$. Representative results from numerical experiments carried out with $c_m = \{0.2, 0.5, 0.8\}$, $\Delta = \{0.2c_m, 0.5c_m, 0.8c_m\}$, $A = \{0.2\Delta, 0.5\Delta, 0.8\Delta\}$, $\eta = \{0.01, 0.1, 1\}$, $\phi = \{0.5, 1, 5\}$ and $k = \{0.2, 0.5, 0.8\}$.

| $c_m$ | $\Delta$ | $A$ | $\eta$ | $k$ | $\phi$ | $p^M$ | $p^R$ | $p^{3P}$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 0.5 | 0.875 | 0.863574 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 1 | 0.875 | 0.862893 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 5 | 0.863169 | 0.862852 | 0.866 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 0.5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 1 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 5 | 0.875 | 0.864123 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 0.5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 1 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 0.5 | 0.867354 | 0.865115 | 0.8725 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 1 | 0.865914 | 0.86428 | 0.87 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 5 | 0.864009 | 0.86302 | 0.866 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 0.5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 1 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 0.5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 1 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 0.5 | 0.865941 | 0.865361 | 0.8675 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 1 | 0.865493 | 0.864969 | 0.866 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 5 | 0.864662 | 0.864323 | 0.866 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 0.5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 1 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 0.5 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 1 | 0.875 | 0.875 | 0.875 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 5 | 0.875 | 0.875 | 0.875 |

Table 13: (Observation 1) Collection rates under different collection channels for $k < 1$. Representative results from numerical experiments carried out with $c_m = \{0.2, 0.5, 0.8\}$, $\Delta = \{0.2c_m, 0.5c_m, 0.8c_m\}$, $A = \{0.2\Delta, 0.5\Delta, 0.8\Delta\}$, $\eta = \{0.01, 0.1, 1\}$, $\phi = \{0.5, 1, 5\}$ and $k = \{0.2, 0.5, 0.8\}$.

| $c_m$ | $\Delta$ | $A$ | $\eta$ | $k$ | $\phi$ | $\tau^M$ | $\tau^R$ | $\tau^{3P}$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 0.5 | 0 | 1 | 0 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 1 | 0 | 1 | 0 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 5 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 0.5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 1 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 5 | 0 | 1 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 0.5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 1 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 0.5 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 1 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 5 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 0.5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 1 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 0.5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 1 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 0.5 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 1 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 5 | 1 | 1 | 1 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 0.5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 1 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 0.5 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 1 | 0 | 0 | 0 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 5 | 0 | 0 | 0 |

Table 14: (Observation 2) Retailer profits under different collection channels for $k < 1$. Representative results from numerical experiments carried out with $c_m = \{0.2, 0.5, 0.8\}$, $\Delta = \{0.2c_m, 0.5c_m, 0.8c_m\}$, $A = \{0.2\Delta, 0.5\Delta, 0.8\Delta\}$, $\eta = \{0.01, 0.1, 1\}$, $\phi = \{0.5, 1, 5\}$ and $k = \{0.2, 0.5, 0.8\}$.

| $c_m$ | $\Delta$ | $A$ | $\eta$ | $k$ | $\phi$ | $\Pi_R^M$ | $\Pi_R^R$ | $\Pi_R^{3P}$ |
|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 0.5 | 0.007813 | 0.004631 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 1 | 0.015625 | 0.013419 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.2 | 5 | 0.093613 | 0.086629 | 0.08978 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 0.5 | 0.007813 | 0.007813 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 1 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.2 | 5 | 0.078125 | 0.018263 | 0.078125 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 0.5 | 0.007813 | 0.007813 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 1 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.2 | 5 | 0.078125 | 0.078125 | 0.078125 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 0.5 | 0.008798 | 0.007797 | 0.008128 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 1 | 0.017979 | 0.016576 | 0.0169 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.5 | 5 | 0.092468 | 0.089679 | 0.08978 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 0.5 | 0.007813 | 0.007813 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 1 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.5 | 5 | 0.078125 | 0.078125 | 0.078125 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 0.5 | 0.007813 | 0.007813 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 1 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.5 | 5 | 0.078125 | 0.078125 | 0.078125 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 0.5 | 0.008986 | 0.008833 | 0.008778 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 1 | 0.018092 | 0.01783 | 0.017956 |
| 0.5 | 0.1 | 0.05 | 0.01 | 0.8 | 5 | 0.091582 | 0.090575 | 0.08978 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 0.5 | 0.007813 | 0.007813 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 1 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 0.1 | 0.8 | 5 | 0.078125 | 0.078125 | 0.078125 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 0.5 | 0.007813 | 0.007813 | 0.007813 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 1 | 0.015625 | 0.015625 | 0.015625 |
| 0.5 | 0.1 | 0.05 | 1 | 0.8 | 5 | 0.078125 | 0.078125 | 0.078125 |

Table 15: Numerical example to demonstrate the impact of diseconomies of scale on manufacturer's reverse channel choice without decoupling. Manufacturer profits under different collection channels are presented for $k > 1$ and $\eta$ such that $\tau^* = 1$.

| $c_m$ | $\Delta$ | $A$ | $\eta$ | $k$ | $\phi$ | $\Pi_M^M$ | $\Pi_M^R$ |
|---|---|---|---|---|---|---|---|
| 0.5 | 0.1 | 0.02 | 104.8576 | 5 | 0.5 | 0.020827 | 0.020192 |
| 0.5 | 0.1 | 0.02 | 6.5536 | 5 | 1 | 0.041654 | 0.040383 |
| 0.5 | 0.1 | 0.02 | 0.010486 | 5 | 5 | 0.208269 | 0.201915 |
| 0.5 | 0.1 | 0.02 | 167772.2 | 8 | 0.5 | 0.020908 | 0.020375 |
| 0.5 | 0.1 | 0.02 | 1310.72 | 8 | 1 | 0.041815 | 0.040751 |
| 0.5 | 0.1 | 0.02 | 0.016777 | 8 | 5 | 0.209077 | 0.203754 |
| 0.5 | 0.1 | 0.05 | 0.1 | 2 | 0.5 | 0.018445 | 0.018 |
| 0.5 | 0.1 | 0.05 | 0.05 | 2 | 1 | 0.03689 | 0.036 |
| 0.5 | 0.1 | 0.05 | 0.01 | 2 | 5 | 0.184451 | 0.18 |
| 0.5 | 0.1 | 0.05 | 65.536 | 5 | 0.5 | 0.018809 | 0.018466 |
| 0.5 | 0.1 | 0.05 | 4.096 | 5 | 1 | 0.037618 | 0.036932 |
| 0.5 | 0.1 | 0.05 | 0.006554 | 5 | 5 | 0.188088 | 0.184662 |
| 0.5 | 0.1 | 0.05 | 104857.6 | 8 | 0.5 | 0.018856 | 0.018581 |
| 0.5 | 0.1 | 0.05 | 819.2 | 8 | 1 | 0.037712 | 0.037162 |
| 0.5 | 0.1 | 0.05 | 0.010486 | 8 | 5 | 0.188561 | 0.185809 |
| 0.5 | 0.1 | 0.08 | 0.04 | 2 | 0.5 | 0.016733 | 0.016569 |
| 0.5 | 0.1 | 0.08 | 0.02 | 2 | 1 | 0.033465 | 0.033137 |
| 0.5 | 0.1 | 0.08 | 0.004 | 2 | 5 | 0.167327 | 0.165686 |
| 0.5 | 0.1 | 0.08 | 26.2144 | 5 | 0.5 | 0.01687 | 0.016753 |
| 0.5 | 0.1 | 0.08 | 1.6384 | 5 | 1 | 0.03374 | 0.033506 |
| 0.5 | 0.1 | 0.08 | 0.002621 | 5 | 5 | 0.168699 | 0.16753 |
| 0.5 | 0.1 | 0.08 | 41943.04 | 8 | 0.5 | 0.016887 | 0.016796 |
| 0.5 | 0.1 | 0.08 | 327.68 | 8 | 1 | 0.033774 | 0.033593 |
| 0.5 | 0.1 | 0.08 | 0.004194 | 8 | 5 | 0.168868 | 0.167965 |

Table 16: The effect of investment cost ($C_L\tau^l$) on the manufacturer's reverse channel choice for different levels of $l$.

| $c_m$ | $A$ | $\phi$ | $\eta$ | $C_L$ | $l$ | $\Delta$ | $\Pi_M^M - \Pi_M^R$ |
|---|---|---|---|---|---|---|---|
| 0.5 | 0.05 | 1 | 0 | 0.1 | 1 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 1 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 1 | 2 | -0.00017 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 1 | 4 | -0.00091 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 2 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 2 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 2 | 2 | -0.00015 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 2 | 4 | -0.00066 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 3 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 3 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 3 | 2 | -0.00015 |
| 0.5 | 0.05 | 1 | 0 | 0.1 | 3 | 4 | -0.0006 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 1 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 1 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 1 | 2 | -0.00057 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 1 | 4 | -0.00354 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 2 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 2 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 2 | 2 | -0.00027 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 2 | 4 | -0.00261 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 3 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 3 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 3 | 2 | -0.00024 |
| 0.5 | 0.05 | 1 | 0 | 0.25 | 3 | 4 | -0.00221 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 1 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 1 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 1 | 2 | -0.00128 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 1 | 4 | -0.00703 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 2 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 2 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 2 | 2 | -0.0006 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 2 | 4 | -0.00497 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 3 | 0.5 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 3 | 1 | 0 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 3 | 2 | -0.00032 |
| 0.5 | 0.05 | 1 | 0 | 0.4 | 3 | 4 | -0.00417 |

Table 17: Scale economies under product differentiation: Comparison of manufacturer and retailer collection scenarios under economies of scale for intermediate $\delta$ values

| $c_n$ | $\delta$ | $\gamma$ | $c_r$ | $C_L$ | $\rho$ | $\Pi_M^M$ | $\Pi_M^R$ |
|---|---|---|---|---|---|---|---|
| 0.4 | 0.6 | 0.8 | 0.192 | 0.008 | 0.8 | 0.0455073 | 0.0455593 |
| 0.4 | 0.6 | 0.8 | 0.192 | 0.008 | 0.4 | 0.0452022 | 0.0452126 |
| 0.4 | 0.6 | 0.8 | 0.192 | 0.005 | 0.8 | 0.0456341 | 0.0457054 |
| 0.4 | 0.6 | 0.8 | 0.192 | 0.005 | 0.4 | 0.0452877 | 0.0453077 |
| 0.4 | 0.6 | 0.6 | 0.144 | 0.008 | 0.8 | 0.0468785 | 0.0470701 |
| 0.4 | 0.6 | 0.6 | 0.144 | 0.008 | 0.4 | 0.0457748 | 0.0458193 |
| 0.4 | 0.6 | 0.6 | 0.144 | 0.005 | 0.8 | 0.0473352 | 0.0475884 |
| 0.4 | 0.6 | 0.6 | 0.144 | 0.005 | 0.4 | 0.0460884 | 0.0461695 |
| 0.4 | 0.5 | 0.8 | 0.16 | 0.008 | 0.8 | 0.045356 | 0.0453932 |
| 0.4 | 0.5 | 0.8 | 0.16 | 0.008 | 0.4 | 0.0451417 | 0.0451488 |
| 0.4 | 0.5 | 0.8 | 0.16 | 0.005 | 0.8 | 0.0454435 | 0.0454947 |
| 0.4 | 0.5 | 0.8 | 0.16 | 0.005 | 0.4 | 0.0452019 | 0.0452159 |
| 0.4 | 0.5 | 0.6 | 0.12 | 0.008 | 0.8 | 0.0463485 | 0.0464882 |
| 0.4 | 0.5 | 0.6 | 0.12 | 0.008 | 0.4 | 0.0455495 | 0.0455797 |
| 0.4 | 0.5 | 0.6 | 0.12 | 0.005 | 0.8 | 0.0466757 | 0.0468627 |
| 0.4 | 0.5 | 0.6 | 0.12 | 0.005 | 0.4 | 0.0457762 | 0.0458326 |
| 0.2 | 0.6 | 0.8 | 0.096 | 0.008 | 0.8 | 0.0801754 | 0.0801962 |
| 0.2 | 0.6 | 0.8 | 0.096 | 0.008 | 0.4 | 0.0800809 | 0.0800865 |
| 0.2 | 0.6 | 0.8 | 0.096 | 0.005 | 0.8 | 0.0802067 | 0.0802314 |
| 0.2 | 0.6 | 0.8 | 0.096 | 0.005 | 0.4 | 0.0801103 | 0.0801203 |
| 0.2 | 0.6 | 0.6 | 0.072 | 0.008 | 0.8 | 0.0806775 | 0.0807557 |
| 0.2 | 0.6 | 0.6 | 0.072 | 0.008 | 0.4 | 0.0803149 | 0.0803377 |
| 0.2 | 0.6 | 0.6 | 0.072 | 0.005 | 0.8 | 0.0808007 | 0.0808936 |
| 0.2 | 0.6 | 0.6 | 0.072 | 0.005 | 0.4 | 0.0804276 | 0.0804665 |
| 0.2 | 0.5 | 0.8 | 0.08 | 0.008 | 0.8 | 0.0801202 | 0.0801347 |
| 0.2 | 0.5 | 0.8 | 0.08 | 0.008 | 0.4 | 0.080056 | 0.08006 |
| 0.2 | 0.5 | 0.8 | 0.08 | 0.005 | 0.8 | 0.0801409 | 0.0801579 |
| 0.2 | 0.5 | 0.8 | 0.08 | 0.005 | 0.4 | 0.0800763 | 0.0800833 |
| 0.2 | 0.5 | 0.6 | 0.06 | 0.008 | 0.8 | 0.0804699 | 0.0805254 |
| 0.2 | 0.5 | 0.6 | 0.06 | 0.008 | 0.4 | 0.08022 | 0.0802359 |
| 0.2 | 0.5 | 0.6 | 0.06 | 0.005 | 0.8 | 0.0805521 | 0.0806173 |
| 0.2 | 0.5 | 0.6 | 0.06 | 0.005 | 0.4 | 0.0802985 | 0.080326 |

Table 18: Scale economies under product differentiation: Comparison of manufacturer and retailer collection scenarios under economies of scale for high and low $\delta$ values

| $c_n$ | $\delta$ | $\gamma$ | $c_r$ | $C_L$ | $\rho$ | $\Pi_M^M$ | $\Pi_M^R$ |
|---|---|---|---|---|---|---|---|
| 0.4 | 0.8 | 0.8 | 0.192 | 0.008 | 0.8 | 0.0459509 | 0.046039 |
| 0.4 | 0.8 | 0.8 | 0.192 | 0.008 | 0.4 | 0.0453641 | 0.0453816 |
| 0.4 | 0.8 | 0.8 | 0.192 | 0.005 | 0.8 | 0.0462186 | 0.046343 |
| 0.4 | 0.8 | 0.8 | 0.192 | 0.005 | 0.4 | 0.0455226 | 0.0455559 |
| 0.4 | 0.8 | 0.6 | 0.144 | 0.008 | 0.8 | 0.0482852 | 0.0486026 |
| 0.4 | 0.8 | 0.6 | 0.144 | 0.008 | 0.4 | 0.0463502 | 0.0464292 |
| 0.4 | 0.8 | 0.6 | 0.144 | 0.005 | 0.8 | 0.0491257 | 0.0495446 |
| 0.4 | 0.8 | 0.6 | 0.144 | 0.005 | 0.4 | 0.0468926 | 0.0470323 |
| 0.4 | 0.2 | 0.8 | 0.16 | 0.008 | 0.8 | 0.045072 | 0.0450783 |
| 0.4 | 0.2 | 0.8 | 0.16 | 0.008 | 0.4 | 0.045025 | 0.0450258 |
| 0.4 | 0.2 | 0.8 | 0.16 | 0.005 | 0.8 | 0.0450944 | 0.0451045 |
| 0.4 | 0.2 | 0.8 | 0.16 | 0.005 | 0.4 | 0.0450371 | 0.0450389 |
| 0.4 | 0.2 | 0.6 | 0.12 | 0.008 | 0.8 | 0.0452844 | 0.0453093 |
| 0.4 | 0.2 | 0.6 | 0.12 | 0.008 | 0.4 | 0.0450994 | 0.0451028 |
| 0.4 | 0.2 | 0.6 | 0.12 | 0.005 | 0.8 | 0.0453721 | 0.0454118 |
| 0.4 | 0.2 | 0.6 | 0.12 | 0.005 | 0.4 | 0.0451472 | 0.0451544 |
| 0.2 | 0.8 | 0.8 | 0.096 | 0.008 | 0.8 | 0.0803678 | 0.080407 |
| 0.2 | 0.8 | 0.8 | 0.096 | 0.008 | 0.4 | 0.0801535 | 0.0801624 |
| 0.2 | 0.8 | 0.8 | 0.096 | 0.005 | 0.8 | 0.0804523 | 0.0805038 |
| 0.2 | 0.8 | 0.8 | 0.096 | 0.005 | 0.4 | 0.0802153 | 0.0802317 |
| 0.2 | 0.8 | 0.6 | 0.072 | 0.008 | 0.8 | 0.0813496 | 0.0814904 |
| 0.2 | 0.8 | 0.6 | 0.072 | 0.008 | 0.4 | 0.0805823 | 0.0806195 |
| 0.2 | 0.8 | 0.6 | 0.072 | 0.005 | 0.8 | 0.0816547 | 0.0818339 |
| 0.2 | 0.8 | 0.6 | 0.072 | 0.005 | 0.4 | 0.0808058 | 0.0808707 |
| 0.2 | 0.2 | 0.8 | 0.08 | 0.008 | 0.8 | 0.0800251 | 0.0800279 |
| 0.2 | 0.2 | 0.8 | 0.08 | 0.008 | 0.4 | 0.0800102 | 0.0800107 |
| 0.2 | 0.2 | 0.8 | 0.08 | 0.005 | 0.8 | 0.0800309 | 0.0800347 |
| 0.2 | 0.2 | 0.8 | 0.08 | 0.005 | 0.4 | 0.0800145 | 0.0800155 |
| 0.2 | 0.2 | 0.6 | 0.06 | 0.008 | 0.8 | 0.0801 | 0.0801111 |
| 0.2 | 0.2 | 0.6 | 0.06 | 0.008 | 0.4 | 0.0800404 | 0.0800425 |
| 0.2 | 0.2 | 0.6 | 0.06 | 0.005 | 0.8 | 0.0801228 | 0.0801378 |
| 0.2 | 0.2 | 0.6 | 0.06 | 0.005 | 0.4 | 0.0800575 | 0.0800616 |