Abstract
Motivated by the mixed results of collaborative forecasting initiatives in the consumer goods sector, this paper investigates the conditions that favor the establishment of collaborative forecasting between a supplier and a retailer. We consider a two-stage supply chain where a single supplier sells to a single retailer who faces the newsvendor problem. Both the supplier and the retailer have forecasting capabilities and both can exert costly effort to improve the quality of their local demand forecasts. We assume that the supplier and the retailer exert effort independently and then pool their local forecasts to form a single shared demand forecast. We start by characterizing the conditions under which a joint forecasting equilibrium exists in a one shot investment game. We then consider a repeated investment game where the supplier and the retailer play the one shot investment game repeatedly. We interpret the existence of an equilibrium in which both parties invest in improving the forecast quality in the repeated investment game as collaborative forecasting being sustainable. We characterize conditions under which such an equilibrium exists. Our results have implications concerning the appropriateness of investing in collaboration technology to extract and use information from both parties in a supply chain.

Key words: supply chain management, demand forecast collaboration, CPFR, information exchange, forecast quality.
1 Introduction

Several industry-specific supply chain initiatives emphasizing the importance of collaboration across levels of the supply chain have emerged over the recent years. Collaborative Planning, Forecasting, and Replenishment (CPFR) is one such initiative in the consumer goods sector. The goal of this initiative is to create standards for inter-firm information exchange and collaboration with specific emphasis on sharing and reconciling demand forecasts. The CPFR website (www.cpfr.org) gives many examples of CPFR implementations. For example, Metro-Procter & Gamble is one initiative where both partners reported positive benefits and implemented CPFR on a large scale (GNX Case Studies 2002). However, most collaboration initiatives in practice have not gone beyond the pilot stage. A recent study (GMA News Releases 2002) conducted for Grocery Manufacturers of America (GMA) reveals that

67% of GMA member companies are engaged in some form of collaborative planning forecasting and replenishment (CPFR) activity, with only 19% moving beyond pilot studies to implement CPFR with their trading partners.

Motivated by this phenomenon in the fast-moving consumer goods sector, this paper studies the conditions that favor the establishment of collaborative forecasting between a supplier and a retailer. Existing research on supply chain forecast collaboration (Aviv 2001, 2002) assumes that the quality of information available to the supply chain partners is exogenously given. Under this assumption, forecast collaboration always leaves both parties better off, which falls short of explaining the lack of wide-spread adoption of CPFR in practice.

The goal of our research is to develop a more complete understanding of conditions that favor the emergence of collaborative forecasting between trading partners. The high initial cost of setting up a collaboration platform is often cited as a barrier to CPFR implementation. The trade literature also underlines the importance of barriers related to the state of internal processes and to the strategic interaction between partners. In particular, in order to create and implement a successful collaboration, firms must internally invest in obtaining relevant data, improving data quality and integrity, and generating forecasts. In addition, since any improvement by one party benefits both parties, reliance on the other party is to be expected. In this paper, we will investigate the impact of the latter two factors, which are more subtle, but potentially as important in explaining the
limited adoption of supplier-retailer collaborative forecasting practices. Our findings point to conditions under which joint forecasting is likely to be successful.

To this end, we consider a supply chain in which a single supplier sells to a single retailer who faces uncertain demand. Our model contributes to the supply chain literature on collaborative forecasting as follows. First, we assume that both the supplier and the retailer have forecasting capabilities and that both can exert costly effort to improve the quality of their local demand forecasts. This assumption allows us to capture the effort needed for obtaining relevant data, improving data quality and generating forecasts, in contrast to the existing research which assumes that the forecast quality is exogenously given (Aviv 2001, 2002).

Second, we assume that the supplier and the retailer exert effort independently and then pool their demand forecasts to form a single shared demand forecast. This assumption mimics the basic CPFR process, where individual forecasts are compared and differences are resolved through information sharing.

Finally, we capture the existence of strategic interaction at the investment stage in contrast to existing research on supply chain games under asymmetric demand information that focuses only on the information sharing stage (Cachon and Lariviere 2001, Özer and Wei 2006, and Miyaoka 2003). In particular, we assume that the effort exerted into forecasting is observable but noncontractible. Building on existing research that identifies contracts that induce truthful demand forecast sharing, our research analyzes supply chain partners’ incentives to invest in developing better demand forecasts assuming contracts that ensure truthful information sharing are in place.

Our analysis proceeds as follows: First, we characterize the equilibria of the one-shot investment game. Second, we characterize the equilibria of the repeated investment game. We interpret the existence of a joint forecasting equilibrium in the repeated investment game as collaborative forecasting being sustainable. We investigate the most important factors that impact the supplier’s and the retailer’s investment decisions and characterize conditions under which collaborative forecasting is sustainable. Our results have implications concerning the appropriateness of investing in collaboration technology to extract and use local information from both parties.

The main contributions of our research to the literature on collaborative forecasting can be summarized as follows: Existing research (Aviv 2001, 2002) implicitly assumes that if both downstream and upstream partners in a supply chain have capabilities to do
forecasting, both would invest and improve the demand forecast. However, we show that when the effort that supply chain partners exert into forecasting is considered as a strategic variable, this is not the case.

Our results can be summarized as follows: First, we find that when the incentives to invest are balanced, joint investments are more likely to be sustained because the effective value derived from participating into joint forecasting is comparable for each partner. Second, we find that joint forecasting is sustainable when supply chain partners’ forecasting capabilities are intermediate. When forecasting capabilities are small, the investments are not justifiable as the cost exceeds the benefits due to a better demand forecast. On the other hand, when forecasting capabilities are large, it might be sufficient for only one of the partners to invest, therefore eliminating the need for joint forecasting. Third, the investment game is more likely to result in a joint forecasting equilibrium when the supply chain partners have complementary information.

The rest of the paper is organized as follows. In §2, we review the related literature and position our research. In §3, we describe the model and discuss our assumptions. Then, in §4 we analyze the one-shot investment game and study the resulting equilibria. In §5, we analyze the repeated investment game and characterize the possible equilibrium outcomes. Finally, §6 discusses the results and managerial implications that can be drawn from our research.

2 Literature Review

There is a vast literature on forecasting and inventory management whose main focus is incorporating forecast information into inventory management decisions. Due to the focus of our paper, we position it with respect to research that incorporates forecast combination and/or collaboration, and game theoretic models with asymmetric demand information.

The value of forecast collaboration has been studied by Aviv (2001, 2002). Aviv (2001) assumes that both supply chain partners have private information regarding the demand and that they jointly maintain and update a single forecasting process. Aviv assumes that the forecast quality at each stage of the supply chain is fixed; the possibility of improving the quality of forecasts is not considered. He also assumes that parties share their private information in a truthful manner. The paper shows that under these conditions, joint forecasting practices may provide substantial benefits to the supply chain and concludes that it is mainly the correlation between trading partners’ forecasts that matters; in other
words, “it is whether or not the trading partners can bring something unique to the table.” Aviv (2002) extends this model to the case of autocorrelated demand. In contrast to these papers, forecast quality is endogenously determined in our model. In addition, the final forecast quality is not only determined by the correlation in forecasts but also the cost of effort, the forecasting capability and the strategic interaction that occurs at the investment stage. Our results complement Aviv’s results regarding the value of collaborative forecasting practices.

The willingness of the supply chain partners to share their demand forecasts truthfully in a collaborative forecasting setting has been studied by Miyaoka (2003). The paper compares the incentives for truthful information sharing under price only contracts, buy back contracts and quantity flexibility contracts. It shows that only a coordinating buy-back contract can achieve credible information sharing and induce each partner to act in the best interest of the supply chain. Building on Miyaoka, we assume that the supply chain partners operate under a contract that gives them an incentive to share their demand forecasts truthfully and focus on their incentives to invest in improving their demand forecasts. In the absence of such a contract structure, joint forecasting is not viable.

In our model, parties’ demand forecasts may differ because each party may choose a different level of investment or have a different capability. Asymmetric demand information has been modelled in Cachon and Lariviere (2001), Özer and Wei (2006), and Lariviere (2002), among others. In these papers, the downstream party has private information regarding the market demand that is not available to the upstream party. The focus is on the design of mechanisms – signalling (Cachon and Lariviere, Özer and Wei) and screening (Özer and Wei, Lariviere) – that induce truthful information sharing. As in our model, Lariviere assumes that the downstream retailer can acquire costly information about the demand. The supplier, in turn, would like to induce the retailer to invest in forecasting and to share that information. Lariviere considers a supplier who offers price-based (buy-back) and quantity-based (quantity-flexibility) contracts to the retailer and compares the relative performance of the two schemes. The paper shows that with a buy-back contract, the supplier must sacrifice some profit whereas with a quantity-flexibility contract, the supplier can screen the retailers that exert effort without sacrificing channel efficiency. In this paper, we assume that a contract structure that incentivizes both parties to truthfully reveal their information is in place and focus on the outcome of the subsequent investment decisions by both partners.
The economics literature has studied the problem of information sharing in oligopolistic markets. Representative papers in this area are Novshek and Sonnenschein (1982), Vives (1984), Gal-Or (1985), and Li et al. (1987). Firms in an oligopoly face uncertain demand for their products and each firm observes a private signal about the demand before choosing the production quantity (or price). All of these papers, except Li et al., assume that the accuracy of the information that is observed by the firms is exogenous and address the question of incentives for information sharing. As in our model, Li et al. assume that information acquisition is costly and address the question of incentives for costly information acquisition. Our model takes a supply chain perspective and assumes that different firms are responsible for production and ordering, whereas the models above assume that production and ordering take place within the same firm.

Another stream of literature related to our research is the literature on free-riding in teams. Free-riding arises when the joint output is the only observable indicator of team members’ inputs and this output has to be shared among the team members. Free riding and non cooperative behavior imply that an inefficient outcome will be achieved. The focus of research (e.g., Holstrom 1982) is to identify ways in which the first best outcome can be achieved. A research stream that is particularly relevant to ours is the research on R&D joint venture formations (see Katz 1995 for a review). Similar to the forecasting efforts in our model, R&D effort is observable but unverifiable, giving rise to the possibility of free riding in R&D joint venture formations. Our focus is different in that we focus on characterizing the conditions under which joint investments into improving the demand forecast are sustainable in a supply chain setting.

Finally, the forecasting model that we employ is based on Clemen and Winkler (1985) and Osband (1989). Clemen and Winkler consider a model where a decision maker is interested in estimating an uncertain parameter. The decision maker has access to dependent information sources (experts) who can provide the decision maker with information regarding the realization of an uncertain event. The model studies the impact of dependence on the precision and value of information sources. Clemen and Winkler assume that the precision of the information from the experts is exogenous, whereas we model the precision endogenously. Osband studies the incentives for forecasting in a principal-agent framework where an agent of unknown expertise is requested to estimate an uncertain outcome and the agent can refine the forecast at a constant marginal cost per unit of precision. The paper studies the principal’s contract design problem. Our forecasting model combines the idea
of information from dependent sources and the idea of refining the precision of the forecast at some cost.

3 The Model

The model we develop is based on the basic CPFR collaboration template. This template works as follows (Seifert 2001): Both the retailer and the supplier enter their sales forecasts into the collaboration platform. If the difference between the supplier’s and the retailer’s forecasts for a given product is more than a predetermined level, the system flags that product and notifies the relevant managers. These managers then discuss their forecasts, share their assumptions, and arrive at a final demand forecast. We use a stylized Bayesian updating model to mimic exception resolution in the CPFR process. Production and stocking are based on this single shared demand forecast. The forecast collaboration is implemented on an ongoing basis in the context of a broader commercial framework concerning pricing and assortment negotiated between the retailer and the supplier, typically on a yearly basis.

We consider a two-stage supply chain where a single supplier sells to a single retailer who faces the newsvendor problem. As discussed in the introduction, our focus is whether forecast collaboration is sustainable. We therefore assume that a platform that allows the supply chain partners to exchange information is already in place and focus on the nature of the subsequent collaboration. The important elements of our model can be summarized as follows: (i) The forecast quality at each stage of the supply chain is endogenous, that is, both the supplier and the retailer can invest and improve the local forecast quality; (ii) The supply chain partners may have different forecasting capabilities. The inclusion of capability parameters allows us to capture that each party in the supply chain may have a different forecasting capability due to their expertise or their technology; (iii) We allow the forecast errors to be correlated, which captures the possibility that some of the information the supplier and the retailer use for forecasting is common; the more common information the two parties use, the less valuable it is to combine information. The next subsection describes a forecasting model that captures these elements.

3.1 The Forecasting Model

The forecasting model that we employ is based on Clemen and Winkler (1985). The supplier and the retailer privately observe imperfect signals drawn from \( \Psi_i = D + \mathcal{E}_i, \) where
D is the demand and E_i’ s are error terms. The prior demand distribution is normal with mean \( \mu \) and standard deviation \( \sigma \) and is common knowledge to both supply chain partners. The error terms, denoted by \( E_S \) and \( E_R \), are normally distributed with unconditional mean and variance, \( \mathbb{E}[E_i] = 0 \) and \( \text{Var}[E_i] = \sigma_i^2 \), for \( i = R, S \) ensuring that the signals are unbiased. The signal errors are uncorrelated with the prior estimation error \( D - \mu \). The error terms are correlated, that is, there exists a dependence between the information that the two signals carry. The correlation between the signals is modelled by the correlation between their estimation errors. We denote this correlation by \( \rho \). Because both the supplier and the retailer might utilize some common data, share common assumptions, or have access to some of each other’s opinions, we restrict our attention to non-negative correlations (i.e., \( \rho \in [0, 1) \)). Observing a signal realization \( \psi_i \) allows party \( i \) to generate a more accurate forecast, \( D|\psi_i \), than having only the prior information about \( D \).

Define the accuracy of the supplier’s and the retailer’s signals as \( p_S = 1/\text{Var}[E_S] \) and \( p_R = 1/\text{Var}[E_R] \). This is a measure of the forecast quality. We assume that the accuracies depend on the supplier’s and retailer’s respective investments into forecasting and their forecasting capabilities. By investing \( I_S \) and \( I_R \), the supplier and the retailer observe signals with accuracies \( p_S(I_S; z_S) \) and \( p_R(I_R; z_R) \), respectively. The accuracy functions are parameterized by the capability parameters \( z_S \) and \( z_R \) which are exogenous and are a measure of how fast the accuracy of the signal increases with the investment level. Finally, there is a cost to investing in improving the forecast quality. The cost of investing \( I \) is \( g(I; \kappa_i), i = S, R \) where the parameter \( \kappa_i \) is a measure of how expensive it is to exert effort into forecasting.

Suppose that the supplier’s and the retailer’s forecasting technologies are characterized by concave accuracy and convex cost functions

\[
p(I; z) = z I^{1/a} \quad \text{and} \quad g(I; \kappa) = \kappa I^{q},
\]

for \( a \geq 1 \) and \( q \geq 1 \). The accuracy as a function of the monetary cost \( m \) is

\[
p(m) = \frac{z}{\kappa^{1/aq}} m^{1/aq},
\]

which means that the accuracy function is concave in the monetary cost since \( aq \geq 1 \). The product \( aq \) is a measure of the effectiveness of the forecasting technology. For tractability, we assume that \( a = 1 \) and \( q \geq 1 \). \( q > 1 \) implies that there are diseconomies in the cost of information acquisition. We assume that the parameter \( q \) is the same for both the supplier and the retailer. The choice of linear accuracy and convex cost can be justified as
follows (Osband 1989). Suppose that we have an initial estimate for an uncertain variable with the initial estimate having accuracy $n_0$. If we take unbiased measurements each of constant accuracy, say $v$, and form the best linear unbiased estimate (BLUE), after taking $nv - n_0$ measurements, the BLUE will have accuracy $nv$ – each measurement will increase the accuracy of the estimate by $v$ units. A convex cost of effort is observed if each additional measurement is more expensive than the previous one.

3.2 Terms of Trade

We assume that terms of trade are such that the supplier and the retailer have an incentive to share their local information truthfully. In particular, we assume that a contract is in effect such that in expectation, each party receives a fixed and predetermined proportion of expected supply chain profits. Mechanisms that achieve such proportional profit sharing are the buy-back and the revenue sharing contracts (Cachon 2003). These contracts can be designed in such a way that the expected supplier profit is a fraction $1 - \lambda$ and the expected retailer profit is fraction $\lambda$ of the total expected supply chain profit (refer to Appendix A to see how a buy-back contract achieves proportional profit sharing). In this case, it is in the best interest of both partners to share information truthfully (Miyaoka 2003): Improving the quality of the final demand forecast will improve the expected supply chain profit, which in turn will improve individual expected profits.

With this assumption, determining the terms of trade means determining $r$ and $\lambda$. In practice, the parameter $\lambda$ would be the result of negotiation between the supplier and the retailer. In retailing, it is reasonable to assume that forecasting and replenishment will take place within a commercial framework that has already been established: Terms of trade are typically negotiated for the whole season by the purchasing department. Operational elements such as demand forecasting, ordering and replenishment are carried out on an ongoing basis within this framework. For this reason, we take $\lambda$ as a given exogenous parameter and focus on the rest of the strategic interaction.

A second assumption that we make is about the nature of effort into forecasting. We assume that forecasting effort is observable to both partners, but is not contractible. In other words, although the supply chain partners can observe whether the other partner has exerted effort or not, this is not provable or is very expensive to prove to a third party such as a court. There is a stream of literature that deals with such observable but non-contractible investment decisions. Grossman and Hart (1986) and Hart and Moore (1990) point out that
certain variables may not be included in contracts between two parties because they are either unforeseen or simply very difficult to describe, although they can be observed by the parties involved in the relationship. In these cases, the parties choose their investments in expectation of what fraction of benefits created by the investments are appropriated back. Investing into improving forecast quality is one example of such investments, with $\lambda$ and $1 - \lambda$ in our model capturing the fraction of benefits appropriated by the retailer and the supplier, respectively. Other examples are investments into new product innovation or investments into improving the quality of a product (e.g., Katz 1995 and Kaya and Özer 2003).

3.3 Sequence of Events

Figure 1 is a sketch of the main events in our model. At time $t_1$, the supplier and the retailer play a simultaneous move game where each player separately, and strategically, decides on whether or not to invest in improving the forecast quality. We refer to this game as the

Investment Game. Let us denote the supplier’s and the retailer’s investment levels by $I_S$ and $I_R$, respectively. To simplify our model, we assume that $I_S \in \{0, T_S\}$ and $I_R \in \{0, T_R\}$. In other words, both the supplier and the retailer can either invest in improving the forecast quality or shirk by not investing.

After the supplier and the retailer invest $I_S$ and $I_R$ and observe signals $\psi_S$ and $\psi_R$, a joint demand forecast is generated at $t_2$. The quality of the final demand forecast depends on the supplier’s and retailer’s investment choices. As discussed earlier, the assumption that the second-stage supply chain profit is shared proportionally between the supplier and the retailer will ensure that both partners share their local forecasts truthfully. Let us denote the updated forecast of demand as $D_J$ and call it the joint demand forecast. Let us also denote the initial accuracy level $1/\sigma^2$ (based on the prior distribution) by $p_0$. Then, $D_J$ is
Given the supplier’s and the retailer’s investment choices, there are four possible cases that we denote by I, II, III and IV. The derivations of the final demand forecasts and parameters for all four cases are given in Appendix B. In case I, both the supplier and the retailer shirk and exert no effort into forecasting and therefore observe uninformative signals about the demand. The final demand forecast in this case is $D^I_j \sim \mathcal{N}(\mu, \sigma)$, which is the prior demand distribution. In case II, the supplier exerts effort $T_S$ and the retailer shirks. In this case, the final demand forecast is $D^{II}_j \sim \mathcal{N}(\mu^{II}_j(\psi_S), \sigma^{II}_j(T_S))$, where $\mu^{II}_j(\psi_S) = x^{II}_S + x^{II}_R$ with $x^{II}_S = 1$ and $\sigma^{II}_j(T_S) = (p_0 + zST_s)^{-1/2}$. Note that the retailer’s signal is given no weight ($x^{II}_R = 0$) in the final demand forecasts since the retailer’s signal is uninformative. In case III, the retailer exerts effort $T_R$ and the supplier shirks. The final demand forecast is $D^{III}_j \sim \mathcal{N}(\mu^{III}_j(\psi_R), \sigma^{III}_j(T_R))$ where $\mu^{III}_j(\psi_R) = x^{III}_P + x^{III}_R\psi_R$ with $x^{III}_P + x^{III}_R = 1$ and $\sigma^{III}_j(T_R) = (p_0 + zRT_R)^{-1/2}$. In this case, supplier’s signal is given zero weight ($x^{III}_S = 0$) as the supplier’s signal is uninformative.

Finally, in case IV, both the supplier and the retailer exert effort $T_S$ and $T_R$, respectively. The final demand forecast denoted by $D^{IV}_j = D_j|\psi_S, \psi_R \sim \mathcal{N}(\mu^{IV}_j(\psi_S, \psi_R), \sigma^{IV}_j(T_S, T_R))$, with mean

$$
\mu^{IV}_j(\psi_S, \psi_R) = x^{IV}_P \mu + x^{IV}_S \psi_S + x^{IV}_R \psi_R
$$

(1)

where $x^{IV}_P$, $x^{IV}_S$, and $x^{IV}_R$ are given in Appendix B and the standard deviation is given by

$$
\sigma^{IV}_j(T_R, T_S) = \left(p_0 + zST_s + zRT_r - 2 \rho \sqrt{zST_s zRT_r} \right)^{-1/2}
$$

(2)

for all $\rho \in [0, 1)$. Note that $x^{IV}_P + x^{IV}_S + x^{IV}_R = 1$ and the posterior mean given in (1) is a convex combination of the prior mean, the supplier’s signal and the retailer’s signal.

After the demand forecast is updated, the retailer decides the stocking quantity at $t_3$. As discussed in §3.2, terms of trade are set in such a way that expected supply chain profit is shared proportionally between the supplier and the retailer: a fraction $\lambda$ of the supply chain profit goes to the retailer and the rest, a fraction $1 - \lambda$, goes to the supplier. For a given $\lambda$, the retailer’s optimization problem is

$$
\max_{Q} \lambda E_{D_j} \left[r \min(Q, D_j) - cQ \right],
$$

where $r$ is the retail price of the product and $c$ is the per unit production cost at the supplier.
The optimal order quantity is

\[ Q^i(I_S, I_R, \psi_S, \psi_R) = \mu^i + \sigma^i z^\alpha, \]

where \( z^\alpha = \Phi^{-1}(\alpha), \alpha = 1 - c/r, \) and \( i = I, II, III, IV. \) Let us define \( \Pi^i(I_S, I_R, \psi_S, \psi_R) = \mathbb{E}_{D_j}[r \min(Q^i, D_j) - cQ^i] \) for \( i = I, II, III, IV. \) Then, \( \Pi^i(I_S, I_R, \psi_S, \psi_R) = (r - c)\mu^i_j - r\sigma^i_j \phi(z^\alpha) \) (refer to Appendix C for the derivation of the expected profit expression). The supplier’s and the retailer’s expected optimal post investment profits for given investment levels \( I_S \) and \( I_R \) and signal realizations \( \psi_S \) and \( \psi_R \) are

\[ \Pi^i_S(I_S, I_R, \psi_S, \psi_R) = (1 - \lambda)\Pi^i(I_S, I_R, \psi_S, \psi_R) \]
\[ \Pi^i_R(I_S, I_R, \psi_S, \psi_R) = \lambda \Pi^i(I_S, I_R, \psi_S, \psi_R). \]

To summarize, given the supplier’s and retailer’s investment choices, there are four possible cases. The expected profits are different in each case because the final demand accuracy depends on the supply chain partners’ investments into improving the demand forecast.

In the next section, we focus on the strategic investment game that is played by the supplier and the retailer at \( t_1. \)

### 4 The Investment Game

The Investment Game (IG) is defined as follows: It consists of two players, the supplier and the retailer. The supplier’s and the retailer’s actions are binary meaning that both can either choose to invest (I) or shirk (S). Let \( I_S = \{0, T_S\} \) and \( I_R = \{0, T_R\} \) denote the supplier’s and retailer’s action sets, respectively. Figure 2 illustrates the two-by-two strategic investment game with payoffs corresponding to each action profile. The retailer is the row player and the supplier is the column player. The supplier’s and the retailer’s expected payoffs at time \( t_1 \) are given by

\[ V^i_S(I_S, I_R) = \mathbb{E}_{\psi_S, \psi_R} [\Pi^i_S(I_S, I_R, \psi_S, \psi_R)] - g_S(I_S) \]
\[ = (1 - \lambda) [(r - c)\mu - r\sigma^i_j \phi(z^\alpha)] - \kappa_S I^i_S \] \hspace{1cm} (3)

\[ V^i_R(I_S, I_R) = \mathbb{E}_{\psi_S, \psi_R} [\Pi^i_R(I_S, I_R, \psi_S, \psi_R)] - g_R(I_R) \]
\[ = \lambda [(r - c)\mu - r\sigma^i_j \phi(z^\alpha)] - \kappa_R I^i_R. \] \hspace{1cm} (4)
for $i = I, II, III, IV$.

Given the supplier’s and retailer’s strategy choices, there are four possible equilibria of the one shot investment game corresponding to the four boxes in Figure 2. We define the equilibrium where both shirk as the NI (no investment) equilibrium. When both partners shirk, there is no improvement in the accuracy of the final demand forecast as both the supplier and the retailer observe uninformative signals that are not given any weight in the final demand forecast. The equilibria where only the supplier or only the retailer invest are defined as equilibria S and R, respectively. When only one of the partners invests, both partners take advantage of the improved demand forecast accuracy but only one of the partners incurs the cost of forecasting. Therefore, we refer to both equilibria R and S as the free-riding equilibria. Finally we define the equilibrium where both partners invest in improving the demand forecast as the JF (joint forecasting) equilibrium.

Let $\Omega_{NI}$, $\Omega_{R}$, $\Omega_{S}$, and $\Omega_{JF}$ denote the set of parameters for which equilibrium NI, R, S, and JF exist, respectively. Equilibrium NI exists if both the supplier and the retailer are better off not investing. The conditions for NI to exist are: $V^{I}_R \geq V^{III}_R$ and $V^{I}_S \geq V^{III}_S$. Equilibrium S exists if the supplier is better off investing, while given that the supplier invests, the retailer is better off not investing. The conditions for equilibrium S to exist are: $V^{II}_R \geq V^{IV}_R$ and $V^{II}_S \geq V^{IV}_S$. Similarly, the conditions for equilibrium R to exist are: $V^{III}_R \geq V^{I}_R$ and $V^{III}_S \geq V^{I}_S$. Finally, equilibrium JF exists if both the supplier and the retailer are better off investing given that the other partner invests. The conditions for the JF equilibrium to exist are: $V^{IV}_R \geq V^{II}_R$ and $V^{IV}_S \geq V^{II}_S$. 

![Figure 2: The investment game](image-url)
We start our analysis with the special case where $\lambda = 0.5$, $\kappa_S = \kappa_R = \kappa$, $T_S = T_R = T$, and $z_S = z_R = z$ and call this game the symmetric investment game. The following proposition fully characterizes the possible equilibria in the symmetric investment game:

**Proposition 1** There exists a threshold

$$z_1 = \frac{1}{T} \left( \frac{1}{A^2} - p_0 \right)$$

where $A = \frac{1}{\sqrt{p_0}} - \frac{\kappa T q}{\lambda r \phi(z)}$

such that for all $z \leq z_1$, NI is the unique equilibrium of the investment game. On the other hand, for $z > z_1$, the following two cases are possible:

**Case 1:** Either both S and R are equilibrium outcomes of the investment game for all $z > z_1$

**Case 2:** There exist $z_1^1$ and $z_2^2$ satisfying $z_1 \leq z_1^1 \leq z_2^1 \leq z_2^2$ such that for all $z \in (z_1, z_1^1]$ and $z > z_2^2$, both S and R are equilibrium outcomes and for $z \in (z_1^1, z_2^1)$, the joint forecasting (JF) equilibrium is the unique equilibrium of the investment game.

**Proof** All proofs are in the Appendix D. ■

![Figure 3](image_url) Illustration of the results in Proposition 1.

Figure 3 illustrates the results in Proposition 1. To summarize, for very small capabilities, NI is the unique equilibrium of the game because the benefits due to having a better demand forecast are not worth the investment cost. For small capabilities, both free-riding equilibria S and R exist because the investment into forecasting can be justified for only one
of the players. For intermediate capabilities, JF may exist because both partners’ investments can be justified. Finally, for very large capabilities, we observe free-riding equilibria again because one of the partners alone can achieve a highly accurate forecast.

Figure 4: Equilibrium regions for the symmetric investment game.

Next, we investigate the equilibrium regions when the forecasting capabilities of the supplier and the retailer differ. Figure 4 demonstrates the equilibrium regions for one set of parameters in the symmetric investment game \( \lambda = 0.5, \kappa_S = \kappa_R = \kappa, \) and \( T_S = T_R = T \) for a range of supplier and retailer capabilities \( z_S \) and \( z_R \). For now, we focus on cases where \( z_S \) and \( z_R \) are of the same order of magnitude as \( p_0/T \), meaning that the accuracy of the signals observed by the supplier and the retailer are comparable to the accuracy of the prior demand forecast. Figure 4(a) and 4(b) differ only in the correlation coefficient \( \rho \). Note also that the equilibria on the \( z_S = z_R \) line were characterized in Proposition 1.

The equilibrium regions in figure 4(a) are in line with our results for Case 2 in Proposition 1. When the supplier’s and the retailer’s capabilities are very small, the equilibrium outcome is not to invest for either player as the benefit due to an improved demand forecast is not worth the investment. For small forecasting capabilities, there exists a region, denoted by \( \Omega_{SR} \), where both equilibria S and R exist. In this region, either one of the players may free ride on the other. When the supplier’s forecasting capability is significantly better than the retailer’s forecasting capability (\( \Omega_S \)), the retailer free-rides on the supplier, and at equilibrium only the supplier invests. Similarly, the supplier free-rides on the retailer.
when the retailer’s forecasting capability is significantly better ($\Omega_R$). Finally, for large and balanced forecasting capabilities, the joint forecasting equilibrium is observed where both partners invest in improving the accuracy of the demand forecast. Note that if we plot the equilibrium regions for very large forecasting capabilities (i.e., $z_i \gg \frac{p_0}{T}$), we would observe another region $\Omega_{RS}$ around the $z_S = z_R$ line, consistent with the second part of Proposition 1.

Based on Figure 4 and other numerical investigations, we make several observations concerning the impact of correlation, negotiation power, and forecast capability on the outcome of the investment game.

**Observation 1** While small correlations make joint forecasting more likely, larger correlations make free-riding more likely.

Figure 4(b) illustrates the effect of increased correlation on the equilibrium regions. The correlation coefficient measures the overlap in the amount of information provided by each player. As the overlap in the information provided by the partners increases, the set of parameters for which the joint forecasting equilibrium exists gets smaller and eventually disappears whereas the set of parameters where S and R coexist gets larger. In other words, while smaller correlations facilitate joint forecasting, larger correlations lead to free-riding.

**Observation 2** Changing the correlation does not affect the NI region, but rather determines whether only one of the partners invests or both invest into improving the forecast.

Comparing the equilibrium regions in figures 4(a) and 4(b), we notice that the region $\Omega_{NI}$ does not change as we increase the correlation. This is supported by Proposition 1 where $z_1$ is independent of $\rho$. All other regions are affected by correlation parameter $\rho$.

**Observation 3** Our numerical studies suggest that (1) negotiation power and forecasting capability are substitutes and (2) the joint forecasting equilibrium is more likely when the incentives to invest for both partners are balanced.

Figure 5 illustrates the equilibrium regions for an asymmetric investment game where the supplier is more powerful ($\lambda = 0.4$ and $\lambda = 0.3$) and appropriates more of the total supply chain profit. One of the significant changes in this asymmetric investment game compared to the symmetric investment game is that the region where equilibrium S exists gets larger while the region where equilibrium R exists gets smaller. Since the supplier appropriates a
bigger fraction of the supply chain profit, the supplier has a stronger incentive to invest in improving the forecast accuracy while the retailer’s incentives are weakened. The second significant change is that the joint forecasting equilibrium is observed for a parameter set where the retailer has higher capability compared to the supplier: Negotiation power and forecasting capability are complements and the joint forecasting equilibrium JF is seen when \( \lambda z_R \) is close to \( (1 - \lambda) z_S \) (which we refer to as the incentives to invest being balanced). On the other hand, if \( \lambda z_R > (1 - \lambda) z_S \), then equilibrium R is more likely as the retailer has a stronger incentive to invest; if \( \lambda z_R < (1 - \lambda) z_S \), then equilibrium S is more likely.

5 Repeated Investment Game

In this section, we focus on the infinitely repeated investment game and interpret the existence of the joint forecasting equilibrium in the repeated investment game as collaborative forecasting being sustainable. In the infinitely repeated investment game, the supplier and the retailer play the one-shot investment game at each discrete time period \( t = 0, 1, 2, 3, \ldots \). The players make their choices simultaneously and at the end of each period, the players can observe the other player’s action.

The one shot investment game may lead the partners to an inefficient equilibrium outcome. In particular, there exists a set of parameters for which the investment game becomes a prisoner’s dilemma type of a game. This class of games is also sometimes referred to as
Equilibrium Regions for $\lambda = 0.5$, $r = 10$, $c = 4$, $\mu = 150$, $\sigma = 50$, $q = 1$, $\rho = 0.1$, $\kappa_S = \kappa_R = 80$, $T_S = T_R = 0.1$

(a) symmetric

(b) asymmetric

Figure 6: Illustration of the prisoners dilemma regions in the one-shot investment game.

partnership games. Figure 6 is representative of equilibrium regions in symmetric and asymmetric investment games as well as the regions where the investment game becomes a prisoner’s dilemma type game. We denote these regions by $\Omega_{PD}$. The conditions defining these regions are $V_{S}^{III} > V_{S}^{IV} > V_{S}^{I} > V_{S}^{II}$ and $V_{R}^{IV} > V_{R}^{I} > V_{R}^{II}$. In a prisoner’s dilemma type game, shirking is a strictly dominant strategy, while higher payoffs are achievable for both players if both exert effort. While it is not possible to escape from the dilemma in the one-shot investment game, the possibility of being able to react to their opponent’s past actions at each time period might allow the supplier and the retailer to achieve the efficient outcome under certain conditions. Our main concern in this section will be to explore this possibility and characterize these conditions.

In the repeated investment game, each player can condition its action at each period on the other player’s action history. Let $a^t_S$ and $a^t_R$ be the actions chosen by the supplier and the retailer at time $t$, respectively. Let also $v^t_S$ and $v^t_R$ be the corresponding payoffs for the supplier and the retailer at time $t$. We assume that each player evaluates the sequence of actions $(a^0_i, a^1_i, a^2_i, \ldots)$ and the corresponding stream of payoffs $(v^0_i, v^1_i, v^2_i, \ldots)$ for $i = S, R$ by the discounted sum of the corresponding payoffs. In other words, each player chooses actions to maximize the discounted sum of its payoffs $\sum_{t=0}^{\infty} \delta^t v^t_i(a^t_S, a^t_R)$, where $\delta \in [0, 1)$ is the common discount factor. If $\delta$ is close to zero, the players are said to be impatient and they care very little about the future. If, on the other hand, $\delta$ is close to one, the players
are said to be patient as the weight given to the later payoffs is significant (Osborne 2003, Chapter 14). We also define the discounted average payoff of the stream \((v^0_i, v^1_i, v^2_i, \ldots)\) as 
\[(1 - \delta) \sum_{t=0}^{\infty} \delta^t v^t_i(a^t_S, a^t_R).\]

A strategy \(s_i\) for player \(i\) specifies the action of player \(i\) for every history \((a^0, a^1, \ldots, a^{t-1})\).

The repeated investment game may have multiple equilibrium strategies. One of the equilibrium strategies in the infinitely repeated investment game is for both players to choose to shirk after every possible history because once a player adopts this strategy, the best response of the other player would be to shirk as well. However, we are interested in a strategy that can be used to sustain the joint forecasting equilibrium. In the rest of this section, we focus on the grim trigger strategy. With the grim trigger strategy each player starts by investing and continues to cooperate until the other player shirks. A player will defect if and only if the other player defects at any earlier period during the game. We also assume that once a player defects, the player defects forever (Osborne 2003, Chapter 14).

The grim trigger strategy is defined as follows: 
\[s_i(\emptyset) = I\]

\[s_i(a^0_j, a^1_j, \ldots, a^{t-1}_j) = \begin{cases} 
\text{invest (I)} & \text{if } (a^0_j, a^1_j, \ldots, a^{t-1}_j) = (I, I, \ldots, I) \\
\text{shirk (S)} & \text{otherwise}
\end{cases}\]

Suppose that one of the players follows the grim trigger strategy. If the other player follows the same strategy, then the equilibrium outcome would be for both players to invest in every period, therefore obtaining a stream of payoffs \(v^t_i = V^{IV}_i\) for \(t = 0, 1, 2, \ldots\) or \((V^{IV}_i, V^{IV}_i, \ldots)\) whose discounted average is \(V^{IV}_i\).

Now suppose that the retailer decides to deviate from the grim trigger strategy and invests in all periods but does not invest in period \(t\). The retailer will obtain \(V^{II}_R > V^{IV}_R\) in period \(t\) and will benefit. The supplier will respond by not investing in period \(t + 1\) and no investment NI will be the outcome of the game from period \(t + 1\) onwards. The retailer’s payoff stream corresponding to this scenario from period \(t\) onward will be \((V^{II}_R, V^{II}_R, V^{II}_R, \ldots)\), whose discounted average is 
\[(1 - \delta)[V^{II}_R + V^{II}_R(\delta + \delta^2 + \delta^3 + \ldots)] = (1 - \delta)V^{II}_R + \delta V^{II}_R\]

Therefore, the retailer cannot benefit by deviating from the grim trigger strategy if
\[(1 - \delta)V^{II}_R + \delta V^{II}_R \leq V^{IV}_R\]
or
\[\delta \geq \delta_{GT} = \frac{V^{II}_R - V^{IV}_R}{V^{II}_R - V^{IV}_R}\]
Therefore, we can conclude that if $\delta \geq \bar{\delta}_{GT}$, the strategy in which each player follows the grim trigger strategy is a Nash equilibrium of the infinitely repeated investment game.

To gain some further insights, we focus on the repeated symmetric investment game where $\lambda = 0.5$, $z_S = z_R = z$, $T_S = T_R = T$, and $\kappa_S = \kappa_R = \kappa$. The following proposition characterizes the equilibrium regions for the repeated symmetric investment game.

**Proposition 2** There exist thresholds

$$z_0 = \frac{1 + \rho}{2T} \left( \frac{1}{A^2} - p_0 \right) \text{ and } z_1 = \frac{1}{T} \left( \frac{1}{A^2} - p_0 \right)$$

where $A = \frac{1}{\sqrt{p_0 - \kappa T q \lambda \phi (z)}}$ satisfying $z_0 < z_1$ for $\rho < 1$ such that for all $z \leq z_0$, NI is the only sustainable equilibrium of the repeated investment game and for all $z \in (z_0, z_1)$, JF is a sustainable equilibrium of the repeated game if $\delta > \bar{\delta}_{GT}$ and NI is the only sustainable equilibrium of the repeated investment game otherwise. For $z > z_1$, the following two cases are possible:

**Case 1:** Either for all $z > z_1$, both S and R are the equilibrium outcome of the repeated investment game or;

**Case 2:** There exist $z^1_2$ and $z^2_2$ satisfying $z_1 \leq z^1_2 \leq z^2_2$ such that for all $z \in (z_1, z^1_2)$ and $z > z^2_2$, both S and R are the equilibrium outcome and for $z \in (z^1_2, z^2_2)$, the joint forecasting equilibrium JF is the unique equilibrium of the repeated investment game.

**Proof** All proofs are in the Appendix D. ■

![Diagram](https://via.placeholder.com/150)

Figure 7: Illustration of the results in Proposition 2.

Figure 7 illustrates the results in Proposition 2. Proposition 2 states that for very small forecasting capabilities, the no investment equilibrium NI is the only sustainable equilibrium. Unlike in the one shot investment game, the joint forecasting equilibrium is
not only sustainable for intermediate forecasting capabilities, but there also exists a range of smaller capabilities for which joint forecasting equilibrium may be sustained with patient players. Finally, for large capabilities, both free riding equilibria can be sustained.

Our results suggest that there are two ways to sustain the joint forecasting equilibrium when the supplier and the retailer interact repeatedly. First, the JF equilibrium can be sustained for small forecasting capabilities \((i.e., \ z \in (z_0, z_1])\) when the discount factor is sufficiently large. A large discount factor implies that the supplier and the retailer are more likely to stay involved in trade with one another in the future, therefore placing high weight on the future payoffs. While the no investment equilibrium NI is the unique equilibrium of the one shot investment game for all \(z \in (z_0, z_1]\), JF can be sustained in this region if the supplier and the retailer interact repeatedly. The grim trigger strategy with a sufficiently high discount factor ensures that the incentives to shirk are overcome by the threat of punishment leading to the possibility of sustained joint forecasting equilibrium.

Second, the JF equilibrium can be sustained for intermediate forecasting capabilities when the correlation between the signals is relatively low. Low correlation implies that the supply chain partners have complementary information. For this range of parameters, the joint forecasting equilibrium is a dominant strategy equilibrium and each player prefers investing into improving the forecast quality irrespective of what the other player does.

6 Conclusions, Limitations and Future Research Directions

Many retailers have initiated collaborative forecasting pilots with their suppliers, however, most of these initiatives have not gone beyond the pilot stage. The trade literature has offered some evidence supporting and criticizing CPFR, however, this evidence is anecdotal in nature. We offer a supply chain modeling framework that sheds light on the conditions that favor the establishment of collaborative forecasting between a supplier and a retailer. In particular, we incorporate (1) the effort that the supply chain partners must exert to obtain relevant data, improve data quality and generate forecasts and (2) the strategic interaction that may arise as a result of the investments into forecasting. Our model also captures the impact of (i) overlap in the information that may be common to supply chain partners; (ii) forecasting capabilities of the partners; (iii) the negotiation power of the supply chain partners.

Our analysis proceeds in two steps. First, we focus on the existence of Nash equilibria and characterize all Nash equilibria of the one-shot investment game as a function of the
system parameters. We find that a joint forecasting equilibrium exists for a limited parameter set. This is due to the following tradeoff: Supply chain partners take into account the cost of their own investment and the benefits that they can appropriate back from the improvement in the quality of the final demand forecast. This tradeoff may result in free-riding in the form of an equilibrium where only one of the partners invests. Second, we focus on the repeated investment game and characterize the conditions under which joint forecasting is sustainable. We say that “joint forecasting is sustainable” if the joint forecasting equilibrium JF is one of the equilibrium outcomes in the repeated investment game. We find that even if the joint forecasting equilibrium JF is not an equilibrium of the one shot investment game, it may be sustained in the repeated investment game.

Our most revealing insight is the following: Although we assumed that parties operate in a framework in which information is truthfully exchanged, which is a strong assumption that is not always true in the traditionally adversarial consumer goods industry, the conditions under which a joint forecasting equilibrium exists and is sustainable are nevertheless quite limited. We conclude that it is indeed not surprising that many of the collaboration initiatives fail.

Our results have several managerial implications. First, we find that when the incentives to invest are balanced, joint investments are more likely to be sustained because the effective value derived from participating into joint forecasting is comparable. One implication of this result is that strong firms (e.g., Metro and P&G) with similar levels of forecasting cost effectiveness are candidates for the emergence of joint forecasting practices.

Second, we concluded that joint forecasting can be sustained when supply chain partners have smaller capabilities but a large discount factor. A large discount factor can be interpreted as a more durable relationship between the trading partners. The implication of this result is that supply chain partners who have lower forecasting capabilities can help the formation of a collaborative forecasting initiative by building stronger relationships with one another. Stronger relationships help supply chain partners mitigate the temptation for free-riding and allows them to sustain collaborative forecasting.

Third, the investment game is more likely to result in a joint forecasting equilibrium when the supply chain partners have intermediate capabilities and complementary information. This is an intuitive result. Its implication is that it is more appropriate to attempt forecasting collaboration with promotional items and new product introductions where partners are more likely to have diversified information compared to regular products where
historical data is available to both partners in the supply chain. However, many firms tend to start collaboration on existing, stable items since past sales data exists and is available to both firms. Clearly, when both firms use the same data to forecast, the benefits from collaboration are limited. Our result could explain why collaborations in which the focus was existing non-promotional products did not go beyond the pilot stage. Indeed, Metro - P&G mostly reported benefits from improved promotion planning. Another implication of this result is that once the supply chain partners agree to collaborate on forecasting, they should coordinate and focus on collecting distinct sets of information so that their information sets can complement each other. For example, while the retailer focuses on analyzing the local market conditions and past sales data for that particular product in that particular location, the supplier might bring its overall view of the marketplace and may provide input on similar promotions taking place at other retailers in the same region that might influence the demand.

We made a number of assumptions that may limit the realism of our model. First, we have assumed that the effort decisions are observable. We expect joint forecasting to be more difficult to sustain if the effort into forecasting is partially unobservable. Second, we assumed that the functional form for the accuracy functions and cost of forecasting functions, $p(T)$ and $g(T)$, are the same for both the supplier and the retailer. We expect that relaxing this assumption by allowing different forecasting technologies and cost functions would work against the joint forecasting equilibrium since we know that the more unbalanced the incentives to invest, the more limited the regions where the joint forecasting equilibrium is observed. Finally, we assumed that the retailer engages in forecast collaboration with a single supplier. This may seem restrictive, however, retailers typically collaborate with only one of their leading suppliers in a category, a practice often referred to as ‘category captainship’, and rely on that manufacturer for information and insights regarding the category as a whole. In this type of arrangement, the collaborative relationships tend to be of a single supplier and single retailer nature, as in this paper.

Finally, we suggest some future research directions. One direction for future research would be to empirically validate the findings of our model. Terwiesch et al. (2004) is an empirical paper that studies forecast sharing in the semiconductor equipment supply chain. In a similar spirit, one can empirically test the findings of our model regarding the conditions under which collaborative forecasting is established between a supplier and a retailer.

We have used a stylized Bayesian updating model to mimic exception resolution in
the CPFR process. One of the concerns raised in practice is the amount of work needed to discuss and resolve exceptions in arriving at a joint demand forecast. For this reason, automation of the resolution of the exceptions is one of the future projects considered by the CPFR initiative (CPFR Future Projects). Various automation rules such as taking the minimum, maximum or the weighted average of individual forecasts have been proposed. An interesting avenue for future research would be to explore and compare the performance of these automation rules.

Acknowledgements: The authors thank Paddy Padmanabhan, Christian Terwiesch, Ludo van der Heyden, Yianis Sarafidis, Luk Van Wassenhove and Robert Winkler for helpful discussions, comments, and suggestions.

References


Appendix

Appendix A

There are various mechanisms that can achieve proportional profit sharing, such as buy-back and revenue sharing contracts (Cachon 2003). For example, with a buy-back contract, the supplier charges the retailer \( w \) per unit purchased, but pays the retailer \( b \) per unit remaining at the end of the season; here \( r > w \geq c \) and \( w > b \geq 0 \), where \( r \) and \( c \) are the unit price and manufacturing cost, respectively. If parties agree on a buy-back contract with buy-back parameters \( \{w, b\} \) such that \( r - b = \lambda r \) and \( w - b = \lambda c \), then the expected supplier profit is a fraction \( 1 - \lambda \) and the expected retailer profit is fraction \( \lambda \) of the total expected supply chain profit.

Appendix B - Updating the demand forecast

We start by deriving the updated demand distribution for the most general case where both invest in improving the forecast quality.

Case IV: Both the supplier and the retailer invest

\((D, \Psi_S, \Psi_R)\) is a multivariate normal random variable with covariance matrix \( \Sigma \), where

\[
\Sigma = \begin{bmatrix}
\sigma^2 & 0 & 0 \\
0 & \sigma_S^2 & \rho \sigma_S \sigma_R \\
0 & \rho \sigma_S \sigma_R & \sigma_R^2 \\
\end{bmatrix} = \begin{bmatrix}
p_0^{-1} & 0 & 0 \\
0 & (z_S T_S)^{-1} & \rho (z_S T_S z_R T_R)^{-1/2} \\
0 & \rho (z_S T_S z_R T_R)^{-1/2} & (z_R T_R)^{-1} \\
\end{bmatrix}.
\]
Then $D|\psi_S, \psi_R \sim \mathcal{N}(\mu_J, \sigma_J)$, where

$$
\mu_J(\psi_S, \psi_R) = (\mu \sigma^{-2} + e^t \Sigma_{22}^{-1} \psi) \sigma^2, \tag{5}
$$

$$
\sigma_J(T_S, T_R) = (\sigma^{-2} + e^t \Sigma_{22}^{-1} e)^{-1/2}, \tag{6}
$$

$e^t = (1, 1)$, $\psi = (\psi_S, \psi_R)$, and

$$
\Sigma_{22} = \begin{bmatrix}
\sigma^2_S & \rho \sigma_S \sigma_R \\
\rho \sigma_S \sigma_R & \sigma^2_R
\end{bmatrix} = \begin{bmatrix}
(z_S T_S)^{-1} & \rho (z_S T_S z_R T_R)^{-1/2} \\
\rho (z_S T_S z_R T_R)^{-1/2} & (z_R T_R)^{-1}
\end{bmatrix}.
$$

For $\rho \in [0, 1]$,

$$
e^t \Sigma_{22}^{-1} e = \frac{z_S T_S + z_R T_R}{1 - \rho} - \frac{\rho}{1 - \rho^2} \left[ \sqrt{z_S T_S} + \sqrt{z_R T_R} \right]^2,
$$

$$
= \frac{z_S T_S + z_R T_R - 2\rho \sqrt{z_S T_S z_R T_R}}{1 - \rho^2},
$$

which allows us to easily compute $\mu_J(\psi_S, \psi_R)$ and $\sigma_J(T_S, T_R)$.

$$
\mu^I_J (\psi_S, \psi_R) = x^I_P \mu + x^I_S \psi_S + x^I_R \psi_R \tag{7}
$$

where

$$
x^I_P = \frac{(1 - \rho^2)p_0}{(1 - \rho^2)p_0 + z_S I_S + z_R I_R - 2\rho \sqrt{z_S I_S z_R I_R}},
$$

$$
x^I_S = \frac{z_S I_S - \rho \sqrt{z_S I_S z_R I_R}}{(1 - \rho^2)p_0 + z_S I_S + z_R I_R - 2\rho \sqrt{z_S I_S z_R I_R}},
$$

$$
x^I_R = \frac{z_R I_R - \rho \sqrt{z_S I_S z_R I_R}}{(1 - \rho^2)p_0 + z_S I_S + z_R I_R - 2\rho \sqrt{z_S I_S z_R I_R}},
$$

and the standard deviation is given by

$$
\sigma^I_J(T_R, T_S) = \left( p_0 + \frac{z_S T_S + z_R T_R - 2\rho \sqrt{z_S T_S z_R T_R}}{1 - \rho^2} \right)^{-1/2}. \tag{8}
$$

**Case I: Both do not invest**

If both partners do not invest, both observe uninformative signals implying that both $z_S T_S \to 0$ and $z_R T_R \to 0$. Both partner’s signals are not given any weight in the final demand forecast. Substituting $z_S T_S \to 0$ and $z_R T_R \to 0$ into (5) and (6), the mean of the updated demand distribution in this case is $\mu$ and the standard deviation is $\sigma$, which are the same as the prior demand distribution.
Case II: Only the supplier invests

In this case, the retailer observes an uninformative signal implying that \( z_{R} T_{R} \rightarrow 0 \). Substituting \( z_{R} T_{R} \rightarrow 0 \) into (5) and (6) with \( \rho = 0 \), we obtain \( \mu_{j}^{\text{II}}(\psi_{S}) = x_{P}^{\text{II}} \mu + x_{R}^{\text{II}} \psi_{S} \) where

\[
x_{P}^{\text{II}} = \frac{p_{0}}{p_{0} + z_{S} T_{S}} \quad x_{S}^{\text{II}} = \frac{z_{S} T_{S}}{p_{0} + z_{S} T_{S}} \quad x_{R}^{\text{II}} = 0
\]

and \( \sigma_{j}^{\text{II}}(T_{S}) = (p_{0} + z_{S} T_{S})^{-1/2} \).

Case III: Only the retailer invests

This case is very similar to the previous case. Now, the supplier observes an uninformative signal implying that \( z_{S} T_{S} \rightarrow 0 \). Substituting \( z_{S} T_{S} \rightarrow 0 \) into (5) and (6) with \( \rho = 0 \), we obtain \( \mu_{j}^{\text{III}}(\psi_{S}) = x_{P}^{\text{III}} \mu + x_{R}^{\text{III}} \psi_{R} \) where

\[
x_{P}^{\text{III}} = \frac{p_{0}}{p_{0} + z_{R} T_{R}} \quad x_{S}^{\text{III}} = \frac{z_{R} T_{R}}{p_{0} + z_{R} T_{R}} \quad x_{R}^{\text{III}} = 0
\]

and \( \sigma_{j}^{\text{III}}(T_{R}) = (p_{0} + z_{R} T_{R})^{-1/2} \).

Appendix C - Derivation of \( \Pi(T_{S}, T_{R}, \psi_{S}, \psi_{R}) \)

The optimal order quantity is \( Q^{*} = \mu_{j} + \sigma_{j} z^{\alpha} \) where \( z^{\alpha} = \Phi^{-1}(\alpha) \) and \( \alpha = (r - c)/r \). Substituting \( Q^{*} \) back into the expected profit expression we obtain

\[
\Pi(T_{S}, T_{R}, \psi_{S}, \psi_{R}) = r E_{D_{j}}[\min(Q_{j}^{*}, D_{j})] - c Q_{j}^{*} = r E_{D_{j}}[D_{j} - (D_{j} - Q_{j}^{*})^{+}] - c Q_{j}^{*}
\]

\[
= (r - c) \mu_{j} - c \sigma_{j} z^{\alpha} - r L(Q_{j}^{*}),
\]

where the loss function is defined as \( L(Q) = \int_{Q}^{\infty} (x - Q) f(x) dx \). For the normal distribution with mean \( \mu_{j} \) and standard deviation \( \sigma_{j} \), \( L(x) = \sigma_{j} L_{0}(z^{\alpha}) \) where \( x = \mu_{j} + \sigma_{j} z^{\alpha} \), hence \( L(Q) = \sigma_{j} L_{0}(z^{\alpha}) \). We also have that \( L_{0}(z^{\alpha}) = z^{\alpha} \Phi(z^{\alpha}) + \phi(z^{\alpha}) - z^{\alpha} \). Combining these results, we obtain

\[
\Pi(T_{S}, T_{R} | \psi_{S}, \psi_{R}) = (r - c) \mu_{j} - c \sigma_{j} z^{\alpha} - r \sigma_{j} L_{0}(z^{\alpha})
\]

\[
= (r - c) \mu_{j} - c \sigma_{j} z^{\alpha} - r \sigma_{j} [z^{\alpha} \Phi(z^{\alpha}) + \phi(z^{\alpha}) - z^{\alpha}]
\]

\[
= (r - c) \mu_{j} - r \sigma_{j} \phi(z^{\alpha}).
\]

Appendix D - Proof of Propositions

Proof of Proposition 1

The conditions for NI to exist are \( V_{R}^{I} \geq V_{R}^{III} \) and \( V_{S}^{I} \geq V_{S}^{III} \). These two conditions are equivalent for the symmetric investment game. Therefore, the condition for the NI equilibrium
to exist is
\[
\lambda(r-c)\mu - \lambda r \phi (z^\alpha) \sigma \geq \lambda(r-c)\mu - \lambda r \phi (z^\alpha)(p_0 + zT)^{-1/2} - kT^q.
\]
Simplifying this condition, we conclude that the NI equilibrium exists if
\[
z \leq z_1 = \frac{1}{T} \left( \frac{1}{A - p_0} \right)
\]
where \(A = \sigma - \kappa T^q / \lambda r \phi\).

The conditions for equilibrium S to exist are \(V_{II}^S \geq V_{I}^S\) and \(V_{II}^R \geq V_{IV}^R\). The first condition \(V_{II}^S \geq V_{I}^S\) is
\[
\lambda(r-c)\mu - \lambda r \phi (z^\alpha) \sigma \leq \lambda(r-c)\mu - \lambda r \phi (z^\alpha)(p_0 + zT)^{-1/2} - kT^q
\]
and is equivalent to \(z \geq z_1\). The second condition \(V_{II}^R \geq V_{IV}^R\) is
\[
\lambda(r-c)\mu - \lambda r \phi (p_0 + zT)^{-1/2} \geq \lambda(r-c)\mu - \lambda r \phi \left( p_0 + \frac{2zT}{1 + \rho} \right)^{-1/2} - \kappa T^q.
\]
Simplifying this condition we get
\[
\lambda r \phi \left[ (p_0 + zT)^{-1/2} - \left( p_0 + \frac{2zT}{1 + \rho} \right)^{-1/2} \right] \leq \kappa T^q.
\]
Let us define
\[
MV = \lambda r \phi \left[ (p_0 + zT)^{-1/2} - \left( p_0 + \frac{2zT}{1 + \rho} \right)^{-1/2} \right]
\]
to be the marginal value of investing for one of the partners given that the other partner already invests. \(MV\) is unimodal in \(z\) because \(\partial MV/\partial z = 0\) has only one solution, implying that the function \(MV\) is increasing first and then decreasing. If \(MV\) is unimodal and \(MV = 0\) for \(z = 0\) and \(MV = 0\) as \(z \to \infty\), then \(MV = \kappa T^q\) may have at most two solutions. The two possible cases are illustrated in figure 8. Let us denote the two possible solutions as \(z_1^1\) and \(z_1^2\) such that \(z_1^1 < z_1^2\).

Equilibrium S exists if \(MV \leq \kappa T^q\). There are two possible cases: (i) \(MV = \kappa T^q\) has no solutions (as in Figure 8(a)); and (ii) \(MV = \kappa T^q\) has two solutions (as in Figure 8(b)). In the first case, equilibrium S exists for all \(z > z_1\). In the second case, equilibrium S exists for all \(z \in (z_1, z_2]\) and all \(z > z_2^2\).

The conditions for equilibrium R to exist are: \(V_{III}^R \geq V_{I}^R\) and \(V_{III}^S \geq V_{IV}^S\). These two conditions are equivalent to the conditions for equilibrium S for the symmetric investment game. Therefore, we can conclude that equilibrium R exists for all parameters where equilibrium S exists.
Finally, the conditions for the JF equilibrium to exist are $V^IV_R \geq V^II_R$ and $V^IV_S \geq V^III_S$. These two conditions are equivalent for the symmetric investment game and can be written as

$$\lambda(r - c)\mu - \lambda r\phi\left(p_0 + \frac{2zT}{1 + \rho}\right)^{-1/2} - \kappa T^y \geq \lambda(r - c)\mu - \lambda r\phi(p_0 + zT)^{-1/2},$$

which can be rewritten as

$$\lambda r\phi\left[(p_0 + zT)^{-1/2} - \left(p_0 + \frac{2zT}{1 + \rho}\right)^{-1/2}\right] \geq \kappa T^y$$

or $MV \geq \kappa T^y$. Again, there are two cases: either (i) there does not exist a $z$ that solves $MV = \kappa T^y$ or (ii) there exist two solutions to $MV = \kappa T^y$. In the first case, there exist no range of parameters for which the condition is satisfied and therefore JF does not exist. In the second case, JF exists for all $z \in (z^1_2, z^2_2]$.

Finally we have to show that $z_1 < z^1_2$. The threshold $z_1$ is characterized by $MV_0(z_1) \doteq \lambda r\phi\left[\sigma - (p_0 + z_1T)^{-1/2}\right]$ which is the marginal value of investing for one player given that the other player does not invest. The threshold $z^1_2$ is characterized by $MV(z_2)$ which is the marginal value of investing for one of the players given that the other player already invests. For the same capability level $z$, $MV_0 \geq MV$ for all $\rho > 0$. Therefore, if $z^1_2$ exists, than $z_1 < z^1_2$.

Figure 8: Example for MV function
Proof of Proposition 2

The proof of proposition 2 is very similar to the proof of Proposition 1. The only difference is the region where the investment game is similar to the prisoner’s dilemma type game. The conditions for the prisoners dilemma are $V_{III} > V_{IV} > V_{I} > V_{II}$ and $V_{I} > V_{III} > V_{IV} > V_{II}$. These two sets of conditions are equivalent for the symmetric investment game.

The condition $V_{II} > V_{IV}$ either holds for all $z$ or is satisfied for all $z < z_1$. The condition $V_{I} > V_{III}$ is satisfied for all $z < z_1$. Finally, the condition $V_{IV} > V_{I}$ is equivalent to

\[ \lambda(r - c)\mu - \lambda r \phi \left( p_0 + \frac{2zT}{1 + \rho} \right)^{-1/2} - \kappa T' > \lambda(r - c)\mu - \lambda r \phi \sigma. \]

Simplifying this condition we get

\[ z > z_0 = \frac{1 + \rho}{2T} \left( \frac{1}{A} - p_0 \right), \]

where $A = \sigma - \kappa T'/\lambda r \phi$. It is straightforward to show that $z_0 < z_1$ for $\rho \in (0, 1)$. Therefore we can conclude that the investment game has a prisoners dilemma type of structure for all $z \in (z_0, z_1]$. 