1. Professor Nutty holds office hours everyday to answer students’ questions. Students arrive at an average rate of 10 per hour. Assume that arrivals are Poisson distributed. Professor Nutty can process students at an average rate of 12 per hour. Assume that the time each student takes is exponentially distributed. What is the average number of students waiting outside Professor Nutty’s office and how long do they wait on an average?

This is an M/M/1 queue.

\[ W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{10}{(12 \times 2)} = 0.416 \text{ hrs} = 25 \text{ min} \]

\[ L_q = \lambda \cdot W_q = 10 \times 0.416 = 4.16 \text{ students} \]

Professor Nutty is concerned that students wait too long. As a remedy, he asks his TA to help him during his office hours. The TA is also able to advise students at the rate of 12 per hour. Students queue at the professor’s office and are answered by whoever becomes available when it’s their turn. What is the average number of students waiting outside Professor Nutty’s office and how long do they wait on an average?

This is an M/M/2 queue. We need to find \( L_q \) for \( s = 2 \) and \( \frac{\lambda}{\mu} = 0.833 \).

Interpolating between \( L_q = 0.1523 \) for \( \frac{\lambda}{\mu} = 0.8 \) and \( L_q = 0.1873 \) for \( \frac{\lambda}{\mu} = 0.85 \) gives \( L_q = 0.1523 + \frac{33}{50} \cdot (0.1873 - 0.1523) = 0.1754 \) students on average.

\[ W_q = \frac{L_q}{\lambda} = \frac{0.1754}{10} = 0.01754 \text{ hrs} = 1.05 \text{ min.} \]

2. The wheat harvesting season in the Midwest is short, and most farmers deliver their truckloads of wheat to a giant central storage bin within a two-week span. Consequently, wheat-filled trucks waiting to unload and return to the fields have been known to back up for about a block, at the receiving bin. The central bin is owned cooperatively, and the farmers have assigned a waiting cost of $18 per hour for each truck and driver, based on the cost of truck rental and idle driver time. The storage bin area is operated 16 hours a day, 7 days a week during the harvest season, and is capable of unloading an average of 35 trucks per hour (assume the service time to be exponentially distributed). Full trucks arrive at an average interval of 2 minutes (assume Poisson arrivals).

(a) What is the average number of trucks and the average time per truck in the system?

\[ L_s = \frac{\lambda}{(\mu-\lambda)} = \frac{30}{(35 - 30)} = 6; \quad W_s = \frac{L_s}{\lambda} = \frac{6}{30} \text{ hr.} = 12 \text{ min.} \]

(b) What fraction of time do you expect to have more than 3 trucks in the system?
\[ 1 - P_0 - P_1 - P_2 - P_3 = 1 - (1 - \rho) \rho^0 - (1 - \rho) \rho^1 - (1 - \rho) \rho^2 - (1 - \rho) \rho^3 = 0.54 \]

(c) The cooperative uses the storage bin heavily for only about 2 weeks per year. Farmers estimate that enlarging the bin would speed up the unloading process, so that each truck can be unloaded in 80 sec. on an average (assume that the unloading time is still exponentially distributed). It will cost about $9,000 to expand the bin. Would it be worthwhile to do so? Explain.

\[ \mu (\text{new}) = 45/\text{hr}; \quad L_s (\text{new}) = 2 \]

Savings from reduced waiting = $18 \times 16 \times (6 - 2) \times 7 \times 2 = $16128 > $9000 so Yes!