Continuous-Time Models for Estimating Picker Blocking in Order-Picking-Systems

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Abstract

Congestion is a problem in order-picking-systems. We developed CTMC models to represent picker blocking in cases including pickers with different exponentially distributed picking times and \( k \) pickers, for narrow and wide aisle systems. The focus was not to produce an all-inclusive model, but to generate models that give designers insight of the system, allowing them to reduce the number of variables considered. We ran instances with various parameters and validated the models against simulation. We found stochastic speeds generate more blockage that deterministic ones, differences in pickers cause more congestion, systems with \( k \) pickers are more complex and have greater blockage.

Keywords
Congestion, Order-Picking-Systems, Picker Blocking, jMarkov, Continuous Time Markov Chains

1. Introduction

One of the most common problems arising in traditional man-to-part order-picking-systems (OPS) is picker congestion. This occurs when a picker is unable to reach her destination for one of two reasons: there is not enough room in the aisle to pass another picker who is in the way (in-the-aisle blocking), or there is enough room, but another picker is picking at the pick-face she is headed for (pick-face blocking). The former occurs in what we’ll consider as narrow aisles and the latter in wide aisles. When a picker gets blocked congestion ensues, creating idle times, reduced efficiency, late deliveries and a reduction of the expected throughput. Existing models which calculate OPS throughput do not consider congestion and may overestimate it, our aim here is to accurately estimate the fraction of time which a picker is blocked in order to adjust throughput calculations by this factor to get a more realistic estimate.

2. Literature Review

While picker congestion had not been studied specifically in the literature until recently, some authors had considered it as a possible inconvenience. Also, simulation studies conducted in an OPS with different configurations clearly showed the presence of congestion due to picker blocking, as in Petersen and Aase [1]. However, the first to take on the problem from an analytical standpoint were Gue, et.al. [2]. They model a narrow aisle OPS, with one-directional travel, where pickers travel the totality of the warehouse each time, so they can model it as a round aisle where pickers go around infinitely without an I/O point, stopping at each pick location with a known constant probability. They also consider the following assumptions: There is a finite number of picking locations, the probability of stopping at a given location is independent of all others, the time to pick at a location or to walk past it is known and constant, there are only two pickers in the warehouse and they are identical and independent of each other. With these assumptions they consider 2 cases, one where pickers pick and walk past a picking location in the same time or when walking takes no time at all and picking takes one time unit. For each case they build discrete time Markov chain (DTMC) models where one time period is the time to make a pick. For the case where the pick:walk speed ratio is 1 they define

\[
X_t = \{\text{Distance between pickers, last action of each.}\} \quad S = \{1_{pp}, 1_{pw}, \text{block}, \ldots, (n-1)_{wp}, (n-1)_{ww}\} \quad (1)
\]

Notice \( w \) is walking and \( p \) is picking. They proceed to define a transition matrix and find a closed-form expression for blocking. In the case of infinite walking speed they define the state of the DTMC as

\[
X_t = \{\text{Distance between pickers.}\} \quad S = \{0, 1, 2, 3, \ldots, n-1, n\} \quad (2)
\]
States $X_t = 0$ and $X_t = n$ represent blocking. Here they also solve the DTMC analytically arriving at a closed form expression for blocking as a function of the stopping probability.

Later Meller and Parikh [3] would conduct a similar analysis for a wide aisle OPS. In this case there is only pick-face blocking. Just like before they consider the same assumptions and the same two cases: pick:walk speed ratio of 1 or infinite walking speed. They construct DTMC models for both, in the case where speeds are the same they define face blocking. Just like before they consider the same assumptions and the same two cases: pick:walk speed ratio of 1 or infinite walking speed. They construct DTMC models for both, in the case where speeds are the same they define the state of the DTMC as

$$X_t = \{\text{Distance between pickers, last action of each.}\} \quad S = \{0_{pp}(\text{blocked}), 0_{pw}, 0_{wp}, \ldots, (n-1)_{ww}\}$$

Notice there is only one blocked state now. For the infinite walking speed case they define the state as

$$X_t = \{\text{Distance between pickers}\} \quad S = \{0, 1, 2, \ldots, n-2, n-1\}$$

Also only one blocked state here. For both cases they build the transition matrices and arrive at analytical results for the first case and numeric results for the second case. They expanded this research to include the possibility of picking multiple SKUs at each pick location in Parikh and Meller [4]. Skufca [5] extends the results of the narrow aisle infinite walking speed model to include $k$ pickers under the same assumptions as Gue, et.al. [2] and arrives at a closed form solution for that case.

### 3. Model Descriptions

Here we will present models which allow studying cases which include differences between pickers, stochastic speeds and several pickers in the aisle. Some assumptions will remain the same for all the models. The pickers walk the entire warehouse on each trip, hence we will continue to model it as Gue et.al. [2] which is a circular aisle the pickers travel through constantly without an I/O point. The warehouse has a finite number of picking locations or pick-faces which we will call $n$. Every picker stops and picks at a given location with a fixed know probability $p$ which is the same for every picker and every location. Picking and walking times, i.e. the time it takes to make a pick or walk past a given picking location, are exponentially distributed random variables and are the same for every location (but may differ between pickers), and they are independent of each other.

#### 3.1 Model for 2 Different Pickers in a Narrow Aisle

Here we consider only in-the-aisle blocking, we will consider that the mean picking rate for picker $i$ is $\mu_i$ and the mean walking rate is $\lambda_i$. We modeled this situation as a Continuous Time Markov Chain (CTMC) with the state definition and state space given by

$$X = \{\text{Distance between pickers and current activity of each}\}$$

$$S = \{0_{ww}, 0_{wp}, 0_{pw}, 1_{ww}, 1_{wp}, 1_{pw}, \ldots, n_{ww}, n_{wp}, n_{pw}, n_{pp}\}$$

Note that states which include distance 0 or $n$ are blocked states, here a picker can be blocked by someone who is picking or simply by a picker that is walking more slowly than they are. In these cases the letter of the blocked picker is not their current action, but the one they intended to perform at the next pick location and will perform when they get there. The transition diagram for the Markov Chain is presented in Figure 1, a transition occurs each time a picker changes pick location. Keep in mind in the graph $p$ stands for the picking probability and $q = 1 - p$.

The result is a model of a $n$ level, 4 phase, Quasi-Birth and Death process (QBD), but in order to reach a closed form solution it would be necessary to invert several matrices which in our particular case are singular, so we ran various scenarios and present the results in the next section.

#### 3.2 Model for 2 Different Pickers in a Wide Aisle

In this case we will consider only pick-face blocking, we will have the same definitions for $\lambda_i$ and $\mu_i$, however since pickers can now overtake each other we run into an issue which we had not considered earlier. In the previously described model the order of the pickers remained the same as passing is not possible in narrow aisles, so, with the previous state definitions whenever someone was blocked one could tell who by looking at the state where blocking occurred and checking if it was 0 or $n$. And since it would be possible for one picker to lap another the distance could go to infinite positions. To resolve the latter issue we fix one picker as number 1 and take the relative distance between her and the other picker, positive ahead and negative behind, so a picker can go from state $(-n/2 + 1)$ to state $(n/2)$.
for an even value of $n$ for example. As for the former, this is resolved by adding an additional possible activity we’ll call blocked, there are only two blocked states since here no one can be blocked if someone’s walking. With these considerations the resulting state definition and space for an even value of $n$ are

$$X = \{ \text{Distance between pickers 1 and 2 and current activity of each} \}$$

$$S = \{ \left( -\frac{n}{2} + 1 \right)_{ww}, \left( -\frac{n}{2} + 1 \right)_{wp}, \left( -\frac{n}{2} + 1 \right)_{pw}, 0_{ww}, 0_{wp}, 0_{pw}, 0_{pb}, 0_{bp}, \ldots, \left( \frac{n}{2} \right)_{ww}, \left( \frac{n}{2} \right)_{wp}, \left( \frac{n}{2} \right)_{pw}, \left( \frac{n}{2} \right)_{pp} \}$$

(6)

Notice that now distance 0 has 5 states associated with it instead of the 4 states the others have. The transition graph is presented in Figure 2. Note that the states in dotted lines have been duplicated for clarity.

Due to the transitions between the outermost states, and the fact that distance zero has a different number of states associated with it than the others this is no longer a QBD, making it particularly hard to solve analytically.

3.3 Model for $k$ Identical Pickers in a Narrow Aisle

This model will be an extension of the one presented in section 3.1. Keeping the same assumption but allowing $k$ pickers in the aisles, since there is no passing we can describe the system with $k$ distances, although strictly only $k - 1$ are necessary using $k$ increases clarity. Also, to complete the description we need $k$ positions to describe each pickers’ current activity. So the state can be a vector of the following form

$$X = \{ (x_1, x_2, \ldots, x_k, a_1, a_2, \ldots, a_k) \}$$

$$x_i = \{ \text{Distance between picker i and the next.} \} \quad a_i = \{ \text{Picker i’s current activity} \}$$

$$0 \leq x_i \leq n ; x_i \in \mathbb{Z} \quad \forall i \in \{ 1..k \} \quad a_i \in \{ w, p \} \quad \forall i \in \{ 1..k \} \sum_{i=1}^{k} x_i = n$$

(7)

All possible combinations of $x_i, a_i$ which satisfy the conditions above will be valid states $X$, forming state space $S$. Note that any zeros appearing would represent a blocked picker, in this case $a_i$ will represent the activity the picker will perform when he is no longer blocked, also several pickers can be blocked at once. To characterize the transitions between states a transition graph is impractical, however using a number of rules it is possible to build a structured transition matrix, which we do not include here because of space constraints, and solve numerically for instances with...
few pickers.

3.4 Model for k Identical Pickers in a Wide Aisle

Using a similar logic as in the previous case we can extend the two picker model to include \( k \) pickers in the aisle.

\[
X = \{(x_1, x_2, \ldots, x_k, a_1, a_2, \ldots, a_k)\}
\]

\[
x_i = \{\text{Distance between picker 1 and } i\} \quad a_i = \{\text{Picker } i\text{'s current activity}\}
\]

\[
-\left(\frac{n}{2} + 1\right) \leq x_i \leq \frac{n}{2} \quad x_i \in \mathbb{Z} \quad \forall i \in \{1..k\} \quad a_i \in \{w, p, b\} \quad \forall i \in \{1..k\}
\]

Notice that the sum constraint is gone, to define the valid states of \( S \) we must determine which \( x_i, a_i \) combinations are possible. First, only combinations with \( x_1 = 0 \), can be valid. Also, there are now 3 activities (walking, picking and blocked), here \( a_i = b \) only if \( \min_j(|x_i - x_j|) = 0 \), thus only a few combinations include \( a_i = b \), although several pickers can be blocked at a time. Using these criteria it is possible to systematically build the state space and at the same time the transition matrix for the CTMC and the solve it numerically, as we will show in the next section.

4. Numerical Results

In order to systematically construct and numerically solve large scale versions of the previous models we used \( jMarkov \), a Java library developed by Riaño et al. [6] for these kind of models.

4.1 Models for 2 Different Pickers

The first thing we wanted to consider was how these models related with previous ones by other authors and the effects of picking probability \( p \), and the ratio between picking and walking speeds of the pickers would have on the fraction of time each picker is blocked. Figure 3 shows a comparison of the percentage of time each picker is blocked for both narrow and wide aisles varying \( p \) and the pick:block speed ratio.

We can see, in Figure 3, that blocking in narrow aisles is always greater than for wide ones, we can also see that these differences become most apparent for lower values of \( p \). Also, in narrow aisles blocking achieves a maximum near 0.3, while for wide aisles it is always an increasing function from 0 to 1. In both cases we see that congestion increases as walking speed increases relative to picking speed, this is quite intuitive since pickers will have greater chances to run into each other and will take longer at each pick. The case for a 1:100 ratio is different from the rest.
Figure 3: Percentage of time each picker is blocked. \( n = 20 \). Changes in the pick:walk speed ratio, identical pickers.

and yet similar to the 1 : \( \infty \) ratio case studied by Gue et.al. [2]. Next, in Figure 4, we consider the case where one picker is faster than the other.

Figure 4: Percentage of time the slower picker is blocked. \( n = 20 \), one \( x\% \) faster at picking.

It is clear from Figure 4 that greater differences between pickers result in increased blockage of the slower picker. It is intuitive that this will also result in reduced blockage of the faster one, however as we can see here, the increase is sometimes greater than 400\% for doubling one pickers speed, hence the reduction of blockage for the faster one does not totally offset this and the end result is increased congestion in the system. This effect is far more dramatic in narrow aisle OPSs than wide aisle OPSs, except for very high values of \( p \) when it is just as critical.

4.2 Models for \( k \) Identical Pickers

For cases where \( k > 2 \) the main interest will be how the increased number of pickers will affect the overall behavior of the congestion within the OPS. Figure 5 shows the comparative effects for 2, 3 or 4 pickers in the aisle.

Figure 5 shows that increasing the number of pickers in either a narrow or wide aisle OPS has the effect of increasing congestion, but without altering its behavior. It also shows that each additional picker bring less congestion to the system than the previous one. This makes it less expensive in terms of congestion to hire additional pickers in busier, larger warehouses (which presumably have many from the start) than at warehouses that only have a few.

5. Conclusion

After verification and validation against simulation we can say that the previously presented models and results adequately represent a realistically simulated warehouse environment where the conditions are similar to those used to develop said models, except in the special case of pick:walk speed ratio equal to 1, in this case the simulation results were somewhat different for more realistic settings. Insights gathered from these can be useful to warehouse designers in making key decisions which affect throughput and congestion.
We observed that our models always predict greater congestion than those by previous authors, this can be explained in our choice of stochastic picking and walking times, though the overall shapes of blocked time vs. picking probability curves remains the same. We noticed wide aisle OPSs present less congestion, regardless of picker speeds, picking probabilities, differences between pickets, etc., the difference however is reduced as picking probability approaches 1. Also, large differences in speed between pickers have considerable immediate effects in OPSs with narrow aisles for any size and picking probability, whereas in wide aisle OPSs these differences only come into play for very high values of \( p \). One could argue that for systems with many short order lists and sparse stops, wide aisle systems should be better in terms of reducing congestion and increasing throughput, particularly if pickers are very different; however in a OPS with longer order lists, multiple stops per trip and more similar pickers the savings in congestion, specially if \( p > .80 \) would probably be far inferior to those obtained by the greater space utilization of narrow aisles.

Just as expected, increasing picking locations decreases blocking, while increasing the number of pickers increases it. An interesting observation is that these variations are not linear, but rather marginally decreasing, i.e. the additional congestion for the \( n \)th picker is always less than for the \( (n-1) \)th, likewise for the number of picking locations. Picking probability \( p \) has a strong effect on congestion, while the picking strategy (SOP, Batch) reflects strongly on \( p \). One can take advantage of this knowledge and avoid choosing a strategy with very high values of \( p \) if the system under consideration has wide aisles and also avoid values of \( p \) close to 0.25 in narrow aisles OPSs, this can be done for example by switching to a picking policy with large order batches.

Future work includes extending the models to remove restrictive assumptions such as a constant picking probability or exponentially distributed times. It would also be useful to try to find closed form approximations of the expected blockage time to make these calculations faster, since these models may be very large and take a long time to solve.

References


