

Technical Notes and Correspondence

Further Results on Linear Nonquadratic Optimal Control

Chih-Hua Hsu and Jeff S. Shamma

Abstract—This note continues an investigation by the authors of minimizing the transient response of a linear system as measured by nonquadratic penalty functions, in particular, penalty functions which have linear growth. First, this note shows that the optimal state feedback which minimizes the transient response in the case of no exogenous inputs also minimizes the induced ℓ^1 norm in case exogenous inputs are present. Second, it considers the case of constrained systems and derives bounds which establish the stability and performance of receding horizon control laws. Finally, this note illustrates the results for scheduling of reliable manufacturing systems.

Index Terms—Disturbance rejection, ℓ^1 , linear programming, optimal control.

I. INTRODUCTION

In [19], the authors considered the so-called linear nonquadratic (LnQ) minimization problem

$$J_{\text{LnQ}}(x_o) = \inf_{u(\cdot)} \sum_{k=0}^{\infty} |z(k)|_1 \quad (1)$$

for the discrete-time linear system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), & x(0) &= x_o \\ z(k) &= Cx(k) + Du(k) \end{aligned} \quad (2)$$

with dimensions $x(k) \in \mathcal{R}^{n_x}$, $u(k) \in \mathcal{R}^{n_u}$, $z(k) \in \mathcal{R}^{n_z}$.

This optimization differs from the popular linear quadratic (LQ) problem with cost function

$$J_{\text{LQ}}(x_o) = \inf_{u(\cdot)} \sum_{k=0}^{\infty} |z(k)|_2^2$$

which has a quadratic penalty function. For some problems, a non-quadratic penalty function better represents the performance objective, e.g., in manufacturing systems [9]. Another advantage is the extension to more general settings, such as LPV systems, where a quadratic penalty function presents certain computational difficulties [20].

In [19], the authors showed how to approximate the optimal infinite horizon feedback by receding horizon implementations of finite horizon optimal control laws. In the end, the optimal control, which is not necessarily unique, takes the form of a nonlinear feedback law

$$u(k) = g_{\text{LnQ}}(x(k)). \quad (3)$$

Manuscript received May 5, 1999; revised April 24, 2000. Recommended by Associate Editor P. Voulgaris. This work was supported in part by the NSF under Grant #ECS-9258005 and in part by the AFOSR under Grant #F49620-97-1-0197.

C.-H. Hsu is with the Department of Industrial Engineering and Management, I-Shou University, Kaohsiung County, Taiwan (e-mail: chhsu@isu.edu.tw).

J. S. Shamma is with the Mechanical and Aerospace Engineering Department, University of California, Los Angeles, Los Angeles, CA 90095 USA (e-mail: shamma@seas.ucla.edu).

Publisher Item Identifier S 0018-9286(01)03610-8.

This paper continues the investigation of LnQ optimal control. First, it is shown the optimal controller which minimizes the LnQ penalty function also minimizes an induced norm in the presence of exogenous disturbances. This result complements the work of [21] which considers induced norm optimization directly. Second, this paper considers the case of LnQ optimization in the presence of state and control constraints. Bounds are derived based on finite-horizon computations which guarantee the stability and performance of a receding horizon implementation of a finite horizon optimal control law. The approach taken here is complementary to the prevailing point of view in receding horizon control. Namely, exploiting prior work in control of constrained systems allows the issues of constraint satisfaction and finite horizon optimization to be taken separately. Finally, the methods are illustrated on a simple control problem for reliable manufacturing systems.

Notation: For $x \in \mathcal{R}^n$, define

$$\begin{aligned} |x|_p &= \left(\sum_{i=1}^n |x_i|^p \right)^{1/p} & |x|_{\infty} &= \max_{i=1, \dots, n} |x_i| \\ |x|_{\max} &= \max_i x_i. \end{aligned}$$

For an infinite sequence, $w = \{w(0), w(1), w(2), \dots\}$, with values in \mathcal{R}^n , define

$$\|w\|_{l^1} = \sum_{k=0}^{\infty} |w(k)|_1 \quad \|w\|_{l^1[0, N]} = \sum_{k=0}^N |w(k)|_1.$$

Finally, define $M(:, i)$ as the i th column of a matrix, M .

II. INDUCED NORM OPTIMALITY

In this section, we consider the following disturbance rejection problem. Consider now the linear system (2), but with an exogenous input $w(\cdot)$

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Lw(k), & x(0) &= 0 \\ z(k) &= Cx(k) + Du(k) \end{aligned} \quad (4)$$

with dimensions as in (2) and $w(k) \in \mathcal{R}^{n_w}$. Our objective is to find state feedback $u(k) = g(x(k))$, which achieves

$$J_{l^1} = \inf_{g(x)} \sup_w \frac{\|z\|_{\ell^1}}{\|w\|_{\ell^1}} \quad (5)$$

i.e., the closed-loop induced ℓ^1 norm.

The problem of induced ℓ^1 norm minimization via linear dynamic feedback is treated in the text [6], but in the context of induced ℓ^{∞} norm minimization. In the case of multivariable linear systems, the two induced norms are related by a simple transpose [8]. However, there is no similar relationship in the case of nonlinear feedback applied to linear systems.

The problem of minimizing the induced ℓ^{∞} norm under full state feedback was considered in [4], [18], where it was shown that the optimal feedback is a nonlinear function of the states, and a constructive procedure was presented. More recently, the problem of minimizing the induced ℓ^1 norm under full state feedback was considered in [21], where a generalization of the bounded real lemma was used to derive the optimal state feedback.

In this section, we will show that the optimal LnQ state feedback for transient response minimization is also the optimal state feedback for induced norm minimization. We also comment that the LnQ objective has been shown to be equivalent to certain classes of stochastic disturbance rejection problems [14].

Assumption 2.1:

- 1) The pair $[A, B]$ is stabilizable.
- 2) The matrix $(C \ D)$ has full-column rank.

These assumptions assure the existence of an optimal LnQ state feedback law [19]. Assumption 2.1.2 simply states that all states and controls are penalized.

Theorem 2.1: The optimal induced ℓ^1 norm satisfies

$$J_{\ell^1} = \max_{i=1, \dots, n_w} J_{\text{LnQ}}(L(:, i))$$

and is achieved by the optimal LnQ feedback $u(k) = g_{\text{LnQ}}(x(k))$.

The remainder of this section is devoted to the proof of Theorem 2.1. The main idea of the proof is to show that the worst case disturbance is an impulse. This is a standard result in the case of linear systems. We will show that this is still the case in the presence of nonlinear feedback. The equivalent effect of an impulse is to set an initial condition for the unforced system (2), and the optimal control action is to minimize the resulting transient response.

First, consider the case where $n_w = 1$, i.e., a scalar disturbance. For any feedback law, the induced ℓ^1 norm is bounded below by $J_{\text{LnQ}}(L)$. This can be seen by inspecting the response to the impulse

$$w^*(k) = \begin{cases} 1, & k = 0 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Now assume that the feedback $g_{\text{LnQ}}(\cdot)$ does not achieve an induced ℓ^1 norm of $J(L)$. Then there exists a disturbance

$$w = \{w(0), w(1), \dots, w(N), 0, \dots\}$$

for some time horizon, N , such that

$$\frac{\|z\|_{\ell^1}}{\|w\|_{\ell^1}} > J_{\text{LnQ}}(L). \quad (7)$$

We can assume without loss of generality that $w(N) \neq 0$. We will show that the alternative disturbance

$$\tilde{w} = \{w(0), w(1), \dots, w(N-1), 0, \dots\}$$

which leads to the alternative response \tilde{z} also satisfies

$$\frac{\|\tilde{z}\|_{\ell^1}}{\|\tilde{w}\|_{\ell^1}} > J_{\text{LnQ}}(L).$$

Since $J_{\text{LnQ}}(\cdot)$ defines a norm on R^{n_x} [19], we have (8) shown at the bottom of the page, where $\alpha = \|z\|_{\ell^1[0, N]} + J_{\text{LnQ}}(Ax(N) + Bu(N))$, $\beta = J_{\text{LnQ}}(L)$, and $\gamma = \|w\|_{\ell^1[0, N-1]}$.

Define the function $f(y) = (\alpha + \beta y)/(\gamma + y)$ with $y > 0$, then $f'(y) = (\gamma\beta - \alpha)/(\gamma + y)^2$. It follows by hypothesis (7) that

$$J_{\text{LnQ}}(L) < \frac{\|z\|_{\ell^1}}{\|w\|_{\ell^1}} \leq \sup_{y>0} f(y).$$

There are two possibilities. First, if $\gamma\beta - \alpha \geq 0$, then f is nondecreasing, and

$$J_{\text{LnQ}}(L) < \sup_{y>0} f(y) \leq \lim_{y \rightarrow \infty} \frac{\alpha + \beta y}{\gamma + y} = \beta = J_{\text{LnQ}}(L)$$

which leads to a contradiction.

Therefore, $\gamma\beta - \alpha < 0$ must hold, which implies that f is monotonically decreasing, and

$$J_{\text{LnQ}}(L) < \sup_{y>0} f(y) \leq \frac{\alpha + \beta y}{\gamma + y} \Big|_{y=0} = \frac{\alpha}{\gamma}.$$

Therefore, we have

$$\begin{aligned} J_{\text{LnQ}}(L) &< \frac{\|z\|_{\ell^1[0, N]} + J_{\text{LnQ}}(Ax(N) + Bu(N))}{\|w\|_{\ell^1[0, N-1]}} \\ &= \frac{\|\tilde{z}\|_{\ell^1}}{\|\tilde{w}\|_{\ell^1}}. \end{aligned}$$

By repeating this procedure, one can progressively drop the last nonzero term in the disturbance, thereby finally leading to the conclusion that

$$J_{\text{LnQ}}(L) < \frac{\|z^*\|_{\ell^1}}{\|w^*\|_{\ell^1}}$$

where w^* is the impulse defined in (6) and z^* is the corresponding response. However, this cannot be the case since the feedback $g_{\text{LnQ}}(\cdot)$, by definition, achieves

$$\frac{\|z^*\|_{\ell^1}}{\|w^*\|_{\ell^1}} = J_{\text{LnQ}}(L).$$

Therefore, the conclusion is that

$$J_{\ell^1} = J_{\text{LnQ}}(L).$$

$$\begin{aligned} \frac{\|z\|_{\ell^1}}{\|w\|_{\ell^1}} &= \frac{\|z\|_{\ell^1[0, N]} + \|z\|_{\ell^1[N+1, \infty)}}{\|w\|_{\ell^1[0, N-1]} + |w(N)|} \\ &= \frac{\|z\|_{\ell^1[0, N]} + J_{\text{LnQ}}(x(N+1))}{\|w\|_{\ell^1[0, N-1]} + |w(N)|} \\ &= \frac{\|z\|_{\ell^1[0, N]} + J_{\text{LnQ}}(Ax(N) + Bu(N) + Lw(N))}{\|w\|_{\ell^1[0, N-1]} + |w(N)|} \\ &\leq \frac{\|z\|_{\ell^1[0, N]} + J_{\text{LnQ}}(Ax(N) + Bu(N)) + J_{\text{LnQ}}(Lw(N))}{\|w\|_{\ell^1[0, N-1]} + |w(N)|} \\ &= \frac{\alpha + \beta|w(N)|}{\gamma + |w(N)|} \end{aligned} \quad (8)$$

Similar arguments hold for the multiple-input case $n_w > 1$. As before

$$J_{\ell^1} > \max_{i=1, \dots, n_w} J_{\text{LnQ}}(L(:, i)).$$

Furthermore, if the feedback $g_{\text{LnQ}}(\cdot)$ does not achieve this level of performance, then there exists an impulsive disturbance, $w^*(k)$, which is active only at $k = 0$ such that

$$\max_{i=1, \dots, n_w} J_{\text{LnQ}}(L(:, i)) < \frac{\|z^*\|_{\ell^1}}{|w^*(0)|_1}$$

where z^* is the response to w^* . However

$$\begin{aligned} \frac{\|z^*\|_{\ell^1}}{|w^*(0)|_1} &= \frac{J_{\text{LnQ}}(Lw^*(0))}{|w^*(0)|_1} \\ &\leq \sum_{i=1}^{n_w} \frac{|w_i^*(0)| J_{\text{LnQ}}(L(:, i))}{|w^*(0)|_1} \\ &= \max_{i=1, \dots, n_w} J_{\text{LnQ}}(L(:, i)). \end{aligned}$$

Again, this leads to a contradiction, and, therefore, the feedback $g_{\text{LnQ}}(\cdot)$ achieves the optimal induced ℓ^1 norm.

III. LNQ OPTIMIZATION WITH CONSTRAINTS

In this section, we again consider transient minimization (1) for the undisturbed linear system (2), but now with state and control constraints

$$(x(k), u(k)) \in \Gamma = \{(x, u) : |Ex + Fu|_{\infty} \leq 1\}. \quad (9)$$

It has been shown in [12] that receding horizon control can be used to approximate the infinite horizon optimal control law. In this note, we will derive bounds based on finite-horizon computations which explicitly bound the infinite horizon performance of a receding horizon control.

A primary motivation for receding horizon control policies for linear systems is the presence of constraints (cf., [17] and references therein), and an important issue is the infinite horizon feasibility of constraints. In this paper, we will make a departure from this viewpoint by *separating* the issues of constraints and optimization. In particular, we will *assume* that the constraints (9) satisfy a sort of invariance property. Such a viewpoint does not sidestep the issue of constraints. Suppose it is desired to maintain

$$|E_{\text{orig}}x(k) + F_{\text{orig}}u(k)|_{\infty} \leq 1. \quad (10)$$

Prior work on constrained systems (e.g., [3], [10], [11], [5], and [18]) can be used to determine whether or not this is achievable over an infinite horizon. If this is achievable, then there exist new constraints

$$|E_{\text{new}}x(k) + F_{\text{new}}u(k)|_{\infty} \leq 1 \quad (11)$$

which capture the infinite horizon feasibility as follows. At any time k , there exists control inputs $\{u(k), u(k+1), u(k+2), \dots\}$ that assure that the original constraints (10) are satisfied if and only if the current control input $u(k)$ satisfies the new constraints (11).¹ In other words, the new constraints represent a necessary and sufficient pointwise-in-time “translation” of the original constraints. With this viewpoint, the issue of infinite horizon feasibility is addressed *a priori*, and receding horizon control is only a means to the end of approximating the infinite horizon optimal control.

¹Note that an infinitesimal relaxation of the constraints (10) may be required in order to obtain a finite collection of constraints (11)

In addition to Assumption 2.1, we will make the following assumption on the constraints (9), which reflect the present viewpoint.

Assumption 3.1:

- 1) Γ is compact.
- 2) Define the convex projection

$$\Xi = \{x : (x, u) \in \Gamma \text{ for some } u\}.$$

There exist positive $\lambda < 1$ and N^* such that for all $x(0) \in \Xi$, there exist control inputs $\{u(0), \dots, u(N^* - 1)\}$ such that $(x(k), u(k)) \in \Gamma$ for $k = 0, \dots, N^* - 1$ and $x(N^*) \in \lambda\Xi$.

The property in Assumption 3.1.2 has been called N^* -step λ -contractiveness in [5], [11]. As discussed earlier, this assumption reflects that the issue of infinite horizon feasibility has already been considered in the formulation of the constraints (9).

We now state two results in preparation for the main result. First, define the finite-horizon optimization

$$J_N(x_o) = \inf_{u(\cdot)} \sum_{k=0}^{N-1} |z(k)|_1 \quad (12)$$

subject to constraints (9).

Proposition 3.1: The finite-horizon optimal costs (12) admit the matrix representation

$$J_N(x_o) = |M_N x_o + m_N|_{\max}$$

for appropriately dimensioned matrices M_N , and column vectors m_N .

Proof: The computation of the M_N and m_N can be done *a priori* through an implementation of standard dynamic programming recursions [2]. ■

Proposition 3.2: The $J_N(x)$ form a uniformly convergent sequence of continuous functions on Ξ .

Proof: The N^* -step λ -contractiveness assumption implies a uniform upper bound on $J_N(x)$ for all N . This implies pointwise convergence of $J_N(x)$ for all $x \in \Xi$. Since Ξ is compact, we have uniform convergence as well. ■

Now, define

$$\alpha_{N-1} = \min\{\alpha : J_N(x) \leq \alpha J_{N-1}(x) \quad \forall x \in \Xi\}.$$

By Proposition 3.2, we have that

$$\lim_{N \rightarrow \infty} \alpha_N = 1.$$

We now state our main result which provides an infinite horizon performance bound for the receding horizon control law

$$\begin{aligned} \phi(x; N) = \arg \min_{\{u : |Ex + Fu|_{\infty} \leq 1\}} \{ & |Cx + Du|_1 \\ & + J_{N-1}(Ax + Bu) \}. \end{aligned}$$

Theorem 3.1: Let

$$\beta = \max\{|Cx + Du|_1 : (x, u) \in \Gamma\}.$$

Define $\|(x, u)\|_{\Gamma}$ as the norm whose unit ball is the set Γ , and let

$$\delta = \max_{(x, u) \in \Gamma} \frac{\|(x, u)\|_{\Gamma}}{|Cx + Du|_1}.$$

The receding horizon control $u(k) = \phi(x(k); N)$ is stabilizing for all N such that

$$(\alpha_{N-1} - 1) \frac{\delta \beta N^*}{1 - \lambda} < 1.$$

Furthermore, the infinite-horizon performance satisfies

$$\sum_{k=0}^{\infty} |z(k)|_1 \leq \frac{J_N(x(0)) - \gamma \left| \begin{pmatrix} x(0) \\ u(0) \end{pmatrix} \right|_1}{1 - \gamma}$$

for

$$\gamma = \frac{\alpha_{N-1} - 1}{\alpha_{N^*-1}} \frac{\delta \beta N^*}{1 - \lambda} < 1.$$

Proof: Let $x(k)$ and $u(k)$ be the state and control trajectory, respectively, resulting from the receding horizon policy $u(k) = \phi(x(k); N)$.

For any k

$$J_N(x(k)) = |Cx(k) + Du(k)|_1 + J_{N-1}(x(k+1)).$$

Therefore, for $x(k) \in \Xi$

$$\begin{aligned} J_N(x(k)) - J_N(x(k+1)) &= |Cx(k) + Du(k)|_1 + J_{N-1}(x(k+1)) - J_N(x(k+1)) \\ &\geq |Cx(k) + Du(k)|_1 - (\alpha_{N-1} - 1) J_{N-1}(x(k+1)) \\ &\geq |Cx(k) + Du(k)|_1 - (\alpha_{N-1} - 1) J_N(x(k)). \end{aligned}$$

Now, for any positive scalar $\rho < 1$

$$J_N(x) \leq \rho J_N\left(\frac{x}{\rho}\right)$$

as long as $x/\rho \in \Xi$. Therefore

$$\begin{aligned} J_N(x(k)) &\leq \left\| \begin{pmatrix} x(k) \\ u(k) \end{pmatrix} \right\|_{\Gamma} J_N \frac{\left(\frac{x(k)}{\rho} \right)}{\left\| \begin{pmatrix} x(k) \\ u(k) \end{pmatrix} \right\|_{\Gamma}} \\ &\leq \left\| \begin{pmatrix} x(k) \\ u(k) \end{pmatrix} \right\|_{\Gamma} \frac{\beta N^*}{1 - \lambda}. \end{aligned}$$

Combining the above inequalities leads to

$$\begin{aligned} J_N(x(k)) - J_N(x(k+1)) &\geq \left(1 - (\alpha_{N-1} - 1) \frac{\delta \beta N^*}{1 - \lambda} \right) |Cx(k) + Du(k)|_1. \end{aligned}$$

The above establishes that $J_N(\cdot)$ provides a Lyapunov function for α_N sufficiently close to one. One can go on further to establish exponential stability since Γ is compact. The details are omitted here.

We will now establish the performance bound. We can bound

$$\begin{aligned} |Cx(0) + Du(0)|_1 &= J_N(x(0)) - J_{N-1}(x(1)) \\ &= J_N(x(0)) - J_N(x(1)) + J_N(x(1)) - J_{N-1}(x(1)) \\ &\leq J_N(x(0)) - J_N(x(1)) + J_N(x(1)) - \frac{1}{\alpha_{N-1}} J_N(x(1)) \\ &= J_N(x(0)) - J_N(x(1)) + \left(\frac{\alpha_{N-1} - 1}{\alpha_{N-1}} \right) J_N(x(1)). \end{aligned}$$

Similarly

$$\begin{aligned} |Cx(1) + Du(1)|_1 &\leq J_N(x(1)) - J_N(x(2)) \\ &\quad + \left(\frac{\alpha_{N-1} - 1}{\alpha_{N-1}} \right) J_N(x(2)). \end{aligned}$$

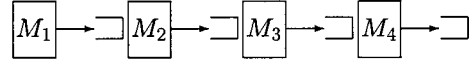


Fig. 1. Four-machine transfer line.

Therefore

$$\begin{aligned} &\sum_{k=0}^{\infty} |Cx(k) + Du(k)|_1 \\ &\leq J_N(x(0)) + \left(\frac{\alpha_{N-1} - 1}{\alpha_{N-1}} \right) \sum_{k=1}^{\infty} J_N(x(k)) \\ &\leq J_N(x(0)) + \left(\frac{\alpha_{N-1} - 1}{\alpha_{N-1}} \right) \\ &\quad \times \sum_{k=1}^{\infty} \frac{\delta \beta N^*}{1 - \lambda} |Cx(k) + Du(k)|_1. \end{aligned}$$

Define $\gamma = (\alpha_{N-1} - 1)/(\alpha_{N-1})(\delta \beta N^*)/(1 - \lambda)$. Note that $\gamma < 1$ for α_{N-1} sufficiently close to one. Rearranging the above inequality then leads to the desired result. ■

IV. NUMERICAL EXAMPLE: TRANSFER LINES

Consider the four machine transfer line of Fig. 1. Material is processed by machines M_1 through M_4 in order to meet the demand, d . Each buffer x_i denotes the cumulative production of the i th machine less the cumulative production of the $(i+1)$ th machine. An exception is buffer x_4 which denotes the cumulative production of the 4th machine less the cumulative demand, i.e., total inventory or backlog.

These dynamics take the state equation form

$$x(k+1) = x(k) + \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} u(k) - \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} d \quad (13)$$

where $u_i(k)$ denotes the production of the i th machine.

The objective is to minimize

$$\sum_{k=0}^{\infty} c_1 x_1(k) + c_2 x_2(k) + c_3 x_3(k) + \max \{c_4^+ x_4(k), c_4^- x_4(k)\}$$

where all coefficients are positive except c_4^- , which is negative to penalize backlog. The constraints on states and controls are

$$0 \leq x(k) \leq x_{\max}$$

and

$$0 \leq u(k) \leq \mu$$

for specified maximum buffer sizes, x_{\max} , and machine capacities, μ . Provided that each machine capacity satisfies $\mu_i > d$, then the above constraints are N^* -step λ -contractive with $N^* = 1$. Note that the above system does not quite fit the formulation in the previous section in that it evolves over the positive quadrant. However, the previous analysis can easily be adapted to this setting.

Reference [15] considered such transfer lines, and showed that the optimal policy for each machine takes the form

$$u(k) = \begin{cases} 0, & k \leq T_i \\ \mu_i, & T_i < k < T_f \\ d, & k \geq T_f \end{cases}$$

where the T_i are ‘‘deferral times’’ to be computed, and T_f is a final time after which the buffers will be cleared, i.e., $x(T_f) = 0$. [15] derives calculations for the deferral times based on a decomposition of the line into sections according to bottleneck machines.

The LnQ procedure was used to compute a feedback control law for this system. The simulation parameters (from [15]) were as follows: Capacities: $\mu = \{1.5, 3, 2, 3\}$; Demand: $d = 1$; Holding costs: $c =$

$\{1, 2, 3, +3, -4\}$; Initial Levels: $x(0) = \{6, 12, 24, -24\}$. The final receding horizon length was $N = 10$.

In order to simplify offline computations, the cost-to-go at each stage was approximated by a simpler cost-to-go with linear growth based on a selected level set. The domain $|x(k)| \leq 1$ was used as a "domain of approximation." Note that the system is expected to operate over a larger domain than that for the cost function approximation.

The result was an explicit state feedback law $u(k) = g_{L\infty Q}(x(k))$ which led to deferral times of $T_0 = 18$, $T_3 = 6$, and $T_4 = 0$. The deferral time T_2 is triggered by the instant buffer x_3 is empty. These are the same deferral times calculated in [15], even though the calculated approximate cost here poorly predicts the actual cost. Note that deferral time T_1 is correct *even though* it occurs after the optimization horizon of $N = 10$.

V. CONCLUDING REMARKS

We conclude with some remarks regarding computations. As stated in Proposition 3.1, the finite-horizon costs take the form

$$J_N(x) = |M_N x + m_N|_{\max}$$

for appropriately dimensioned matrices M_N , and vectors m_N . It is possible to compute these matrices offline, and therefore to numerically verify the conditions in Theorem 3.1 *a priori*. However, this approach is computationally intractable for high dimension systems.

One approach toward alleviating the computational burden is to employ real-time optimization to compute the receding horizon control law. This significantly reduces offline computations at the cost of real-time computations involving possibly large linear programs. In the case of manufacturing scheduling problems, it is possible to exploit special structures of these linear programs in order to streamline computations [13], [16], [7].

Another possibility is to approximate the optimal cost function in performing the dynamic programming iterations. This concept of "approximate" dynamic programming is discussed in detail in [1]. The advantage here is a "closed-form" expression for the control law, and a lighter real-time computational burden. The offline computational burden, while less than that of direct dynamic programming, can still be significant.

REFERENCES

- [1] D. Bertsekas and J. Tsitsiklis, *Neuro-Dynamic Programming*. Belmont, MA: Athena, 1996.
- [2] D. P. Bertsekas, *Dynamic Programming and Optimal Control*. Belmont, MA: Athena, 1995.
- [3] D. P. Bertsekas and I. B. Rhodes, "On the Minimax reachability of target sets and target tubes," *Automatica*, vol. 7, pp. 233–247, 1971.
- [4] F. Blanchini and M. F. Sznajer, "Persistent disturbance rejection via static-state state feedback," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 1127–1131, June 1995.
- [5] M. Cwikel and P.-O. Gutman, "Convergence of an algorithm to find maximal state constraint sets for discrete-time linear dynamical systems with bounded controls and states," *IEEE Trans. Automat. Contr.*, vol. AC-31, no. 5, pp. 457–459, 1986.
- [6] M. A. Dahleh and I. J. Diaz-Bobillo, *Control of Uncertain Systems: A Linear Programming Approach*. Englewood Cliffs, NJ: Prentice-Hall, 1995.
- [7] P. d' Alessandro and E. De Santis, "General closed loop optimal solutions for linear dynamics systems with linear constraints and functional," *J. Math. Syst., Estim., Control*, vol. 6, no. 2, 1996.
- [8] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. New York: Academic, 1975.
- [9] S. B. Gershwin, *Manufacturing Systems Engineering*. Englewood Cliffs, NJ: Prentice-Hall, 1994.

- [10] J. D. Glover and F. C. Scheppe, "Control of linear dynamic systems with set constrained disturbances," *IEEE Trans. Automat. Contr.*, vol. AC-16, pp. 411–422, May 1971.
- [11] P.-O. Gutman and M. Cwikel, "Admissible sets and feedback control for discrete-time linear dynamical systems with bounded controls and states," *IEEE Trans. Automat. Contr.*, vol. AC-31, pp. 373–376, Apr. 1986.
- [12] S. S. Keerthi and E. G. Gilbert, "An existence theorem for discrete-time infinite-horizon optimal control problems," *IEEE Trans. Automat. Contr.*, vol. AC-30, pp. 907–909, May 1985.
- [13] X. Luo and D. Bertsimas, "A new algorithm for state-constrained separated continuous linear programs," *SIAM J. Control Optim.*, vol. 37, no. 1, pp. 177–210, 1998.
- [14] S. Meyn, "Stability and optimization of queueing networks," in *Mathematics of Stochastic Manufacturing Systems*, G. Yin and Q. Zhang, Eds. Providence, RI: Amer. Math. Soc., 1997, vol. 33, AMS Lectures in Applied Mathematics, pp. 175–200.
- [15] J. R. Perkins and P. R. Kumar, "Optimal control of pull manufacturing systems," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 2040–2051, Dec. 1995.
- [16] M. C. Pullan, "Forms of optimal solutions for separated continuous linear programs," *SIAM J. Control Optim.*, vol. 33, no. 6, pp. 1952–1977, 1995.
- [17] J. B. Rawlings and K. R. Muske, "The stability of constrained receding horizon control," *IEEE Trans. Automat. Contr.*, vol. 38, pp. 1512–1516, Oct. 1993.
- [18] J. S. Shamma, "Optimization of the ℓ^∞ -induced norm under full state feedback," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 533–544, Apr. 1996.
- [19] J. S. Shamma and D. Xiong, "Linear nonquadratic optimal control," *IEEE Trans. Automat. Contr.*, vol. 42, pp. 875–879, June 1997.
- [20] —, "Set-valued methods for linear parameter varying systems," *Automatica*, vol. 35, no. 6, pp. 1081–1089, 1999.
- [21] J. Yu and A. Sideris, "Optimal induced l^1 -norm state feedback control," *Automatica*, vol. 35, no. 5, pp. 819–827, 1999.

Fault Accommodation of a Class of Multivariable Nonlinear Dynamical Systems Using a Learning Approach

Marios M. Polycarpou

Abstract—This note presents a learning approach for accommodating faults occurring in a class of nonlinear multi-input–multi-output (MIMO) dynamical systems. Changes in the system dynamics due to a fault are modeled as unknown nonlinear functions of the measurable state variables. The closed-loop stability of the robust fault accommodation scheme is established using Lyapunov redesign methods. A simulation example, based on a model of a jet engine compression system, is used to illustrate the fault accommodation design procedure.

Index Terms—Fault accommodation, fault diagnosis, learning approach, neural network, nonlinear systems.

I. INTRODUCTION

The demand for increased productivity leads to more challenging operating conditions for many modern engineering systems. Such conditions increase the possibility of system failures, which are character-

Manuscript received October 28, 1999; revised May 24, 2000 and August 16, 2000. Recommended by Associate Editor J. Si.

The author is with the Department of Electrical and Computer Engineering and Computer Science, University of Cincinnati, Cincinnati, OH 45221-0030 USA.

Publisher Item Identifier S 0018-9286(01)03611-X.