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Competition, matching, and geographical clustering at early stages of the industry life cycle

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Abstract

We analyze the matching-based centripetal force leading to spatial clustering of firms within industries undergoing the early stages of their evolution. In these industries, the quality of the match between the characteristics of the inputs offered by suppliers and sought by assemblers' are uncertain. The model also incorporates the centrifugal force preventing spatial concentration of industry. The model shows that depending on the relative strength of the centripetal and the centrifugal forces of spatial distribution, the equilibrium outcome can be either concentration of all industry workers, suppliers and assemblers in one location or dispersion. We analyze the conditions, which determine the relative strength of the two forces.

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1. Introduction

Theoretical models of geographical clustering of economic activity go back at least to Marshall (1980). More recently economists interested in understanding the economics of spatial distribution of economic activity focused their attention on the composition of industries within clusters and how the mechanisms of geographical clustering is shaped by that composition. One line of thought (closely related to the Marshall's original explanation for agglomeration) emphasizes localization economies, referring to the benefits generated

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by the proximity of firms producing similar goods (see Glaeser et al., 1992). Another line of thought stresses the importance of urbanization economies, which account for the advantages associated with the overall level of activity prevailing in a particular area (see Jacobs, 1969). The proponents of the latter view argue that it is diversity rather than specialization which is the operative mechanism of geographical clustering.

In this paper, we suggest that the roles of diversity and specialization as factors of spatial distribution of economic activity varies with the evolution which industries undergo as they move through different stages of their life cycle. The early stages of the evolution of most industries are characterized by experimentation and a high rate of innovation by the firms populating the industries. Experimentation and innovation often imply that vertically-linked assembler and supplier firms have to interact in the absence of the leading design of the industry's end product. Under these circumstances, the lack of a dominant interface standard between the supply parts used in the assembly of the end product means that upstream suppliers have to produce intermediate parts to the specifications of individual downstream assemblers. This creates uncertainty with regard to the match between the needs of assemblers and the characteristics of the inputs provided by the suppliers. Greater diversity of firms in both groups (i.e., assemblers and suppliers) improves the quality of the matching process between them.

In contrast, after the emergence of the dominant end-product design, the relationship between assemblers and suppliers acquires greater certainty; at mature stages of industry evolution, firms rarely introduce new products and most of the time routinely produce standardized goods and services. The existence of stable interface standards between the segments of the industry's value chain implies that upstream firms can mass-produce intermediate products to satisfy standardized downstream demand, instead of customizing them to individual specifications of downstream firms. At the same time, the downstream firms are able to pick standardized inputs "off the shelf" without worrying how well they will fit in the assembly of the final product. Greater certainty in the vertical relationships at mature stages of industry evolution allows firms within each segment of the vertical value chain to compete in terms of cost rather than quality.¹

The main focus of this paper is the mechanism for the geographic concentration of relatively young industries characterized by the lack of stable interface standards in the vertical value chain. Therefore we develop an analytical framework in which spatial clusters emerge

¹ Modeling agglomeration effects in mature industries has been the main focus of the "new economic geography" literature that adopted an approach based on the Dixit–Stiglitz framework of monopolistic competition with iceberg trade costs (see Dixit & Stiglitz, 1977). However, this approach is not quite suitable for a study of agglomeration effects in innovative activity. Within this framework, geographical concentration emerges as equilibrium only at the intermediate level of trade costs because it is inefficient to service the entire geographical area from one location when trade costs are high. When they are low, the importance of geographical proximity to markets and suppliers is less important than the congestion effects due to the inelastic supply of immobile local resource. However, for industries at the early stages of their life cycles, trade costs are unlikely to be an important factor of spatial distribution because the highly innovative and experimental end products produced at these stages are characterized by high value-to-weight ratios. Therefore, the model of monopolistic competition with iceberg trade costs would predict that high-technology firms should be more-or-less spatially dispersed. But this prediction is in disagreement with the geography of high-growth and high-technology industries which demonstrate a strong tendency towards agglomeration.

as a result of the matching process among vertically-linked assemblers and suppliers, rather than as a result of traditional scale economies in manufacturing of standardized products. Specifically, in the analytical framework introduced in [Section 2](#), we show that competition in terms of quality among upstream input suppliers (e.g., providers of producer services) and downstream assemblers' uncertainty about the quality (productivity) of the inputs create matching-based external economies leading to geographical clusters. Our model assumes that all firms in the local upstream supplier industry have the same marginal cost and each of them can customize its products to specifications of each of the local downstream firms. Therefore, the main uncertainty in the matching process is over compatibility between the customized inputs offered by suppliers and the assemblers' needs for these inputs.²

Since marginal costs are identical across suppliers, quality competition among them can be interpreted as competition in "real prices," which are defined by the difference between prices and match quality parameters (see [Tirole, 1988](#)). To formalize assemblers' uncertainty, we adopt the order-statistics modeling framework of ex post quality competition among input suppliers. Specifically, we assume that the parameters characterizing the quality of the match between inputs offered by assemblers and the needs of suppliers are independent random draws from a probability distribution.

Our model uses the idea of matching under uncertainty to illustrate the interdependence between the location decisions of upstream suppliers and downstream assemblers. The downstream firms choose their location based on their expectations about the quality of the inputs, which they will be able to buy from the upstream firms at a given location. Since the realized input quality is the outcome of the matching process, the average quality is higher in the location with the greater density of the upstream firms. Therefore, locations with more diverse populations of heterogeneous suppliers attract more downstream assemblers. Similarly, the suppliers are attracted to locations with the greater density of assemblers because a higher probability of the good match in such locations translates into greater expected local demand for suppliers' products. Therefore, the matching process between suppliers and assemblers combined with competition among firms within segments of the vertical value chain creates an agglomerative force leading to concentration of all firms from both segments in one location.

In [Section 3](#), we extend the baseline framework considered in [Section 2](#) by adding labor as another segment to the vertical value chain, which in [Section 2](#) included only assemblers and suppliers. In this extended version of the model, an increase in assemblers' demand for inputs translates into greater suppliers' demand for labor that raises local wage. Since higher local wage increases the costs of all local input suppliers, prices for inputs also increase. This reduces the profits of assemblers who are no longer indifferent to the presence of other assemblers in the same location. Therefore, in addition to the centripetal force based on the matching externality due to the co-location of assemblers and suppliers, in the

² In another paper (see [Klimenko, 2003](#)), we consider an alternative framework in which suppliers are indistinguishable in terms of quality but may have different marginal cost realizations and compete in Bertrand fashion while assemblers search over the price offers of suppliers. In that model, suppliers' marginal costs are the main uncertainty that determines the location decisions of all agents in the economy. Using a different specification of the assembler's decision problem, we show that in that setting the centripetal force of agglomeration dominates the centrifugal congestion force and agents at all segments of the industry concentrate at the same location.

framework considered in Section 3, we have a centrifugal force based on the congestion effect due to the inelastic supply of local labor.³ In this setting, geographical concentration of firms representing all segments of the industry's value chain in one location is no longer assured. Depending on the relative strength of the centripetal and the centrifugal forces, the equilibrium outcome can be either concentration of all workers, suppliers and assemblers in one location or dispersion. We analyze the conditions, which determine the relative strength of the two forces.

There are a number of earlier theoretical papers which are close to our paper. In the models considered in Stahl (1982) and Wolinsky (1983), heterogeneous quality of products offered by sellers induces buyers to engage in comparison shopping. In this setting, the entry of new sellers creates a positive externality on the demand for the varieties offered by the incumbent sellers because buyers are attracted to the locations with greater variety of the goods offered by the sellers. As a result, sellers may have an incentive to cluster together. However, in these models geographical concentration of sellers does not lead to more competition and lower prices. Building on this earlier work, Schulz and Stahl (1996) analyze the linkage between the number of sellers and the price elasticity of demand at a given location and show that when the number of incumbent sellers is not too large, the positive demand externality from the entry of additional sellers outweighs the negative pressure on the profits due to the increase in competition among the sellers. Therefore, their model predicts that geographical clusters of vertically-linked industries will be of moderate sizes.

The ideas of the model are related to Dudey (1990), who considers the subgame perfect equilibria of a multi-stage game in which sellers of a homogenous commodity choose location, buyers decide which location to visit by estimating the price level at each location based on the number of local competing sellers, and sellers compete in quantities at each location. His analysis predicts that under certain conditions there is an equilibrium outcome in which sellers are able to mitigate the hold-up problem by choosing the same location. The buyers are, therefore, attracted to that location too. As a result, all buyers and sellers concentrate at a single location. Our model analyzes a mechanism which is similar in spirit to Dudey's (1990), but based on competition in terms of quality between the sellers and buyers' uncertainty about the quality of non-tradable products offered by the sellers. Therefore, in terms of its theoretical contribution, our paper complements Dudey's analysis of agglomeration, which is based on quantity competition between suppliers. Moreover, given the focus of our paper, the mechanism incorporating quality competition and supplier–assembler matching under uncertainty more accurately describes the agglomerative forces characterizing spatial distribution of the highly innovative industries at the early stages of their life cycle.

1.1. Review of supporting empirical literature

Two strands of the empirical literature on economic geography lend support to our theoretical model. First, there is empirical evidence suggesting that the variation in spatial

³ Centrifugal forces in the emerging industries are essentially the same as in the mature ones (transportation and congestion costs). But distance is a less important factor in the emerging high tech industries because the output is often either a service (i.e., completely non-tradable) or a blueprint, which has zero transportation cost. Therefore, in the paper we considered only the congestion cost.

clustering of mature manufacturing activity does not explain much of the geographic concentration of innovative activity characteristic of newer industries. Second, our theoretical framework is consistent with the empirical findings established in the literature documenting the role of diversity in geographical distribution of economic activity.

Evidence regarding the difference in the patterns of spatial clustering of mature manufacturing and innovative industrial activity was presented, for example, in [Audretsch and Feldman \(1996\)](#). In that paper, the maturity of the industry is identified by corresponding number of new commercial product announcements made by the firms populating the industry. Audretsch and Feldman measured the extent of spatial clusterization by calculating the industry's "locational Gini coefficient," using a method that was introduced in [Krugman \(1991\)](#). By analyzing the mean locational Gini coefficients for value-added, employment, and commercial product announcements for two-digit SIC manufacturing sectors, Audretsch and Feldman found evidence that the variation in geographical concentration of non-R&D value added and employment does not explain much of spatial clustering of new commercial products announcement. They concluded that the propensity for innovative activity to cluster varies across industries even after accounting for the geographic concentration of mature manufacturing.⁴

Since our model is focused on the link between diversity of the local input suppliers and innovativeness of the industry in general, the empirical support for our theory can be found in the literature assessing the relative importance of diversity and specialization as factors of geographical distribution of emerging industrial and innovative activity. For example, detailed micro evidence with regard to the evolution of urban employment patterns in US cities presented in [Glaeser et al. \(1992\)](#) showed the positive effect of diversity on agglomeration. Pursuing similar line of research, [Henderson et al. \(1995\)](#) presented empirical evidence suggesting that the type of agglomeration externalities vary by stage of the industry life cycle. They analyzed data describing employment growth patterns in eight specific manufacturing industries in 224 metropolitan areas between 1970 and 1987. The five traditional manufacturing industries in their sample were machinery, electrical machinery, primary metals, transportation, and instruments. The three "new" high-tech industries were computers, electronic components, and medical equipment. They found that while the new high-tech industries are attracted to large diverse metro areas with a diversified skill base, the mature industries tend to decentralize to smaller, more specialized metro areas, with lower wage and land costs.

[Harrison et al. \(1996\)](#) and [Kelley and Helper \(1999\)](#) took a different approach and studied the locational context, which determines the innovativeness of US firms belonging to the SIC's three-digit machine-making industries (ranging from heating equipment and plumbing fixtures to guided missiles and aircraft). They showed that diversity and heterogeneity of inputs available locally (including the richness of local infrastructure and the breadth of the skill mix of local labor) contribute significantly towards the growth of the newer industries characterized by a high rate of innovation, while narrow specialization hinders it.

⁴ According to their survey sectors exhibiting the greatest geographic concentration of manufacturing include primary metals, transportation equipment, textiles, food and beverages, leather and chemicals while sectors with the greatest tendency for spatial clustering of innovative activity are transportation equipment, instruments, and electronics.

Similarly, using a data set on new product introductions compiled from over one hundred technology, engineering and trade journals in the US, [Feldman and Audretsch \(1999\)](#) found that local diversity across complementary economic activities tends to promote innovative output more than does the local specialization within just one single industry. Moreover, they found that the degree of local competition within a city is more conducive to innovative activity than is local monopoly.

The existing literature documents the pervasiveness of the fact that while newer highly innovative industries are attracted to diverse locations, more mature industries tend to cluster in locations specialized in producing inputs specific to these industries. For example, [Henderson \(1988\)](#) found empirical evidence of this phenomenon in Brazil, India, and the US, as did [Combes \(2000\)](#) in France.⁵

1.2. Organization of the paper

The rest of this paper is organized as follows. [Section 2](#) presents the baseline model, capturing the role of competition and supplier-assembler matching under uncertainty on the location decisions of input suppliers and assemblers. In [Section 3](#), we consider an extension of the baseline model, which analyzes the matching-based agglomeration mechanism in the presence of the congestion effect due to inelastic supply of labor force. Concluding remarks are presented in [Section 4](#).

2. Basic model

In this section, we consider a partial equilibrium framework in which the mechanism of agglomeration is based on uncertainty about the quality of the match between input suppliers and assemblers and non-tradability of inputs. In this environment, suppliers benefit from larger concentration of assemblers because such concentration increases the probability of a good match between the quality characteristics of the input offered by a supplier and the needs of an assembler. This, in turn, raises the ex ante expected revenue and profit of the supplier. On the other hand, assemblers benefit from locating their production facilities in the areas with higher concentration of potential input suppliers because of the ex ante uncertainty about the match quality of the inputs. Competition between the suppliers in terms of quality ensures that the inputs, which the assemblers buy from the suppliers, are better matched to the needs of the assemblers. Therefore, suppliers have reasons to establish production facilities in the areas with greater concentration assemblers and assembler are

⁵ [Combes \(2000\)](#) analyzed a dataset representing 52 industrial and 42 service sectors in France during the period of 1984–1993. He hypothesized that to the extent that services are more diversified in their input consumption and in the sectors they sell to, they benefit more from diversity than industries do. Based on the dataset for France, Combes found that industrial and service sectors react to density and diversity in an opposite way. He concluded that the positive impact of these factors on employment growth in local services could be explained by the matching effect arising from the local presence of large and diversified input and output markets and by inter-sectoral information spillovers. On the other hand, according to Combes, the negative effect of density and diversity on employment growth in industry can be attributed to congestion (i.e., higher prices for local inputs, land and transportation).

attracted to locations with a greater number of suppliers. These effects may lead to spatial concentration of the entire vertical value chain in a single location if the competition effect does not deter the firms from co-location.

There are n identical suppliers of non-tradable inputs and N assemblers of tradable outputs in the economy. Following Krugman (1991) we assume that the total revenue function of an assembler is: $R_i = (\beta + s_i)q_i - (1/2)q_i^2$, where q_i is the quantity of the input used by the assembler i . We view the random variable s_i as a random quality (or productivity) characteristic of the input, purchased by assembler i . Each supplier can customize its product at no cost to specifications of any of the local assemblers (i.e., it can produce as many varieties of input as there are local assemblers). Thus, suppliers have diversified production and once their production facilities are established at a certain location, there is no extra fixed cost of producing an additional variety of input.⁶ Therefore, the input quality parameter s_i can also be interpreted as the parameter describing the match between the needs of assembler i and the characteristics of the input variety offered by the supplier chosen by the assembler.

Although the suppliers have differing ex post realizations of the quality of the input offered to the assemblers, they all have the same marginal cost, which they know ex ante with certainty. Similar to Farrell et al. (1998), we assume that the identity of the input supplier chosen by assembler i is determined by competition among suppliers with ex ante uncertain quality (i.e., match) parameters but identical unit cost c of producing the input. The ex post values of the match parameter s_i for the input varieties offered by the potential suppliers to an assembler i at location j are determined as realizations of n_j independent random draws from the distribution $F_i(\cdot)$, such that $F_i(0) = 0$.⁷

Given that the number of assemblers at location j is N_j , a supplier who sets up a production facility at location j can potentially supply up to N_j varieties of input (i.e., customize its product to the specifications of up to N_j local assemblers). Having chosen the location, the supplier learns the match parameter between the characteristics of its own product and the needs of assembler i ($i = 1, \dots, N_j$) as well as the parameters characterizing the match between the input varieties offered by other $n_j - 1$ suppliers and the needs of the same assembler. In each of N_j assembler-specific markets in location j , the same group of n_j suppliers compete for the assemblers' demands.⁸

⁶ IT consulting and software development are examples of industries with such characteristics. Software companies incur large fixed costs to establish regional offices, which then provide a variety of customer-specific services (or supply different versions of the same software) to a large number of clients in the area. While customizing for each new client may require fixed costs, these costs are negligible compared to the fixed cost of opening and staffing a regional office.

⁷ The assumption of a group of suppliers engaging in competition for differentiated unit demands of several assemblers makes our framework essentially equivalent to the many-to-one assignment models used in describing job matching between firms and workers (see, e.g., Crawford & Knoer, 1981). The outcome of Crawford and Knoer's salary-adjustment process in the context of job matching is very similar to the outcome that can be reached in our framework when a group of n suppliers participates simultaneously in N independent rounds of quality competition for the demands of N assemblers.

⁸ To be more specific, we can assume that events unfold in the following sequence, which is similar to the one considered by Dudey (1990). First, suppliers simultaneously choose the location. Then assemblers observe the spatial distribution of suppliers and make their own location decisions. In the next stage, suppliers observe their match quality parameters and choose their prices. Finally, assemblers observe the prices and the match quality parameter realizations and choose the suppliers with the best price/quality realizations. However, there

From the assembler revenue maximization, it follows that the assembler's demand for the input depends both on the input's match parameter and its price; i.e., omitting the assembler-specific subscript, we have $p = \beta + s - q$. Therefore, when suppliers choose prices they take the quality (i.e., the match variable) of their products into account. Since we assume that marginal costs are identical across the suppliers, they essentially compete in what [Tirole \(1988\)](#) calls "real prices" $p - s$ rather than simply in prices. To win in this quality competition, a supplier now needs to have the lowest "net unit cost" (i.e., cost minus match quality realization).

In this setting, the match parameter of the input variety supplied in equilibrium to assembler i is the largest of n draws from the distribution $F_i(\cdot)$. Using the notation of order statistics and omitting assembler's subscript, we denote this largest draw by $s_{(n)}$. Therefore, the supplier with the highest realization of match quality $s_{(\max)} = s_{(n)}$ will be willing to lower his quality adjusted price (i.e., price minus quality) to the level of the "net marginal cost" of the supplier with the second highest realization of quality parameter, $s_{(n-1)}$: $p - s_{(n)} = c - s_{(n-1)}$; or equivalently the highest quality supplier will set the following price: $p = c + s_{(n)} - s_{(n-1)}$.⁹

The following diagram represents the described market structure ([Fig. 1](#)).

In order to maximize their profits, the assemblers choose the amount of input, which sets marginal revenue product of the input equal to its price: $\beta + s_{(n)} - q = c + s_{(n)} - s_{(n-1)}$. Therefore, we can derive the quantity of the highest quality input, which the typical assembler will employ in equilibrium:

$$q = \beta + s_{(n-1)} - c \quad (1)$$

Hence, the equilibrium profit of the assembler is:

$$\begin{aligned} \Pi^{\text{As}} &= (\beta + s_{(n)})(\beta + s_{(n-1)} - c) - \frac{1}{2}(\beta + s_{(n-1)} - c)^2 \\ &\quad - (\beta + s_{(n-1)} - c)(c + s_{(n)} - s_{(n-1)}) = \frac{1}{2}(\beta + s_{(n-1)} - c)^2 \end{aligned} \quad (2)$$

The profit of the highest quality supplier is:

$$\Pi^{\text{Sup}} = (p - c)q = (s_{(n)} - s_{(n-1)})(\beta + s_{(n-1)} - c). \quad (3)$$

is an important difference in the approaches taken in [Dudey \(1990\)](#) and in the present paper. Dudey makes an assumption (see [Dudey, 1990](#), condition (5), p. 1096), which ensures that concentration of all firms and consumers at a single location is the only subgame perfect equilibrium in his model. In contrast, as discussed further in this section, we do not make such a restrictive assumption and allow for the possibility of multiple equilibria including dispersion. Instead, we use the Marshallian argument to show why only the complete concentration equilibrium will survive a plausible adjustment dynamics process to become the stable equilibrium outcome.

⁹ The full description of the best response strategy of a firm in the homogenous good Bertrand pricing game should take into account the possibility that the lowest price of the second-highest quality firm can be higher than the price that the highest quality firm would charge if it were a monopolist. However, for simplicity, we would like to rule out this possibility. This can be achieved by assuming that the price $p = c + s_{(n)} - s_{(n-1)}$ does not exceed the price chosen by a monopolistic supplier with the quality $s_{(n)}$: $c + s_{(n)} - s_{(n-1)} < 0.5(\beta + s_{(n)} + c)$. Therefore, we impose the condition: $s_{(n)} - 2s_{(n-1)} < \beta - c$. If we assume that ex-ante distribution of the quality parameter s is Uniform[0, 1], the required condition is ensured by the parameters satisfying the inequality: $1 < \beta - c$.

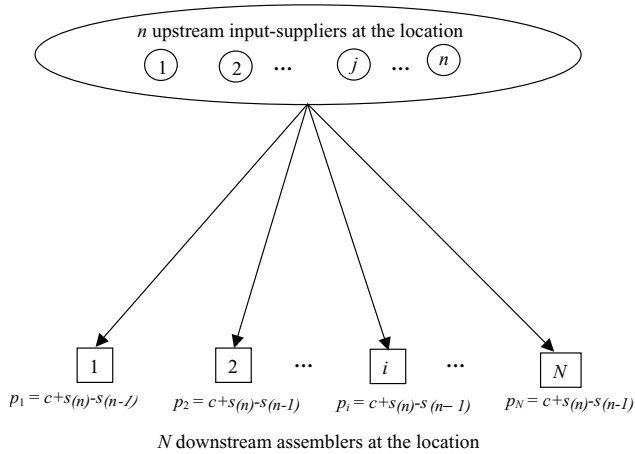


Fig. 1. Ex post quality-adjusted price competition at a location with N assemblers and n inputs suppliers.

For simplicity, we further assume that ex ante quality/match parameters for the inputs offered by the suppliers to assembler i have the same standard uniform distribution: $s_i \sim F_i(\cdot) = \text{Uniform}[0, 1]$ ($i = 1, \dots, N$). Then the ex ante expected profits of the assembler and the supplier are determined by the following equations:^{10, 11}

$$E(\Pi^{\text{As}}) = \frac{1}{2} \left[(\beta - c)^2 + 2(\beta - c) \frac{n - 1}{n + 1} + \frac{n(n - 1)}{(n + 2)(n + 1)} \right] \tag{4}$$

and

$$E(\Pi^{\text{Sup}}) = \frac{N}{n} \left[(\beta - c) \frac{1}{n + 1} + \frac{n - 1}{(n + 2)(n + 1)} \right]. \tag{5}$$

More intense competition among suppliers in the location (i.e., larger n) increases the expected profit of the typical local assembler by allowing him to obtain the input of a higher match quality at a lower price. At the same time, more intense competition among the local suppliers reduces their own equilibrium expected profits. By contrast, the increase in the number of potential buyers (assemblers) of the inputs in the location leads to higher expected profits for each local supplier.

¹⁰ Given the assumption that the quality parameters of inputs produced by n suppliers are elements of a random sample from Uniform[0, 1] population, we can calculate the following moments for the n th and $(n - 1)$ th order statistics obtained from this sample (Arnold et al., 1992, p. 14, 20): $E(s_{(n-1)}) = (n - 1)/(n + 1)$; $E(s_{(n-1)}^2) = n(n - 1)/[(n + 1)(n + 2)]$; $E(s_{(n)}s_{(n-1)}) = (n - 1)/(n + 2)$. We use these moments to calculate expected profits (4) and (5).

¹¹ Although we chose the standard uniform distribution to present the results of this section, these results can be generalized to other distribution functions. It is well known that if U is a Uniform[0, 1] random variable, then for any distribution with continuous cumulative distribution function $F(x)$ the transformed random variable $F^{-1}(U)$ has distribution function F . Moreover, the order statistics $X_{(1)}^{(n)}, X_{(2)}^{(n)}, \dots, X_{(n)}^{(n)}$ in a sample from any continuous distribution with cumulative distribution function $F(x)$ can be transformed by the order-preserving probability integral transformation $U = F(x)$ into the order statistics $U_{(1)}^{(n)}, U_{(2)}^{(n)}, \dots, U_{(n)}^{(n)}$ from the standard uniform distribution (see Arnold et al., 1992).

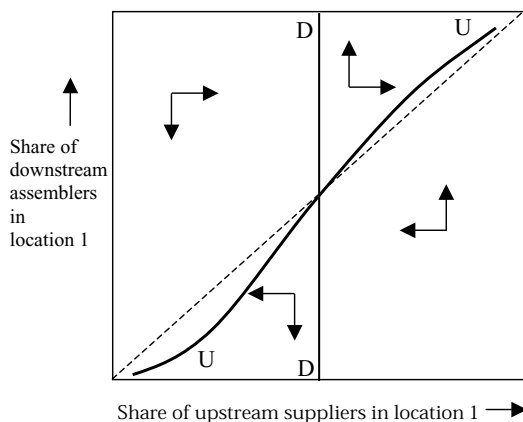


Fig. 2. Dynamics of spatial distribution of suppliers and assemblers.

Similar to [Krugman \(1991\)](#) and [Rauch \(1993\)](#), we assume the Marshallian adjustment dynamics in spatial distribution of suppliers and assemblers to analyze the equilibrium outcomes of firms' location decisions. The geographical area under consideration consists of two locations, denoted 1 and 2. N_j and n_j ($j = 1, 2$) denote respectively the number of assemblers and suppliers in location j . The total number of assemblers (suppliers) in the economy is defined by $N_1 + N_2 = N$ ($n_1 + n_2 = n$). The location choice game unfolds through time (period by period) with the parameters defining the quality of the match randomly drawn every period at both locations. Therefore, the identity of suppliers that win competition at a given location may change every period. Since the expected assembler and supplier profits defined by [Eqs. \(4\) and \(5\)](#) are for one period only, all firms are confronted with the location decision every period of the location choice game.¹² All firms are forward-looking and base their location decisions on the expectations about the behavior of other suppliers and assemblers.

A phase diagram for the model illustrates the possibility of multiple equilibria. In [Fig. 2](#), we have the share of assemblers in one of the two locations on the vertical axis and the share of suppliers in that location on the horizontal axis. The center of the rectangle represents an even split of both types of firms between the two locations, which makes them equally attractive for any firm. Points on the curves DD and UU represent the distributions of downstream assemblers and upstream suppliers, which make the typical assembler and supplier indifferent between the two locations. The curves intersect at the center of the rectangle: if firms are evenly divided between the two locations, they earn the same profits in either of the locations.

The location indifference curve for the typical assembler is vertical: its expected profit, defined by [Eq. \(4\)](#), does not depend on the number of other assemblers in the location because of the assembler-specificity of the input suppliers' match parameters and the lack

¹² The expected profits given in [Eqs. \(4\) and \(5\)](#) are assumed to be sufficiently large to cover the fixed costs that may be involved in redrawing the variable cost parameters between the periods. There is no other fixed cost of changing locations.

of competition or congestion effects on the assemblers' side in this framework. It is also apparent from the examination of Eq. (5) that the upstream supplier location indifference curve UU is positively sloped and is steeper than the 45° line at the point of even distribution of firms between the two locations. If we move away from the point of even split of suppliers and assemblers between the two locations by increasing the number of both types of firms in one of the locations, the number of assemblers in that location should increase faster than the number of suppliers to compensate for the higher order negative effect of the number of suppliers on the supplier expected profit.

The arrows in Fig. 2 indicate the dynamics of firms' location decisions. The intersection of the loci UU and DD at the center corresponds to the unstable equilibrium with even distribution of economic activity between the two locations. We are unlikely to observe it if there is even infinitesimal amount of noise in the system. Therefore, we should observe the eventual convergence to the stable equilibrium, which corresponds to concentration of all assemblers and suppliers either in location 1 or 2 depending on initial conditions.¹³ Thus, the stable equilibria in this model are the points in the two corners of the diagram corresponding to the complete concentration of all assemblers and suppliers in either one or the other location.

3. A three-tier model of industry's spatial concentration

In the basic model considered above, we abstracted from the existence of congestion forces that may deter industry concentration at a single location. In this section, we overcome this modeling shortcoming by assuming that the production of non-tradable inputs depends on local labor, which is supplied inelastically. This allows us to introduce congestion externality among assemblers through the effect of labor market equilibrium on the prices of inputs produced by suppliers. To determine whether geographical concentration of the entire vertically-linked industry in one location will emerge as the equilibrium outcome in this setting, we need to take into account the interaction of location decisions made by workers, suppliers and assemblers.

We maintain the assumptions about the assembler's revenue function, $R_i = (\beta + s_i)q_i - 1/2q_i^2$, and about the supplier's ability to customize their products to specifications of each of the assemblers at the location. Therefore, each supplier at location 1 (2) can produce N_1 (N_2) assembler specific varieties of inputs. The suppliers of the ij -specific input varieties (i.e., the input varieties customized to the specifications of assembler i situated in location j) use identical technologies, which require x units of labor to produce one unit of variety ij . If the wage rate at the location is w_j , then the marginal cost of the input i supplier at that location is xw_j . There are no fixed costs.

As explained in the previous section, the quantity of the highest quality input, which the typical assembler i will employ in equilibrium is given by:

$$q_{ij} = \beta + s_{i(n_j-1)} - xw_j, \quad j = 1, 2 \quad (6)$$

¹³ Klimenko (2003) uses the idea of replicator dynamics from evolutionary game theory to discuss the convergence process for assemblers and suppliers using two strategies (location A or B).

where $s_{i(n_j-1)}$ is the second-highest realization of the match quality parameter among all n_j input varieties offered to the assembler i at location j . The ex post total labor demand at location j is equal to the sum of labor demands \hat{L}^i generated by the suppliers who win in the N_j local assembler-specific rounds of competition:

$$\sum_{i=1}^{N_j} \hat{L}^i = \sum_{i=1}^{N_j} q_{ij}x = \sum_{i=1}^{N_j} (\beta + s_{i(n_j-1)} - xw_j)x = N_j\beta x - N_jx^2w_j + x \sum_{i=1}^{N_j} s_{i(n_j-1)}, \quad j = 1, 2 \quad (7)$$

Assuming that within the location the labor market clears and denoting the number of workers concentrated at location j by L_j we have:

$$L_j = \sum_{i=1}^{N_j} \hat{L}^i, \quad j = 1, 2, \quad (8)$$

Therefore, the equilibrium wage at location j is determined ex post by:

$$w_j = \frac{1}{x} \left(\beta - \frac{L_j}{N_jx} + \frac{\sum_{i=1}^{N_j} s_{i(n_j-1)}}{N_j} \right), \quad j = 1, 2. \quad (9)$$

Combining Eqs. (6) and (9), we obtain the ex post equilibrium quantities of inputs acquired by assemblers at location j :

$$q_{ij} = s_{i(n_j-1)} + \frac{L_j}{N_jx} - \frac{\sum_{i=1}^{N_j} s_{i(n_j-1)}}{N_j}, \quad j = 1, 2. \quad (10)$$

Maintaining the assumption that ex ante match quality parameters of all inputs offered to the assembler i , s_i , have the same standard uniform distribution, we can obtain the expected equilibrium wage at location j :

$$E(w_j) = \frac{1}{x} \left(\beta - \frac{L_j}{N_jx} + \frac{n_j - 1}{n_j + 1} \right), \quad j = 1, 2 \quad (11)$$

where we used the same properties of the order statistics from the standard uniform distribution as in the previous section.

From Eq. (11) it is apparent that the expected wage rate at the given location increases in both the size of the supplier (n_j) and the assembler (N_j) industries at this location. An increase in the number of suppliers improves the matching process in terms of input quality between the assemblers and suppliers at the location. This leads to a larger average size of the input order, which a supplier receives from an assembler at the location. An increase in the number of assemblers at the location translates into a greater number of such orders. Both of these effects lead to a greater labor demand at the location.

The profit of the top-quality (i.e., best matched) input supplier of a given assembler-specific input variety in location j is:

$$\pi_j = (p_j - xw_j)q_j = (s_{(n_j)} - s_{(n_j-1)})(\beta + s_{(n_j-1)} - xw_j) \quad (12)$$

where we omitted the assembler-specific subscripts.

The profit of a representative assembler in location j is given by:

$$\begin{aligned} \Pi_j &= (\beta + s_{(n_j)})(\beta + s_{(n_{j-1})} - xw_j) - \frac{1}{2}(\beta + s_{(n_{j-1})} - xw_j)^2 \\ &\quad - (\beta + s_{(n_{j-1})} - xw_j)(xw_j + s_{(n_j)} - s_{(n_{j-1})}) \\ &= \frac{1}{2}(\beta + s_{(n_{j-1})} - xw_j)^2. \end{aligned} \tag{13}$$

Given the assumption about the standard uniform distribution of the input quality parameter, the ex ante expected profits of the supplier and the assembler are determined by the following equations:

$$E(\pi_j) = \frac{1}{n_j(n_j + 1)} \left[\frac{(1 - n_j)(N_j - 1)}{(n_j + 1)(n_j + 2)} + \frac{L_j}{x} \right], \tag{14}$$

$$E(\Pi_j) = \frac{L_j^2}{2N_j^2x^2} + \frac{n_j - 1 + (N_j - 2)N_j(n_j - 1)(n_j - 1 + n_j^2)}{(N_j - 1)N_j(n_j + 2)(n_j + 1)^2}. \tag{15}$$

(The details concerning the derivations of Eqs. (14) and (15) are relegated to the [Appendix A.](#))

To describe the dynamics of location choices of the firms and workers we introduce additional notation. Denote the location 1's shares of assemblers, suppliers and workers, respectively, by α , β , and γ . Then the absolute number of assemblers, suppliers and workers at locations 1 and 2 can be represented as $N_1 = N$, $n_1 = n$, $L_1 = L$, $N_2 = (1 - \alpha)N$, $n_2 = (1 - \beta)n$, and $L_2 = (1 - \gamma)L$. The share vector (α, β, γ) can be interpreted the state of the economy at any given time. Therefore, we can analyze the dynamics of the expected profits and wages as a function of the economy's state. The evolution of the spatial structure of the economy is governed by the sign of the expected income differentials between the locations:

$$\begin{aligned} \Pi_\Delta(\alpha, \beta, \gamma) &= E(\Pi_1(\alpha, \beta, \gamma) - \Pi_2(\alpha, \beta, \gamma)); \\ \pi_\Delta(\alpha, \beta, \gamma) &= E(\pi_1(\alpha, \beta, \gamma) - \pi_2(\alpha, \beta, \gamma)); \\ \omega_\Delta(\alpha, \beta, \gamma) &= E(\omega_1(\alpha, \beta, \gamma) - \omega_2(\alpha, \beta, \gamma)). \end{aligned} \tag{16}$$

when Π_Δ , π_Δ and ω_Δ are positive, location 1 is more attractive for assemblers, suppliers and workers and, therefore, the shares α , β and γ will increase. The converse is true if Π_Δ , π_Δ and ω_Δ are negative.

Although system of Eq. (16) governing the evolution of the economy's pattern of spatial distribution is too complex to study analytically, it can be quite easily solved numerically for a variety of parameters. [Figs. 3 and 4](#) make the point by illustrating the system's dynamics using a set of two-dimensional phase diagrams. The sides of the rectangles on the figures represent the shares of assemblers, suppliers, and workers in location 1. The location indifference curves within the rectangles plot the zero expected profit and wage differentials for the specified total populations of firms (N and n) and workers (L) and the given unit labor requirement parameter x : $\Pi_\Delta(\alpha, \beta, \gamma) = 0$, $\pi_\Delta(\alpha, \beta, \gamma) = 0$, and $\omega_\Delta(\alpha, \beta, \gamma) = 0$. Since all assemblers, suppliers and workers are identical, the location indifference curves intersect at the center of the rectangles indicating that the even distribution of both types of firms and workers between the two locations makes the locations equally attractive.

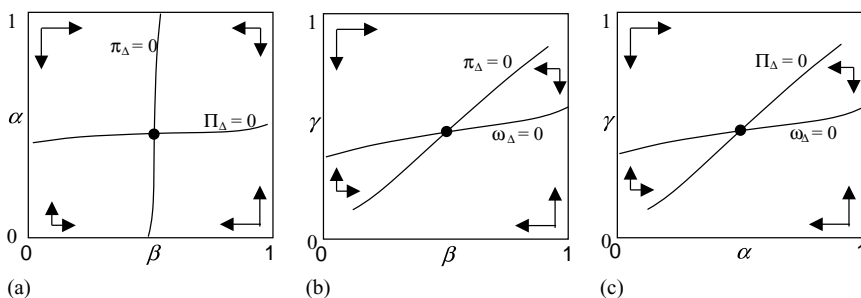


Fig. 3. Phase diagrams of the adjustment dynamics with a strong congestion effect. (a) Location choices of assemblers and suppliers. (b) Location choices of workers and suppliers. (c) Location choices of workers and assemblers.

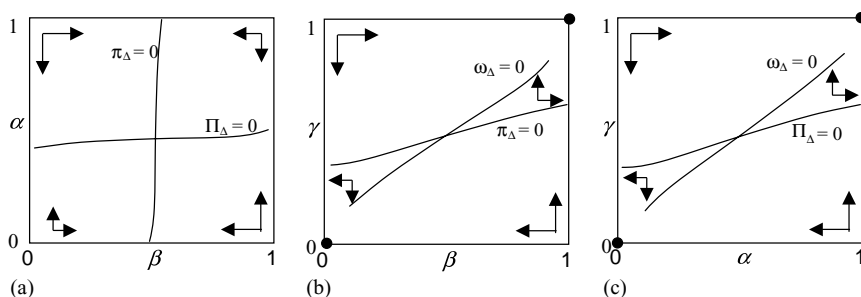


Fig. 4. Phase diagrams of the adjustment dynamics with a weak congestion effect. (a) Location choices of assemblers and suppliers. (b) Location choices of workers and suppliers. (c) Location choices of workers and assemblers.

Arrows in the figures indicate the dynamics of the model. The congestion externality due to the inelastic total supply of labor is the force pushing the system toward divergence. The agglomeration externality arising from the positive effect of spatial concentration on the efficiency of the matching process among suppliers and assemblers is the force working toward convergence. The diagrams indicated that depending on the parameter values we may have either regional convergence or regional divergence.

The full representation of the results would involve phase diagrams in the three-dimensional space, $\alpha \times \beta \times \gamma$. In practice, however, essential results can be represented using two-dimensional diagrams, because for each distribution of agents in one segment of the vertical value chain (i.e., the supplier, assembler or labor segments), there is a two-dimensional stable subset in the complementary space (which is spanned by the remaining two segments) that contains all possible equilibria to which all adjustment paths are directed.¹⁴

¹⁴ For example, for each distribution of the assembler population given by $(\alpha, 1 - \alpha)$, $0 < \alpha < 1$, there is a two-dimensional stable equilibrium subset in the worker–supplier space $\gamma \times \beta$. Since concentration of all workers and suppliers at location 1 is a stable equilibrium for all allocations of assemblers, we, therefore, need only to set $\gamma = 1$ and $\beta = 1$ and determine whether the force pushing assemblers to disperse (i.e., pushing α away from 1) is stronger than the force pushing them into the same location where all suppliers and workers are. Similarly, with

For any values of the model's parameters and a given distribution of workers between the two locations ($\gamma, 1 - \gamma$), $0 < \gamma < 1$, the centrifugal force due to congestion is stronger than the centripetal force due to the matching-based agglomeration effect. Concentration of assemblers and suppliers in one location increases the efficiency of the matching between them. When inputs are better matched to the assemblers' needs, the assemblers increase their orders for inputs produced by suppliers, who, in turn, increase their demand for labor. For any fixed supply of workers in the location, the positive effect of assembler and supplier concentration on their profits due to the better matching between them is dominated by the negative effect due to the increase in the local wage rate. Therefore, if workers don't move, the assemblers and suppliers tend to disperse between the two locations. This is illustrated in Figs. 3a and 4a, in which the equilibrium outcome is the even distribution of all firms between the two locations.¹⁵

The equilibrium outcome of the location dynamics among workers and assemblers (or suppliers) for a fixed distribution of suppliers (or assemblers) also depends on the strength of the congestion effect relative to the agglomeration effect. The workers are attracted to the location with the larger cluster of assemblers and suppliers because greater concentration of firms of either type leads to a greater labor demand and higher wages. At the same time, the arrival of new workers to the location lowers the wage rate and attracts even more assemblers and suppliers. When the congestion effect due to competition for scarce labor resource is strong, the centrifugal force leading to dispersion dominates the centripetal force leading to concentration. Therefore, the workers and firms tend to spread evenly between the two locations. This possibility is illustrated in Fig. 3b and c. In this case, the dispersion forces characterizing the dynamic interactions among workers and assemblers (Fig. 3c) and among workers and suppliers (Fig. 3b) reinforce the dispersion outcome that arises from the interaction between assemblers and suppliers when the allocation of workers is fixed. By contrast, Fig. 4b and c illustrates an example of the weak congestion effect. Since the matching-based agglomeration externality dominates the negative externality due to congestion, we would observe the eventual convergence of workers and firms in same location.

The strength of the congestion effect is determined by how sensitive the wage rate is to the share of worker at the location, γ . Eq. (11) suggests that when unit labor requirement

respect to suppliers, we need to set $\gamma = 1$ and $\alpha = 1$ and verify that the positive effect of agglomeration pushing β to 1 is stronger than the negative effect of congestion, which pushes β to 1/2. To confirm that this is indeed the case, we use Eqs. (14)–(16) to verify that $\Pi_{\Delta}(\alpha, 1, 1) > 0$ for $\alpha \in [1/2, 1]$ and $\pi_{\Delta}(1, \beta, 1) > 0$ for $\beta \in [1/2, 1]$.

¹⁵ Although the location indifference curves $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$, $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ and $\omega_{\Delta}(\alpha, \beta, \gamma) = 0$ in Figs. 3 and 4 were constructed by numerical simulation based on the standard uniform distribution of the quality parameter, we can show analytically that the curve $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ is always steeper than the curve $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$. By examining Eqs. (14) and (15) we verify that in order to maintain the condition of the supplier indifference $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ as we move away from the point of even distribution of both types of firms between the two locations ($\alpha = 1/2, \beta = 1/2$), an increase in N_j must occur in a greater proportion than an increase in n_j . At the same time, in order to maintain the condition of the assembler indifference $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$, the increase in n_j must outpace the increase in N_j . This means that in the vicinity of the point ($\alpha = 1/2, \beta = 1/2$), the curve corresponding to the equation $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ is steeper than the 45° line, while the curve plotting the points corresponding to $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$ is less steep than the 45° line. A similar argument can be used to show that the slope of the location indifference curve for workers, $\omega_{\Delta}(\alpha, \beta, \gamma) = 0$, relative to the location indifference curves for assemblers and suppliers depends on the unit labor requirement parameter x .

x is high, the expected wage rate is less sensitive to the change in the location's share of workers.¹⁶ This means that the congestion effect, which arises due to supplier competition for workers, is less severe when x is high. This observation can be illustrated by plotting the curves $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$, $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ and $\omega_{\Delta}(\alpha, \beta, \gamma) = 0$ for different values of x . Fig. 3b and c shows the position of the curve $\omega_{\Delta}(\alpha, \beta, \gamma) = 0$ relative to the curves $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ and $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$ for the low value of x . Given that the assembler and the supplier location indifference curves are steeper than the workers' location indifference curve, the arrows indicate that the basin of attraction lies at the center of the graph. In other words, since for low x the congestion effect is stronger than the agglomeration effect, the equilibrium outcome to which the system will evolve in the long-run is the dispersion of the workers and firms between the two locations.

In Fig. 4b and c, the curves are drawn for the high value of x , which corresponds to a weak congestion effect. At this value of x , the centripetal force due to the matching-based agglomeration effect is stronger than the centrifugal force due to the congestion effect. Since in Fig. 4b and c the curve $\omega_{\Delta}(\alpha, \beta, \gamma) = 0$ is steeper than the curves $\pi_{\Delta}(\alpha, \beta, \gamma) = 0$ and $\Pi_{\Delta}(\alpha, \beta, \gamma) = 0$, the basin of attraction shifts from the center of the graph to the upper-right and lower-left corners of the graph. Therefore, the phase diagram indicates that for the high value of x the long-run equilibrium is concentration of all workers and firms at a single location.

Overall, our analysis of the equilibrium location patterns in the model incorporating the labor market suggests that competition for scarce resources and congestion reduce the strength of the matching-based agglomeration externality. Although the discrete nature of location decisions by agents in our model precludes the comparative statics analysis of the equilibrium patterns of geographical distribution of industries, our analysis shows that under fairly standard conditions complete spatial dispersion can emerge as the stable equilibrium outcome. Therefore, our model can serve as a theoretical explanation for the empirically observed periodic spells of deagglomeration in the evolution of urban areas in many countries.¹⁷

4. Concluding remarks

This paper has considered the microeconomic foundations of the agglomeration economies characterizing industries at the early stages of the industry life cycle. Given the novelty and continuous experimentation with the end-product designs in such industries, there is substantial uncertainty in the relationships between the vertically-linked segments of the industrial value chain. In this environment, scale economies and cost competition between firms is a less important factor of spatial distribution of economic activity than quality competition and the efficiency of the matching process between upstream and downstream

¹⁶ The sensitivity of the wage to the location's share of workers is inversely related to the elasticity of the demand for labor at the location. The inverse labor demand function at the location is defined by Eq. (14).

¹⁷ For example, Kanemoto et al. (1996), using the data on Japanese metropolitan areas empirically demonstrated that once the city size exceeds a certain limit the advantages associated with the agglomeration economies are outweighed by the disadvantages associated with congestion.

firms in the industry value chain. The diversity and heterogeneity of firms at all segments of the value chain increases the likelihood of the positive outcome of the matching process, which brings together intermediate product suppliers and assemblers of the end products.

We have shown that the efficiency of the matching process among firms within a regional cluster is closely related to competition between input suppliers in terms of quality. Combined with the assemblers' uncertainty about the quality of the input match, competition between suppliers increases the expected quality of the local inputs for the assemblers. Therefore, assemblers are attracted to locations with more diverse supplier bases. At the same time, entry of additional assemblers increases the location's attractiveness for suppliers because their expected profits rise. This mutual interdependence of the location decisions of upstream and downstream firms leads to the creation of geographical clusters of vertically-linked industries.

While our model is necessarily stylized, we believe it captures the main features of the agglomeration mechanism for newly emerging industries with a high rate of innovation. This mechanism of agglomeration appears to be what [Porter and Stern \(2001\)](#) had in mind when they wrote about the concept of a specialized industrial cluster, which they identified as a critical mass of interconnected companies and institutions in a particular location:

Clusters offer potential advantages in perceiving both the need and the opportunity for innovation. Equally important, however, is the flexibility and capacity clusters can provide to act rapidly to turn new ideas into reality. A company within a cluster can often more rapidly source the new components, services, machinery and other elements necessary to implement innovations . . . Reinforcing these advantages for innovation is the sheer pressure—competitive pressure, peer pressure, customer pressure and constant comparison—that is inherent within a cluster. ([Porter & Stern, 2001](#), p. 29)

The matching efficiency-enhancing benefits from co-location should be balanced against the costs due to congestion. In our model, the congestion effect arises because input producers compete for labor, which is supplied elastically. In this setting, whether the location equilibrium outcome is concentration or dispersion depends on the relative strength of the matching-based centripetal force and the congestion-related centrifugal force. Our model shows that one of the factors that determine the relative strength of these forces is the sensitivity of the local wage rate to the availability of workers at the location. When the local wage is less sensitive, the congestion effect is weaker and, therefore, the equilibrium outcome is more likely to be concentration. The labor demand elasticity and the institutional structure of the labor market are among the factors that determine the wage sensitivity.

One aspect of geographical concentration of economic activity that our model does not address is that of distance and transportation costs. Although transportation costs are relatively low for high-technology firms populating newly emerging industries, they are non-negligible. Our model can also be made richer (albeit at a cost of greater complexity) by assuming that firms can choose among more than two locations. Despite these concessions for the sake of tractability, our model highlights the importance of diversity for location decisions by firms in industries without stable interface standards. It is a matter of further research to create an analytical multi-location framework combining transportation costs and assembler-supplier matching under uncertainty.

Appendix A. Derivation of Eqs. (14) and (15)

Given the assumption that the quality parameters of inputs produced by n suppliers are elements of a random sample from Uniform[0, 1] population, we can calculate the following moments for the n th and $(n-1)$ th order statistics obtained from this sample (Arnold et al., 1992, pp. 14, 20): $E(s_{(n-1)}) = (n-1)/(n+1)$; $E(s_{(n-1)}^2) = n(n-1)/[(n+1)(n+2)]$; $E(s_{(n)}s_{(n-1)}) = (n-1)/(n+2)$. We use these moments to calculate expected profits in Eqs. (14) and (15):

$$\begin{aligned}
 E(\pi_j) &= \frac{N_j}{n_j} E((s_{(n_j)} - s_{(n_j-1)})(\beta + s_{(n_j-1)} - xw_j)) \\
 &= \frac{N_j}{n_j} \left[E((s_{(n_j)} - s_{(n_j-1)})s_{(n_j-1)}) - \frac{1}{N} E\left(\sum_{i=1}^{N_A} s_{(n_j-1)}(s_{(n_j)} - s_{(n_j-1)})\right) \right. \\
 &\quad \left. + \frac{L_j}{N_j x} E(s_{(n_j)} - s_{(n_j-1)}) \right] \\
 &= \frac{N_j}{n_j} \left[E((s_{(n_j)} - s_{(n_j-1)})s_{(n_j-1)}) - \frac{1}{N_j} E(s_{(n_j-1)}(s_{(n_j)} - s_{(n_j-1)})) \right. \\
 &\quad \left. - \frac{N_j - 1}{N_j} E(s_{(n_j-1)})E(s_{(n_j)} - s_{(n_j-1)}) + \frac{L_j}{N_j x} E(s_{(n_j)} - s_{(n_j-1)}) \right] \\
 &= \frac{N_j - 1}{n_j} \left[E((s_{(n_j)} - s_{(n_j-1)})s_{(n_j-1)}) - E(s_{(n_j-1)})E(s_{(n_j)} - s_{(n_j-1)}) \right. \\
 &\quad \left. + \frac{L_j}{n_j x} E(s_{(n_j)} - s_{(n_j-1)}) \right] \\
 &= \frac{1}{n_j(n_j + 1)} \left[\frac{(1 - n_j)(N_j - 1)}{(n_j + 1)(n_j + 2)} + \frac{L_j}{x} \right]
 \end{aligned}$$

and

$$\begin{aligned}
 E(\Pi_j) &= \frac{1}{2} E(\beta + s_{(n_j-1)} - xw_j)^2 = \frac{1}{2N_j^2} E\left(N_j s_{(n_j-1)} + \sum_{i=1}^{N_j} s_{(n_j-1)}^i + \frac{L_j}{x}\right) \\
 &= \frac{L_j^2}{2N_j^2 x^2} + \frac{n_j - 1 + (N_j - 2)N_j(n_j - 1)(n_j - 1 + n_j^2)}{(N_j - 1)N_j(n_j + 2)(n_j + 1)^2}.
 \end{aligned}$$

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