Generalized Aharonov-Bohm effect and topological states in graphene *nanorings*: Particle-physics analogies *beyond* the massless Dirac fermion

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APS March 2013

Supported by the U.S. DOE (FG05-86ER45234)
Hexagon vs. Triangle

臂折

$w=14$

Tight Binding (TB)

Same Edge

Different Shape

$\times 10^{-2}$

$\varepsilon(t)$

$\Phi/\Phi_0$

$\delta_1$

$\delta_2$

$2718$

$2142$

(a)

(b)

(c)

$\Delta_0$
Tight Binding (TB)

Armchair vs. Zigzag

Hexagon

Same Shape
Different Edge

w=14

w=16
1D Generalized Dirac equation

\[ [E - V(x)] \Psi + i\hbar v_F \alpha \frac{\partial \Psi}{\partial x} - \beta \phi(x) \Psi = 0 \]

\[ \Psi = \begin{pmatrix} \psi_u \\ \psi_l \end{pmatrix} \]

**Dirac-Kronig-Penney Superlattice**

a single side/ 3 regions

\((V1, m1)\) \hspace{1cm} \((V2, m2)\) \hspace{1cm} \((V3, m3)\)

**Transfer matrix method**

\[
\Omega_K(x) = \begin{pmatrix}
ed^{iKx} & e^{-iKx} \\
\Lambda e^{iKx} & -\Lambda e^{-iKx}
\end{pmatrix}
\]

\[ K^2 = \frac{(E - V)^2 - m^2v_F^4}{\hbar^2 v_F^2} \]

\[ \Lambda = \frac{\hbar v_F K}{E - V + mv_F^2} \]
DKP Results: Hexagon/ armchair

$m_0 = 0$

$m_0 = 0.01t/v_F^2$

$m_0 = 0.30t/v_F^2$

Polyacetylene

Dimerization/ Kekule

TB results

Two Domains

Corner

Domain Wall
DKP Results: Triangle/ armchair

$m_0 = 0$

$m_0 = 0.02t/v_F^2$

TB results

One Domain

Polyacetylene

Corner/ scatterer
DKP Results: Hexagon/ zigzag

\[ \tilde{E}(t) = E - \mathcal{M}v_F^2 \]

\[ \mathcal{M} = \frac{42.06t}{v_F^2} \]

\[ \mathcal{M}_e = \frac{(2.10t)}{v_F^2} \]

nonrelativistic behavior similar to the 1D quantum ring in the reczag trigonal flake
Relativistic quantum-field-theory Lagrangian

\[ \mathcal{L} = \mathcal{L}_f + \mathcal{L}_\phi \]

\[ \mathcal{L}_f = -i \hbar \Psi^\dagger \frac{\partial}{\partial t} \Psi - i \hbar v_F \Psi^\dagger \alpha \frac{\partial}{\partial x} \Psi - \phi \Psi^\dagger \beta \Psi \]

\[ \Psi = \begin{pmatrix} \psi_u \\ \psi_l \end{pmatrix} \]

\( \alpha \) and \( \beta \): any two of the three 2x2 Pauli matrices

electrostatic potential

scalar field / position-dependent mass \( m(x) \)

Yukawa coupling

fermionic
scalar field

\[ \mathcal{L}_\phi = -\frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 - V(\phi) \]

\[ V(\phi) = \frac{\xi}{4} (\phi^2 - \zeta^2)^2 \]

Euler-Lagrange equation

\[ -\frac{\partial^2 \phi}{\partial x^2} + \xi (\phi^2 - \zeta^2) \phi = 0 \]

1. \( \phi_0 \) (Symmetry breaking)/ constant mass Dirac fermion

2. kink soliton/ zero-energy fermionic soliton

kink soliton

\[ \phi_k(x) = \zeta \tanh \left( \sqrt{\frac{\xi}{2}} \zeta x \right) \]

zero-energy fermionic soliton (Dirac eq.)

\[ \Psi_S(x) \propto \left( \exp \left( -\int_0^x \phi_k(x')dx' \right) \right) \]
Conclusions

1) The 1D Dirac-Kronig-Penney superlattice model provides a unifying interpretation of the tight-binding spectra (as a function of $B$) of planar graphene rings.

2) The spectra are sensitive to the topology (edge and shape) of the rings.

3) In the DKP, the topology is captured by general, position-dependent scalar fields (mass terms), beyond the massless Dirac-Weyl fermion.

4) A Lagrangian formalism establishes rich analogies with 1D quantum-field theories, e.g., fermionic solitons, mass generation, nonrelativistic behavior.
Aharonov-Bohm oscillations

Armchair (linear)

Zigzag (quadratic)