Rapidly rotating few bosons with long or short range repulsion: An exact diagonalization study

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Hamiltonian in the LLL

We diagonalize:

\[ H_{LLL} = \hbar (\omega_\perp - \Omega) \hat{L} + \sum_{i < j}^{N} v(\mathbf{r}_i - \mathbf{r}_j) \]

where \( v(\mathbf{r}) = g \delta^2(\mathbf{r}) \) OR \( v(r) = \frac{c}{|r|} \)

using the basis of symmetrized wave functions (Permanents):

\[ \Phi_A = \sqrt{\frac{n_1! n_2! \cdots n_M!}{N!}} \sum_P u_{P_{\alpha_1}}(1) u_{P_{\alpha_2}}(2) \cdots u_{P_{\alpha_N}}(N) \]

**Filling factor:** \( \nu = \frac{N(N-1)}{2L} \)
Ground states & Magic numbers

**COULOMB INTERACTION N=6**

- **Angular momentum**
- **Rotational frequency**

**Magic Numbers**
Ground states & Magic numbers

DELTA INTERACTION N=6

Angular momentum

Rotational frequency

Magic Numbers
Formation of Rotating Boson Molecules (RBMs): Small fractions ($\nu<1/2$)

Increasing angular momentum

N=6

(L=30)

$\nu=1/2$

(L=60)

$\nu=1/4$

(L=120)

$\nu=1/8$
RBMs, isomers, and the spectrum
Larger size \( N \): multiple rings (Coulomb)

COULOMB INTERACTION \( N=11 \) \((3,8)\) \( I_{gs}=110+8 \) \((\nu<1/2)\)

Particles on different rings rotate independently

Reference point

RBM

\((3,8)\)
Higher order correlation functions

N-point correlation is calculated by:

\[ P(r; r_1, r_2, ..., r_{N-1}) = |\Psi(r; r_1, r_2, ..., r_{N-1})|^2 \]

Reveals formation of RMB at low angular momenta (large fractions: \( \nu > 1/2 \)) even when 2-point (CPD) does not

a) 2-POINT
   (a) CPD
   (b) N-POINT1
   (c) N-POINT2

COULOMB INTERACTION N=6 \( L_{gs} = 15 \) (\( \nu = 1 \))
$N=9$; Zeros of N-point correlation

**Delta** interaction

**Coulomb** interaction

Fixed particle positions (●)

L = 72

Laughlin ($\nu=1/2$)

(2,7) RBM

Calculated position of the 9-th particle (■)

Zeros for the 2 different interactions differ in charge
Conclusions

• The emerging crystalline structure (RBMs) in finite systems of rotating bosons does not depend strongly on range of interaction.

• For crystalline structures with multiple rings the rings rotate independently of each other.

• For larger fractions (smaller angular momenta) identification of crystalline structures requires higher-order (e.g. N-point) correlation functions.

• The ground states for various interaction ranges may be distinguished by the behaviour of the zeros of the N-point correlation.