

SIMULATION OF THE IMPACT OF PRODUCT VARIETY ON LOGISTICS PERFORMANCE

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Abstract

We study the impact of product variety on the performance of a simple integrated production-distribution system equivalent to the stochastic economic lot-scheduling problem. We show that, keeping the total demand constant, the expected cost of inventories and backorders increases linearly with the number of products. This result is contrary to the conventional wisdom – based on pooling economies – whereby the expected cost would increase as the square root of the number of products. The linear relationship stems from the increase in replenishment lead time induced by an increase in product variety. In a systematic simulation study we show the phenomenon to be quite robust, as it does not depend on load, flexibility, or processing variability.

* In memoriam

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1. Introduction

The trend toward increasing the breadth of product lines is well known (Pine, 1993). For example, in the packaged consumer goods industry, from 1986 to 1996, the number of new product introductions has doubled from 12,000 to 24,000 (Market Intelligence Service, 1997). Similarly, Safeway reports that the number of products available in large supermarkets has increased thirty-fold from an average of 1000 in the 1950's. One frequent argument for a broader product line is that customized products command premium prices in the marketplace (Fortune, 1993). Examples include Levi Strauss's make-to-order jeans and Ingersoll-Rand's production of different prismatic parts in lot sizes of one (Goldhar and Lei, 1995). Rapid technology changes and globalization (or expansion of customer base) are also cited as key drivers of product proliferation (Lee, 1996). Rapid technological innovation pushes a company to sell multiple versions of the same product simultaneously. Globalization, on the other hand, imposes additional requirements on the product due to differences in language, local regulations, and technical considerations such as voltage, frequency, and plug conventions.

The breadth of the product line has also been considered as one of the key drivers of logistics performance (e.g., Magee, Copacino, and Rosenfield, 1985). Thonemann and Bradley (2002) assert that high product variety impairs supply chain performance, particularly in terms of replenishment lead times and cost. They report that, for a hard drive manufacturer, the average lead time for product lines with high product variety is longer than the average lead time for product lines with low product variety. Longer average lead times, in turn, force the retailers to hold more inventory, increasing their costs. Other companies such as Procter & Gamble and Ford have reduced the number of variants of their products in an effort to control the cost of high product variety. In spite of increasing anecdotal evidence, no analytical or empirical evidence seems to be available to confirm this conventional wisdom, let alone assess the magnitude of the impact. On the theoretical side, with a few exceptions discussed below, no systematic analysis of the phenomena relating product variety to logistics performance appears in the literature. The impact of product variety on manufacturing costs (e.g., Banker et al., 1988), on manufacturing cell performance (e.g., Kekre, 1987), and on marketing (e.g., Kekre and Srinivasan, 1990) has been the object of more attention.

A first answer on the impact of product variety on logistics performance is provided indirectly by Eppen (1979). In a multi-location newsvendor problem, he shows that the benefits of the *centralization* of safety stocks are proportional to the square root of the number of locations (in the symmetric case, where all locations have the same cost parameters and demand characteristics). The reinterpretation of this model in terms of *rationalization* of the product line leads to the conclusion that the cost of variety should be proportional to the square root of the number of products.

Zipkin (1995) reemphasizes that result and generalizes it. He shows that the square root effect applies to a multi-product integrated production-distribution system when the production facility is perfectly flexible (i.e., exhibits instantaneous setups).

Gupta and Srinivasan (1998) study a cyclic exhaustive production system and derive conditions under which the total work-in-process inventory actually decreases with increasing product variety. Based on these conditions, they propose control strategies to manage demand and processing rates.

In this paper, we consider a simple integrated production-distribution system with positive setup times. The proposed model is equivalent to the stochastic economic lot-scheduling problem most recently studied by Federgruen and Katalan (1996). We argue that the performance of the system deteriorates significantly as the number of products increases, keeping the *total* demand rate constant. In particular, the expected cost of inventory and backorder increases linearly in the number of products. The phenomenon does not depend on the load of the system, its flexibility, or the variability of processing requirements.

The significance of the result is best appreciated in light of the percentage of revenues that would typically be affected. A survey of logistics performance in Europe, by A.T. Kearney, Inc. (1993), reports that inventory and warehousing costs alone amount to 5.2% of revenues on average. No statistics are available on the costs of backorders and lost sales, which consist in large part of hard to evaluate opportunity costs, but their impact is likely to be significant. Suppose, for illustration purposes that inventory and backorder costs combined amount to 10% of turnover in a particular company. The margin is 5%. Our analysis suggests that if the company were doubling the number of its products, these costs would increase to reach 20% of

revenue. Starting with a margin of 5%, such an increase in costs would bring the company to a dramatic loss situation.

A few comments are in order. First, the analysis suggests that costs would increase linearly if the design of the production and distribution system is not improved to support the increase in variety. Second, an increase in variety should typically be associated with an increase in revenues. Our analysis focuses on the cost impact only. We further assume that the total demand rate remains constant, implying that individual product demand rates decrease as variety is added. This is quite typical in highly competitive markets where new products are introduced to maintain market share rather than increasing the total market size (Berry and Cooper, 1999). Third, the increase in inventory and backorder costs and the associated phenomenon of increase in replenishment lead time is not likely to be easily traceable by standard accounting systems as it would impact indirect operating costs in terms of overheads and opportunity costs.

The rest of the paper is organized as follows. Section 2 introduces the model and notation. A simple heuristic argument describing the phenomenon is offered in Section 3. Section 4, the main body of the paper, reports the results of our simulation study. The paper closes in Section 5 with comments and conclusions.

2. The model

We consider an integrated production-distribution system with n different products facing independent stochastic demand and sharing a common production facility with setups. Figure 1 schematically depicts such a system. Demand is met from inventory carried at the distribution facility; unmet demand is backordered. The transportation delay between the production and distribution facilities is assumed to be negligible (e.g., both facilities are on the same site). The problem is equivalent to the stochastic economic lot-scheduling problem. Following a recent analysis of this model by Federgruen and Katalan (1996), we focus on cyclical exhaustive base-stock policies. Under such a policy, the production facility rotates between the different products in a fixed sequence; once initiated, the production of a particular item continues until inventory reaches a prespecified base-stock level. This operating policy, called the *cyclic exhaustive polling policy* in the queueing literature, is easy to implement; moreover, it minimizes the amount of unfinished work in the system.

The total demand generated by the different products is assumed to follow a Poisson process. For simplicity, we restrict the analysis to the symmetric case in which all the products have identical demand and production parameters. Let λ denote the aggregate demand rate. The demand rate of any particular product is then given by $\mu_D = \lambda/n$, while the standard deviation is given by $\sigma_D = \sqrt{\lambda/n}$.

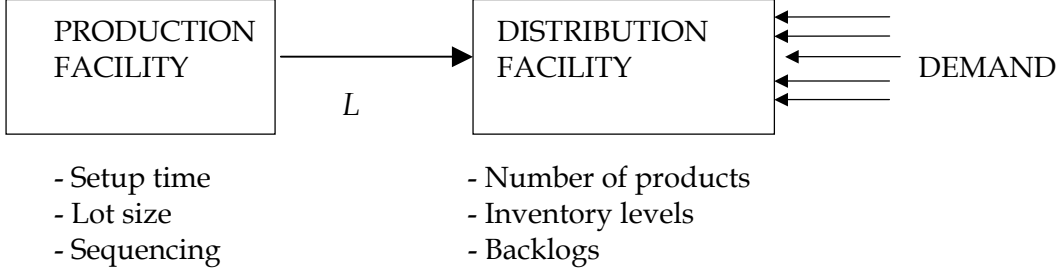


Figure 1. *Production-Distribution System*

The production facility rotates between the different products. The processing of a particular product starts with a setup of duration r followed by production at a rate μ until the inventory level at the distribution facility reaches the specified base stock level s . (We do not consider direct setup costs; transportation is instantaneous with a transfer lot-size of one.) In the simulation study of Section 4, we consider the cases of both deterministic and stochastic processing times.

We evaluate the performance of the system in terms of the expected level of inventory, I , and the expected level of backorders, B . Let P denote the (discrete) probability distribution of demand during the replenishment lead time, L . The sequence of orders, setups and production is not affected by changes in the base stock level. Thus, the distribution P is independent of the base stock level s and the expected performance of the system can be evaluated as in a newsvendor problem (Theorem 1 in Federgruen and Katalan, 1996). We then have

$$I = \sum_{x=0}^s (s-x)P(x), \text{ and}$$

$$B = \sum_{x=s}^{\infty} (x-s)P(x).$$

In Section 4, the above relations are used to evaluate the performance of the system in a simulation study. By the above argument, it suffices to simulate the

system for a particular value of the base stock, construct the distribution P , based on that distribution calculate the performance of the system for other values of the base stock, and finally plot the resulting tradeoff curves. (The constructed distribution can also be used to optimize the base stock level for given values of the inventory holding cost and shortage cost.)

To recap, we focus on the impact of product variety on performance, keeping the aggregate demand rate λ constant. A possible increase in demand triggered by a broader product line is not considered here. Rather, we are interested in the impact of other parameters (setup time, processing rate and the resulting load factor, and variability of processing) on the relation between performance and variety. The base-stock level does not play a direct role in the analysis as we compare tradeoff curves (i.e., we compare performance for all possible values of the base-stock level).

3. Heuristic argument

The combination of two phenomena shapes the impact of an increase in product variety on the performance of the above system. The first phenomenon is the loss of *pooling economies* associated with the disaggregation of demand into smaller segments. This effect is equivalent to the decentralization of inventories in a multi-location newsvendor problem, as first analyzed by Eppen (1979). The second phenomenon is the increase in *replenishment lead time* associated with an increase in variety. An increase in product variety affects lot-sizing decisions and production cycles, which in turn affect lead times, as first emphasized by Karmarkar (1987).

The following heuristic argument suggests that the performance of the production-distribution system, measured in terms of inventory and backorder costs, might deteriorate linearly in the number of products. This intuitive argument is supported by the simulation study of Section 4.

Let σ_R denote the standard deviation of the replenishment lead time demand for an individual product. If we approximate the distribution of the replenishment lead time demand by a normal distribution, the expected inventory and backorder cost is proportional to σ_R . Let μ_L and σ_L denote the expected value and standard deviation, respectively, of the replenishment lead time. Assuming independence of the demand, D , and the replenishment lead time, L , (clearly an approximation in the integrated production-distribution system considered in Section 2) we have (see Silver and Peterson, 1985)

$$\sigma_R \equiv \sqrt{\mu_L \sigma_D^2 + \mu_D^2 \sigma_L^2}.$$

As argued in Section 2, μ_D and σ_D^2 are proportional to $1/n$. For a fixed lead time, this yields the square root effect of Eppen (1979). The key point in the argument that follows is that a change of product variety should also affect replenishment lead times.

The effect of a change in n on μ_L is easy to predict. Let q denote the expected lot-size. The effective utilization of the production facility is given by $\lambda(r/q + 1/\mu)$. A cyclical base stock policy with no planned idle time will typically yield an effective utilization of 100%. Setting the above expression to 1 and solving for q yields

$$q = \frac{\lambda r}{1 - \rho},$$

where $\rho = \lambda / \mu$ denotes the system load. On average, the replenishment lead time is approximately equal to half the duration of a production cycle. After substitution of q from the above expression, this yields

$$\mu_L \approx \frac{n}{2} \left(r + \frac{q}{\mu} \right) = \frac{n}{2} \frac{r}{(1 - \rho)}.$$

The effect of a change in n on σ_L is harder to predict intuitively. One reasonable first guess is that the coefficient of variation of L would not be affected by n so that σ_L would increase linearly in n . Putting together the impact on D and L , we conclude that σ_R should not be affected by a change in n . In other words, the expected inventory and backorder cost associated with one particular product should remain unchanged when the number of products, n , is increased.

As the demand rate associated with individual products decreases with an increase in variety, the cost per unit increases linearly. Thus, we expect the aggregate performance of the system, measured as the sum of the inventory and backorder cost of the individual products, to increase linearly in the variety of the product line. This heuristic argument is validated in the simulation study of Section 4.

4. Integrated production-distribution systems

4.1 The Simulation Study

We now analyze the production-distribution system through discrete event simulation to assess the impact of product variety on logistics performance. Recall that we are considering a system where n different products with independent

stochastic demand share a common production facility with setups. The production facility is modeled as a polling system with cyclic exhaustive service. Under such a service policy, the production facility rotates between the different products in a fixed sequence; once initiated, the production of a particular item continues until the inventory reaches a prespecified base-stock level. A non-zero set-up time is incurred when the facility switches to another product.

Other variants of the service policy exist. In *gated* service, as opposed to *exhaustive* service, the server processes only those entities that were present at the instant where it switches to a particular queue. Federgruen and Katalan (1996) assert that gated service is inferior to exhaustive service. It is also possible to use *polling tables* instead of a cyclic service policy. This variant is especially useful for modeling asymmetric products or different priority schemes.

We investigate the impact of product variety on logistics performance under varying degrees of system load (the ratio of demand rate and production rate), flexibility (measured by setup times), and variability of the production process. Demand follows an independent Poisson stream for each product; service times are independent and exponentially distributed. Setup times are deterministic. Parameter values for the arrival rate, the service rate, and the set up time ensure an effective system utilization that is strictly less than 100%. This, in turn, guarantees the achievement of steady-state conditions in the simulation experiments.

The model is implemented in SLAMSYSTEM (Pritsker, 1987). After a warm-up period of 10,000 service completions, each replication was run for 200,000 units of demand for each product under steady-state conditions. Five independently seeded replications were conducted for each scenario; common random numbers were used across scenarios to reduce estimator variance.

4.2 Observations

Figure 2 depicts the average replenishment lead times for 2, 4, 8, and 16 products at high, medium, and low levels of system load. We observe that the average replenishment lead time grows linearly as the number of products increases. Moreover, this growth appears to be independent of the system load. This observation is consistent with the heuristic argument that an increase in product variety affects lot sizing decisions and production cycles, which, in turn, affect lead times.

The increase in replenishment lead time is coupled with the loss of pooling economies as logistics performance deteriorates with increasing product variety. Figure 3 shows the inventory-service trade-off with 2, 4, 8, and 16 products. Unlike the case with exogenous (fixed) lead times, where the aggregate cost increases as the square root of n , the combined effect of endogenous replenishment lead times and the loss of pooling economies results in a cost increase that is linear in n . As the system load gets lighter, however, the effect tends to the square root phenomenon.

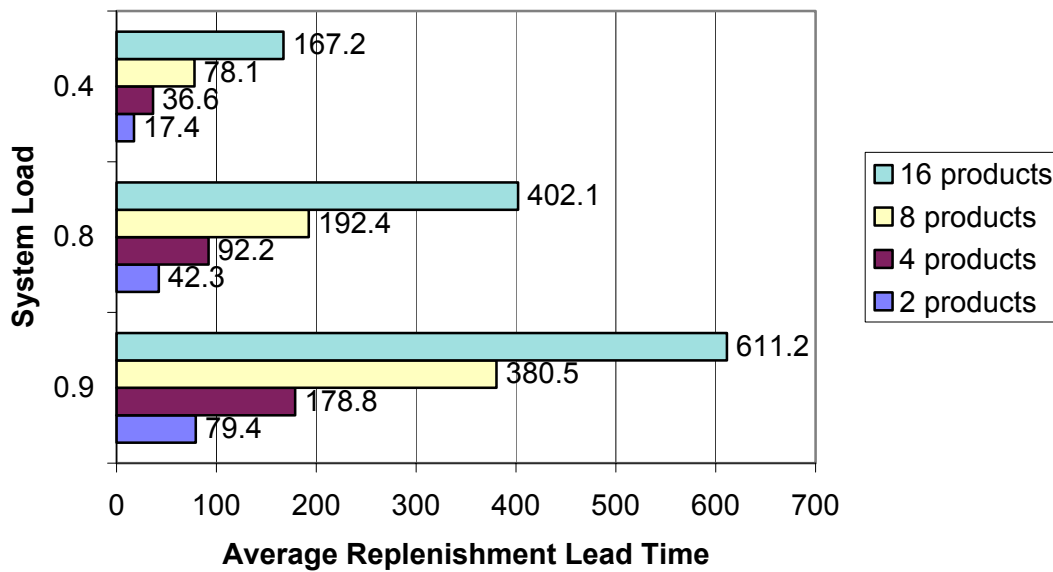


Figure 2. *Replenishment Lead Time and Product Variety*

In addition to the linear deterioration in the first-order effects, the standard deviation of the replenishment lead time also increases linearly with the number of products, n . This is shown in Figure 4. Given that the standard deviation of the replenishment lead time is one of the key drivers of safety stocks, a broader product line should then result in higher expected inventory levels. We therefore conclude that both the expected lead time and its standard deviation increase linearly with the breadth of the product line. Since the former increases the cycle stock and the latter increases the safety stock, the overall inventory levels must grow at least linearly with the number of products, n .

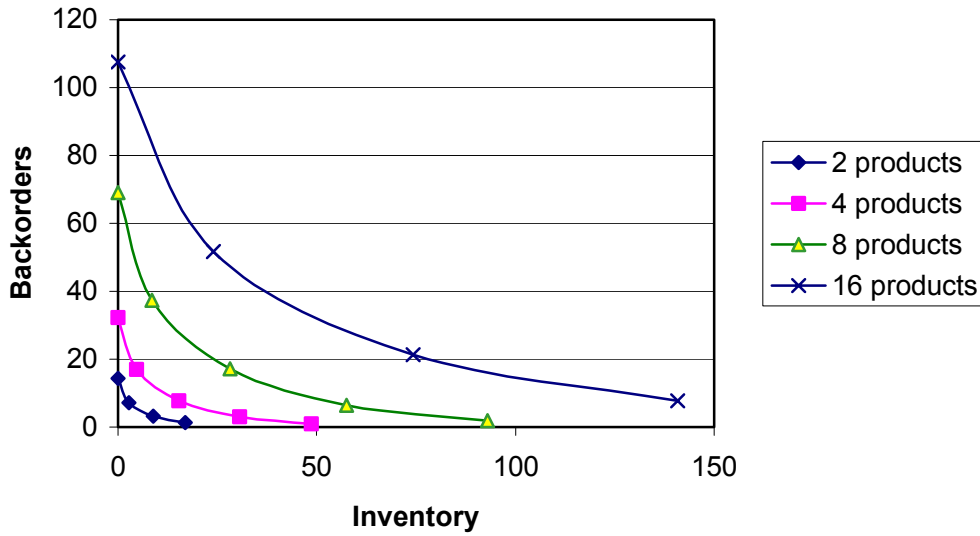


Figure 3. *Inventory-Service Trade-off (Load=0.90)*

4.3 Sensitivity Analysis

To assess the robustness of the observations and to understand the impact of various factors on system performance, we conducted a full factorial experiment. Table 1 lists the system parameters used in this investigation. They include the breadth of the product portfolio (number of products), the utilization of the system (system load), the length of the set-up time, and the volatility in the demand and manufacturing processes. The parameter values have been selected so as to ensure coverage of a wide valid experimental region. Within the 2^5 factorial experiment, we replicated each design point five times; each replication was run for 200,000 units of demand for each product, following an initial warm-up period. Common random numbers were used across design points to reduce the variance of the estimators. The performance metrics of interest include expected total inventory (expressed in terms of the number of units), expected total backlog (number of units), and the expected replenishment lead time (number of time periods).

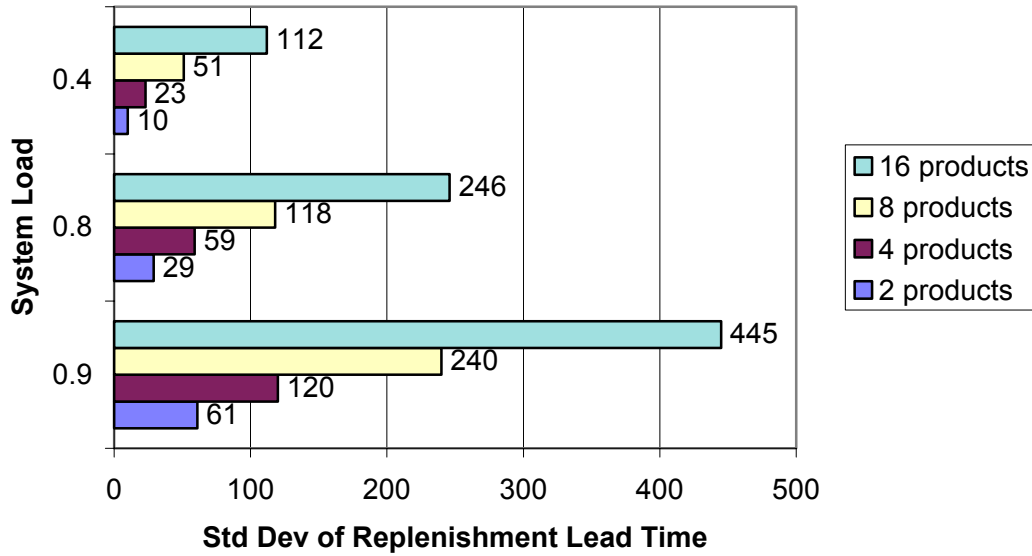


Figure 4. Standard Deviation of Replenishment Lead Time

FACTOR	LOW LEVEL	HIGH LEVEL
no. of products, n	4	16
system load	0.80	0.90
setup time	10.0	40.0
CV (demand)	0 (deterministic)	1 (exponential)
CV (manufacturing)	0 (deterministic)	1 (exponential)

Table 1. The Factorial Experiment

Tables 2 and 3 show the results of the experiment for the main effects and two-way interactions, respectively. The responses marked with an asterisk (*) represent statistically significant effects based on a 90% confidence interval for that response value. As expected, the breadth of the product line has a significant impact on both operating costs (in terms of the expected total inventory) and customer service (in terms of expected total backlogs and the replenishment lead time). More specifically, under the given parameter values, broadening the product portfolio from 4 products to 16 products increases inventories by more than 37 units, backlogs by more than 108 units, and replenishment lead times by more than 1016 time periods. An increase in system load leads to a decrease in expected total inventories as the production facility falls behind in meeting customer orders, a phenomenon indicated by an increase in both expected total backlogs and expected replenishment lead times. More specifically, under the given parameter values, increasing system load from 0.80 to 0.90 reduces inventories by more than 14 units, increases backlogs by

more than 77 units, and increases replenishment lead times by more than 464 time periods. An increase in setup times or in the variability of the manufacturing process has a similar impact on system performance. Bradley and Conway (1998) offer a crisp description of this phenomenon.

	#Products, n	Load	Setup	CVdmd	CVmfg
Inventory	37.72 *	-14.65 *	-36.79 *	0.71	-3.74
Backlog	108.56 *	77.78 *	153.40 *	3.45 *	20.79 *
Lead Time	1016.07 *	464.32 *	956.32 *	13.46 *	122.67 *

Table 2. *Main Effects: Impact on Inventory, Backlog, and Lead Time*

$n \times$	Load	Setup	CVdmd	CVmfg
Inventory	-7.55 *	-22.19 *	0.83	-3.19
Backlog	45.17 *	97.99 *	0.56	18.22 *
Lead Time	265.93 *	605.68 *	-1.21	105.08 *

Table 3. *Two-Way Interactions between No. of Products and Other System Characteristics*

From the two-way interactions, namely the interactions between the breadth of the product portfolio and other system characteristics such as system load, set up times, and the variability in demand and supply, we further observe that a broad product line coupled with an increase in system load or in setup times or in the variability of the manufacturing process leads to increased operating costs and poorer service. More specifically, under the given parameter values, broadening the product portfolio from 4 products to 16 products along with an increase in setup times from 10 time units to 40 time units reduces inventories by more than 22 units, increases backlogs by 99 units, and increases replenishment lead times by more than 605 time periods. It is interesting to note that the breadth of the product line and the variability of the demand process have no significant combined effect on system performance.

5. Conclusion

Product proliferation creates a major operational challenge in manufacturing. In this paper, we have captured some essential drivers of the impact of product variety on

logistics performance through a simple model of an integrated production-distribution system. In particular, we have established that, keeping total inventories constant, the expected cost of inventories and backorders increases linearly with the number of products due to the loss of pooling economies. Furthermore, the phenomenon appears to be quite robust, as it does not depend on load, flexibility, or processing variability. Our results therefore quantitatively support earlier intuition on the operational challenges of product proliferation (Miller and Vollman, 1985).

In addition, a preliminary analysis of the data collected through the Best Factory Award (an initiative undertaken by several European Business Schools, and business magazines) has indicated that our results are indeed valid (Loch et al., 2003).

In practice, there have been several initiatives to better manage the operational challenge created by product proliferation. Companies have invested in information technology to enhance the transparency of supply chains and to reduce the order fulfillment cycle. Other initiatives include the redesign of products (e.g., promoting part commonality, component standardization) and/or the restructuring of the manufacturing process (e.g., postponement, modular design) to mitigate the inventory-service trade-off. From a unifying perspective, these initiatives are all instances of risk pooling, the use of a pool of resources to satisfy several classes of demands. More broadly, risk pooling includes strategies such as centralization of inventory (Eppen, 1979), transshipments (Herer, Tzur, and Yücesan, 2002), delayed differentiation (Lee and Tang, 1997), component commonality (Gerchak, Magazine, and Gamble, 1988), product substitution (Parlar and Goyal, 1984), and flexible capacity (van Mieghem, 1998).

The current study focuses exclusively on cost. However, an increase in product variety should hopefully be associated with an increase in revenues. There exists some empirical evidence suggesting that the increase in manufacturing costs due to higher product variety is outpaced by an increase in revenues, yielding higher profitability (Kekre and Srinivasan, 1990). An integrated production-distribution model with explicit cost and revenue components should shed further light on this important interaction.

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