Shareholder Monitoring with Strategic Investors

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September 21, 2011

Abstract

This paper provides a generalization of the theory of shareholder monitoring, originally developed by Admati, Pfleiderer, and Zechner (1994). An activist shareholder can trade with a finite number of strategic passive investors. If there are only a few investors, then their strategic behavior leads to an allocation of shares that increases the activist investor’s incentive to monitor, which is socially desirable. This is because they take into account the effect of their purchases on the incentives of the large shareholder, so they buy fewer shares. On the other hand, a large number of investors leads to free-riding and less monitoring. In the limit, as the number of investors grows to infinity, a similar equilibrium as in Admati, Pfleiderer, and Zechner (1994) emerges. The model formalizes the idea that a financial market with a small number of shareholders provides stronger incentives for shareholder monitoring. The prediction is consistent with empirical findings which show that in countries where monitoring is important, e.g. because of weak legal protection of investors, shareholder concentration is high and stock market participation is low.

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1 Introduction

In a highly influential paper, La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1998) study how the legal origin of a country influences the ownership structure of large corporations. They find that ownership concentration is higher in countries where the legal protection of shareholders is weak. High concentration is costly because of the lack of diversification in investors’ portfolios. But it is necessary in the sense that in countries where managers can expropriate outside investors, shareholder concentration provides stronger incentives for investors to engage in monitoring. However, large block holders are unlikely to emerge in a competitive financial market. As Admati, Pfleiderer, and Zechner (1994) show, a risk-averse, large shareholder has an incentive to sell his shares on the market, because he pays the full cost of monitoring and receives only part of the benefits. Through trading his block shrinks and he has no incentive to provide the public good of monitoring. Thus the question arises how concentrated ownership emerges in the first place. This paper argues that a smaller financial market, i.e. a lower stock market participation, can lead to endogenous large block holders. The intuition is best understood in the extreme case with only one activist shareholder, and one passive investors. The passive investor understands that if he cuts back his demand for shares, trade will result in a larger allocation to the activist shareholders. The latter will have stronger incentives to monitor, which will increase the expected payoff of all shares. In other words, the passive investor is strategic and understands that his own shares will be worth more if he cuts back his demand. This basic intuition generalizes to multiple passive investors. As the number of investors in the economy increases, however, the passive investors start to free-ride on each other and the strategic effect diminishes. In the limit, as the number of investors grows to infinity, a similar result as in Admati, Pfleiderer, and Zechner (1994) emerges. Recent empirical evidence is consistent with the main predictions of the model. Guiso, Sapienza, and Zingales (2008) show that countries with weak protection of shareholder rights, i.e. the countries with the largest need for shareholder monitoring, are associated with a low stock market participation.

The main difference to Admati, Pfleiderer, and Zechner (1994) is that the present paper does not assume a continuum of passive investors. Instead, the active shareholder can trade with a finite number of strategic investors. This new assumption has the advantage that strategic effects can be studied, with interesting economic implications. At the same time the results of Admati, Pfleiderer, and Zechner (1994) emerge in the limit as the number of investors grows to infinity. The present model allows to study how the number of investors in the economy affects allocations and incentives, while this is not possible under the assumption of a continuum of investors.

The paper is related to the literature on block holders in large corporations. Admati, Pfleiderer, and Zechner (1994) predict that risk aversion together with the fact that large shareholders only reap a fraction of the benefits from monitoring induce them to sell their shares. However, even in countries with dispersed ownership structures we observe that most corporations have several block holders, see Dlugosz, Fahlenbrach, Gompers, and Metrick (2006). One simple explanation for the existence of controlling block holders is private benefits of control. This assumption drives the emergence of blocks in Zwiebel (1995), Bennedsen and Wolfenzon (2000), and Bloch and Hege (2003). There is empirical evidence for significant
private benefits of control, for instance in Zingales (1994) and Doidge, Karolyi, Lins, Miller, and Stulz (2009). But even if those benefits are small, there might be reasons for block holders. Maug (1998) shows that trading with uninformed investors allows an activist investor to make profits that cover his private monitoring costs. Edmans and Manso (2011) extend this argument to a framework with multiple block holders, where the trading decisions of activist investors move the stock price closer to the fundamental value. This increases the manager’s incentive to exert effort, which in turn increases firm value. Again, block holders recover their monitoring costs by trading against uninformed noise traders. The present framework, on the other hand, does not assume private benefits of control or noise traders. It is shown that an activist large shareholder may emerge even in the absence of private benefits. Additionally, each player is rational and maximizes expected utility, as opposed to models with noise traders. Finally, it should also be mentioned that large shareholders can create inefficiencies. Burkart, Gromb, and Panunzi (1997) show that although monitoring by a large shareholder is ex post efficient, it can be inefficient ex ante by reducing the manager’s incentive to invest in firm specific human capital.

The remainder of the paper is structured as follows. Section 2 introduces the basic model and provides a closed form solution for the case with a simple monitoring technology. Section 3 shows that the extensive form of the basic model is comparable to Admati, Pfleiderer, and Zechner (1994), and considers an alternative model with a different monitoring technology. Section 4 concludes.

2 The Model

The model is a non-cooperative game with perfect information. The players are a firm owner $O$ and a finite number of outside investors, indexed with $i \in \{1, \ldots, N\}$. The owner is endowed with a firm which is modeled as a claim to a normally distributed dividend per share

$$D \sim N(\mu, \sigma^2), \quad \text{with} \quad \mu \geq 0, \sigma > 0.$$  

Further he can choose a monitoring level $m \in M \subseteq \mathbb{R}_+$, which changes the distribution of the dividend to

$$D' \sim N(\mu + m, \sigma^2).$$

Monitoring causes a private cost $c(m)$, which is an increasing function of the monitoring level. It is assumed that monitoring is a non-contractible action, and the total number of shares of the firm is $S \geq 1$. The order of moves is as follows. First the firm owner selects a price $p \in \mathbb{R}_+$ at which he is willing to sell some of his shares. After that the investors simultaneously choose how many shares to buy, $x_i \in \mathbb{R}_+$, with $i \in \{1, \ldots, N\}$. Subsequently the owner decides how much to monitor, $m \in M \subseteq \mathbb{R}_+$. At the end of the game the payoff of the firm realizes, the cost of monitoring is paid, and consumption takes place. The timeline of events is summarized in Figure 1. It is assumed that the owner has no wealth except his ownership in the firm, and that the initial wealth of every outside investor is zero. This is without loss of generality in a CARA-normal model, because initial wealth has no effect on the demand for stocks. Also, any residual wealth not invested in the firm can be invested in a risk free asset with a normalized interest rate of zero.
Owner selects price $p \in \mathbb{R}_+$

Investors choose quantity $x_i \in \mathbb{R}_+, \ i \in \{1, ..., N\}$

Owner’s monitoring decision $m \in M \subseteq \mathbb{R}_+$

Payoffs realize

Figure 1: Timeline of events in the model.

All players maximize expected utility of terminal wealth, $W_T$, and their Bernoulli utility function has constant absolute risk aversion (CARA):

$$U(W_T) = E[u(W_T)] = E[-e^{-aW_T}].$$

The constant $a > 0$ denotes the coefficient of absolute risk aversion and is assumed to be the same for all players. Note that terminal wealth is a random variable which depends on the stock price $p$, the number of shares owned by a player and the monitoring decision of the owner. Additionally, $W_T$ is normally distributed, which allows to use the moment generating function of the normal distribution to simplify the utility of any player to

$$E[-e^{-aW_T}] = -\exp\left\{-aE[W_T] + \frac{a^2}{2}\text{Var}[W_T]\right\}.$$

Maximizing the owner’s utility is then equivalent to maximizing

$$U_O(p,m) = xp - c(m) + (1 - x)m - \frac{a\sigma^2}{2} (1 - x)^2,$$  \hspace{1cm} (1)

where $x = \sum_{i=1}^{N} x_i$. Analogously, maximizing an outside investor’s utility is equivalent to maximizing

$$U_i(x_i) = x_i(\mu + m - p) - \frac{a\sigma^2}{2} x_i^2, \ i \in \{1, ..., N\}.$$  \hspace{1cm} (2)

Note that the parameters $a$ and $\sigma^2$ appear together in the payoff functions. To simplify algebraic expressions, it will be useful to define $\alpha = a\sigma^2$, without loss of generality, as a single parameter which measures both risk aversion and risk. The following subsections assume simple parameterizations for the cost of monitoring function $c(m)$ and the set of possible monitoring levels $M$, for which the model is then solved in closed form.

### 2.1 A Simple Example

In this subsection a simple example illustrates the intuition behind the main results. Assume the firm owner can choose a monitoring level $m \in M = \mathbb{R}_+$, and incurs cost of monitoring $c(m) = \gamma m^2/2$, with $\gamma > 0$. The total supply of shares is $S = 1$, and for simplicity $\mu = 0$. Then the owner maximizes

$$U_O = xp - \frac{\gamma m^2}{2} + (1 - x)m - \frac{\alpha}{2}(1 - x)^2,$$
and each outside investor maximizes

\[ U_i = x_i(m - p) - \frac{\alpha}{2} x_i^2. \]

Backwards induction is used to find a subgame-perfect Nash equilibrium of the game. First, the optimal monitoring decision of the firm owner is derived. This is followed by the optimal demand for stocks of the outside investors. Finally, the stock price that maximizes the owner’s utility is found.

The optimal monitoring level, given \( p \) and \( x \), is \( m = (1 - x)/\gamma \). Substitute this into \( U_i \) to find the optimal demand of investor \( i \). Note that the investors are in a strategic situation: every price \( p \) initiates a subgame in which the investors play against each other. Investor \( i \) chooses \( x_i \), given the demand of the others, \( x_{-i} = \sum_{j \neq i} x_j \). The best reply function of investor \( i \) is

\[ x_i = \frac{1 - x_{-i} - \gamma p}{2 + \alpha \gamma}. \]

Similarly to standard Cournot competition, the game is one of strategic substitutes. The analogy to Cournot competition also highlights an important feature of the game between investors. When thinking about how many shares to buy, investor \( i \) only takes into account the negative effect of his own demand on the monitoring decision of the owner. Therefore, each investor demands more shares than the amount which would be optimal for the investors as a group. In other terms, there is a free-rider problem between the investors. They would demand fewer shares if they could collude to choose demands collectively.\(^1\)

Assuming a symmetric Nash equilibrium in pure strategies for the game between investors, aggregate demand is

\[ x(p) = \frac{N(1 - \gamma p)}{N + 1 + \alpha \gamma}. \]  

(3)

Figure 2 illustrates the aggregate demand function in (3). It shows that the demand curve becomes steeper as the number of investors grows. This happens for two reasons. First, as there are more investors, the aggregate risk bearing capacity of outside investors in the economy increases, so aggregate demand for the risky asset goes up. In the limit, outside investors collectively behave like a risk neutral investor. The second reason is more subtle. Every investor takes into account the effect of his own demand on the monitoring incentives of the owner. But when \( N \) is large, every investor free-rides on the others and demands a few more shares. In the aggregate, this free-rider problem leads to a steeper demand function. Finally, the optimal price \( p \) can be found by substituting the optimal monitoring level and the aggregate demand function to the owner’s objective function \( U_O \) and solving the first order condition, which yields

\[ p = \frac{1/\gamma}{1 + \alpha \gamma} + \frac{\alpha(1 - \alpha \gamma)}{(N + 2)(1 + \alpha \gamma)}. \]  

(4)

The corresponding equilibrium demand is

\[ x = \frac{\alpha \gamma N}{(2 + N)(1 + \alpha \gamma)}. \]  

(5)

\(^1\)The formal proof that they indeed demand fewer shares when it is possible for them to collude and the conditions under which they prefer to do so are available from the author.
Let the number of outside investors $N$ grow to infinity, then

$$\lim_{N \to \infty} p = \frac{1/\gamma}{1 + \alpha \gamma} \quad \text{and} \quad \lim_{N \to \infty} x = \frac{\alpha \gamma}{1 + \alpha \gamma}.$$ 

These expressions are very intuitive in the sense that they capture the tradeoff between risk sharing and monitoring incentives. If monitoring is very costly, i.e. if $\gamma$ is large, then the owner sells a large fraction of the firm at a low price. This is because risk sharing is relatively more attractive to the owner, hence he sells a large fraction of the firm. His remaining holdings are very small so he has no incentive to monitor, which yields a low price. On the other hand, if $\gamma$ is small, monitoring becomes relatively more attractive, while risk sharing is less important to him. This results in a small number of shares sold, but at a high price. Also, if the firm is riskier, the owner sells a higher fraction of the firm because risk sharing is more important to him. In equilibrium the stock price is decreasing in firm risk because of two reasons. First, the lower ownership share of the owner leads to less monitoring. Second, outside investors require a risk premium through a discounted price.

The following Proposition summarizes these findings and generalizes them to an arbitrary number of investors $N$.

**Proposition 1.** Both a higher risk parameter $\alpha = a \sigma^2$ and a higher cost of monitoring parameter $\gamma$ imply that in equilibrium

1. the stock price $p$ is lower,
2. the fraction of the firm sold $x$ is higher,
3. the owner’s monitoring level $m$ is lower.

This also holds for the limit case when $N \to \infty$, where additionally the stock price $p$ equals the expected payoff $m$, i.e. there is no discount in $p$ due to risk. Also, if there are more outside investors in the economy, the owner sells a larger fraction of the firm and monitors less.
The intuition for this result, in particular the effect of the number of outside investors, is best understood by looking at Figure 2. As \( N \) increases, the free-rider problem between outside investors becomes stronger. They demand more shares because they only partly internalize the costs of their demand. This stronger demand for shares makes risk sharing relatively more attractive to the owner than monitoring. As a result he sells more shares and monitors less.

### 2.2 The general case

In the simple example of the previous subsection, outside investors collectively behaved like a risk neutral agent when \( N \to \infty \). This is because the supply of the risky asset was fixed at \( S = 1 \), but the risk bearing capacity of the economy increased with \( N \). But in Admati, Pfleiderer, and Zechner (1994), the outside (or passive) investors have positive aggregate risk aversion. A simple solution to this problem is to let the supply of the risky asset increase with \( N \), a technique also used by Garcia and Urosevic (2010) in a different context. Therefore, assume that the supply of risky assets equals the number of outside investors in the economy, \( S = N \). At the same time it is necessary to assume that the cost of monitoring is increasing in \( N \). Otherwise the equilibrium level of monitoring in unbounded as \( N \to \infty \). Therefore, assume that the cost of monitoring is

\[
c(m) = \frac{\gamma N m^2}{2},
\]

with \( \gamma > 0 \). The firm’s expected payoff per share without monitoring is \( \mu \geq 0 \). Then the owner’s objective function is

\[
U_O(p, m) = xp - \frac{\gamma N m^2}{2} + (N - x)(\mu + m) - \frac{\alpha}{2}(N - x)^2,
\]

where \( x = \sum_{i=1}^{N} x_i \). Analogously, outside investor \( i \) maximizes

\[
U_i(x_i) = x_i(\mu + m - p) - \frac{\alpha}{2} x_i^2, \quad i \in \{1, \ldots, N\}.
\]

The same backwards induction as in the previous section yields an optimal monitoring intensity of \( m(x) = (N - x)/(\gamma N) \), given that \( x \) shares have been sold. Solving the subgame played between outside investors provides the aggregate demand for shares,

\[
x(p) = \frac{N^2(1 + \mu \gamma - \gamma p)}{1 + N + \alpha \gamma N}.
\]

Finally, the equilibrium price \( p \) and quantity \( x \) can be calculated, as well as the equilibrium level of monitoring \( m \). The following proposition summarizes the results in the general case.

**Proposition 2.** In equilibrium, the share price is

\[
p = \frac{1 + \mu \gamma + \alpha \mu \gamma^2 N}{\gamma (1 + \alpha \gamma N)} + \frac{\alpha N (1 - \alpha \gamma N)}{(2 + N)(1 + \alpha \gamma N)},
\]

\footnote{The formal proof of this statement is available from the author.}
the number of shares sold is
\[ x = \frac{\alpha \gamma N^3}{(2 + N)(1 + \alpha \gamma N)}, \]
and the owner’s monitoring intensity is
\[ m = \frac{1}{\gamma} - \frac{\alpha N^2}{(2 + N)(1 + \alpha \gamma N)}. \]

As the number of outside investors grows, the owner sells more shares and monitors less.

The last part of Proposition 2 says that if there are more investors in the economy, then the owner finds it optimal to sell more shares and engage in less monitoring. This is a result that cannot be derived in the framework of Admati, Pfleiderer, and Zechner (1994), because they do not have a finite number of investors. The intuition for the result is the same as in the simple example of the previous section. If there are more investors then the demand for risky assets is stronger, because each of them only takes into account the effect of his own demand on the owner’s incentives. This makes risk sharing relatively more attractive for the owner compared to monitoring.

Admati, Pfleiderer, and Zechner (1994) show that the initial owner does not trade all the way to the point of optimal risk sharing, i.e. there is a large shareholder after trading. A look at \( x \) in Proposition 2 might suggest that there is no large shareholder after trade, because \( \lim_{N \to \infty} x/N = 1 \). In other words, the fraction of the firm sold converges to one. But this is simply a consequence of the way the limiting economy is constructed. As \( N \) grows, the economy becomes bigger, and in the limit the owner’s risk bearing capacity is negligible. A better way to determine whether there is a large shareholder after trade is to look at the firm owner’s absolute shareholdings relative to the shares of a typical passive investor. The following corollary establishes that there exists a large shareholder after trade, even in the limiting economy.

**Corollary 1.** The owner’s absolute shareholdings relative to the shares held by a typical outside investor after trade converge to
\[ \lim_{N \to \infty} \left( 2 + \frac{1}{\alpha \gamma} + \frac{2}{\alpha \gamma N} \right) = 2 + \frac{1}{\alpha \gamma}. \]

In this sense there exists a large shareholder after trade, even in the limiting economy.

It is interesting to examine the three terms in Corollary 1, before the limit is taken. The first term is the number of shares held by the owner is monitoring is not possible. It is larger than one because the owner effectively is a monopolist in the market for risky assets. The second term \( 1/(\alpha \gamma) \) adds a positive amount to the owner’s holdings after trade, because monitoring increases firm value so he wants to sell fewer shares. The third term, \( 2/(\alpha \gamma N) \), is a correction that comes from the fact that outside investors are strategic. They understand that their demand decreases the owner’s monitoring incentives, so they reduce demand. This induces the owner to keep more shares for himself and to monitor more. However, the strategic effect becomes negligible as the number of investors grows to infinity. In the limit it disappears, and the result becomes comparable to Admati, Pfleiderer, and Zechner (1994).
3 Robustness

3.1 Extensive Form

Naturally the question arises whether the extensive form of the game proposed here is comparable to the anonymous Walrasian trading mechanism in Admati, Pfleiderer, and Zechner (1994). In principle there could be other extensive forms which are closer the original framework. To show that the present framework fits well to the work of Admati, Pfleiderer, and Zechner (1994), one can assume a continuum of agents and at the same time keep the current extensive form, i.e. the order of moves and so on. If the trading mechanism proposed here is comparable to Admati, Pfleiderer, and Zechner (1994), then the result should be similar to the equilibrium in their model. Let’s index investors with $i \in [0, 1]$, define $x = \int_0^1 x_i di$, normalize the supply of the risky asset to one, and keep the remaining notation from the previous section. Then in the symmetric Nash equilibrium of the game between investors, the aggregate demand function is

$$x(p) = \frac{1 - \gamma p}{1 + \gamma \alpha}.$$ 

Substituting this demand function to the owner’s objective function allows to derive the equilibrium price and quantity

$$p = \frac{1 + 2\alpha \gamma - \alpha^2 \gamma^2}{\gamma(1 + 3\alpha \gamma)} \quad \text{and} \quad x = \frac{\alpha \gamma}{1 + 3\alpha \gamma}.$$ 

Note that the quantity is the same as in Admati, Pfleiderer, and Zechner (1994). Adapting their notation to the present one, the authors write that in equilibrium the number of shares sold is $x = \alpha \gamma/(1 + 3\alpha \gamma).^3$ The prices are identical as well, which can be seen after some rearrangements to the results in Admati, Pfleiderer, and Zechner (1994). Therefore, the two approaches are equivalent in the sense that they yield the same prices and quantities. We conclude that the extensive form proposed in the current paper is appropriate to analyze the effect of a finite number of passive investors on shareholder monitoring.

3.2 Discrete monitoring choice

In the previous sections the assumptions of the model are kept as closely as possible to Admati, Pfleiderer, and Zechner (1994). In particular, the owner is endowed with a continuous monitoring technology and a quadratic cost function. The current section presents a model with a discrete monitoring technology, but otherwise the extensive form of the game is the same. It turns out that a discrete monitoring technology leads to surprising and seemingly counter intuitive results. In particular, it can happen that the availability of a monitoring technology makes all players worse off.

The owner can only choose between two levels of monitoring, $m \in M = \{0, \bar{m}\}$, with $\bar{m} > 0$. Analogously to the model with continuous monitoring, $m$ measures the shift in the distribution of the firm’s payoff. The cost of monitoring is $c \in (0, \bar{m})$.\(^4\) The supply of stocks

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\(^3\)Page 1109, with $\omega = 1$, $\alpha_A = 1 - x$, and $\tau = \rho = 1/a$.

\(^4\)Note the slight abuse of notation, as $c$ is both the name of the cost of monitoring function in the general model and also the constant cost of monitoring in the version of the model with discrete monitoring. Also, the symbol $\bar{m}$ is used as the name of the action “monitoring” and as the positive number by which the payoff distribution is shifted if the owner monitors.
in set to $S = 1$. Backwards induction is used to find a subgame-perfect Nash equilibrium of the game. First, the optimal monitoring decision of the firm owner is derived. This is followed by the optimal demand for stocks of the outside investors. Finally, the stock price that maximizes the owner’s utility is found.

It is optimal for the owner to monitor if his utility with monitoring is at least as large as without monitoring. This is equivalent to the inequality

$$xp - c + (1 - x)(\mu + \bar{m}) - \frac{\alpha}{2}(1 - x)^2 \geq xp + (1 - x)\mu - \frac{\alpha}{2}(1 - x)^2.$$  

This inequality reduces to $(1 - x)\bar{m} \geq c$, which is simply the statement that the owner’s share of the benefits from monitoring has to be at least as large as his costs. From this it follows that the monitoring decision only depends on $x$, the aggregate demand of the outside investors. If aggregate demand lies below the threshold $1 - c/\bar{m}$, then the owner monitors. If $x \geq 1 - c/\bar{m}$ then it is optimal not to monitor. Formally the best reply correspondence of the owner is

$$m(x) = \begin{cases} 
\text{monitor} & \text{if } x < 1 - c/\bar{m} \\
\text{monitor, not monitor} & \text{if } x = 1 - c/\bar{m} \\
\text{not monitor} & \text{if } x > 1 - c/\bar{m}.
\end{cases}$$ (7) 

The next step is to derive the optimal demand for stocks of the outside investors. Each price $p$, chosen by the owner, initiates a subgame where the investors play against each other and simultaneously decide how many shares to buy. The investors understand that if their aggregate demand exceeds the threshold $1 - c/\bar{m}$, then the owner monitors. If $x \geq 1 - c/\bar{m}$ then it is optimal not to monitor. Formally the best reply correspondence of the remaining investors is

$$x_i = \begin{cases} 
(\mu - p)/\alpha & \text{if } U_i(p, 1 - c/\bar{m} - x_{-i}, \bar{m}) \leq \max_{x_i} U_i(p, x_i, 0), \\
1 - c/\bar{m} - x_{-i} & \text{if } U_i(p, 1 - c/\bar{m} - x_{-i}, \bar{m}) \geq \max_{x_i} U_i(p, x_i, 0) \\
\frac{\mu + \bar{m} - p}{\alpha} & \text{otherwise},
\end{cases}$$ (8) 

for a given demand of the remaining investors $x_{-i} = \sum_{j \neq i} x_j$. The three parts of the correspondence agree with the three cases in Figure 3. Writing this best reply correspondence
Figure 3: These figures show the utility of investor $i$ as a function of the number of stocks $x_i$ he acquires. In each case, the solid parabola depicts his utility if the owner monitors, whereas the dashed parabola depicts his utility when the owner does not monitor. Case 1 obtains if the stock price $p$ is small. As $p$ increases the position of the parabolae changes relative to the threshold $1 - c/m - x_i$. Case 2 and case 3 obtain for intermediate and high values of $p$, respectively.

Explicitly yields

$$x_i = \begin{cases} 
(\mu - p)/\alpha & \text{if } (\mu + \bar{m} - p)(1 - c/\bar{m} - x_{-i}) - \alpha(1 - c/\bar{m} - x_{-i})^2/2 \leq \frac{(\mu - p)^2}{2\alpha}, \\
1 - c/\bar{m} - x_{-i} & \text{if } (\mu + \bar{m} - p)(1 - c/\bar{m} - x_{-i}) - \alpha(1 - c/\bar{m} - x_{-i})^2/2 \geq \frac{(\mu - p)^2}{2\alpha}, \\
(\mu + \bar{m} - p)/\alpha & \text{otherwise}.
\end{cases}$$

The game between the outside investors has both symmetric and asymmetric Nash equilibria in pure strategies. The following Lemma is based on symmetric equilibria and presents the aggregate demand correspondence of the investors.

**Lemma 1.** In a symmetric Nash equilibrium in pure strategies of the game between the $N$ investors there are two thresholds $\underline{p} < \bar{p}$ such that the aggregate demand for shares $x$ is a piecewise linear correspondence

$$x(p) = \begin{cases} 
N(\mu - p)/\alpha & \text{if } p \leq \underline{p}, \\
1 - c/\bar{m} & \text{if } \underline{p} < p \leq \bar{p}, \\
N(\mu + \bar{m} - p)/\alpha & \text{if } \bar{p} \leq p
\end{cases}$$

with a single discontinuity at $\underline{p}$. The thresholds

$$\underline{p} = \mu - \frac{\alpha(1 - c/\bar{m})}{N} + \frac{\bar{m}}{N} - \frac{\bar{m}}{N^2} - \frac{\sqrt{2\alpha N(\bar{m} - c) + \bar{m}^2(N - 1)^2}}{N^2}$$

and

$$\bar{p} = \mu + \bar{m} - \frac{\alpha(1 - c/\bar{m})}{N}$$

are increasing in $N$ and $c$, with $\lim_{N \to \infty} \underline{p} = \mu$ and $\lim_{N \to \infty} \bar{p} = \mu + \bar{m}$. 

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Figure 4: Aggregate demand correspondence of the investors for different values of the stock price $p$. The two vertical dashed lines depict the thresholds $p_L$ and $\bar{p}$, respectively.

Figure 4 depicts the aggregate demand correspondence in stylized form. It shows that it is piecewise linear, weakly decreasing, and discontinuous at $p$.

The remaining step to solve the model is to find the optimal stock price $p$. By choosing $p$ the owner effectively picks a point on the demand correspondence in Figure 4. The owner’s optimal price lies on one of the three segments of the correspondence. Suppose for now that at $p$ aggregate demand is $(\mu - p)/\alpha$, i.e. at the upper end of the discontinuity. Informally, if the cost of monitoring $c$ is large relative to the benefit of monitoring $\bar{m}$, the optimal price will lie on the first segment of the demand correspondence. This is because both $p$ and $\bar{p}$ are large, so the first segment extends over a large set of prices. In this equilibrium the owner sells a fraction of the firm at a relatively low price and does not monitor. On the other hand, for small values of $c$, the third part of the demand function dominates. In this case the optimal price is on the third segment, so the owner sells a fraction of the firm at a relatively high price and monitors. For intermediate values of $c$, the optimal price is on the middle segment of the demand function in Figure 4, and the owner monitors in equilibrium. The following Proposition presents the subgame-perfect Nash equilibria of the game for different values of exogenous parameters.

**Proposition 3.** There are two thresholds $c \leq \bar{c}$, where $c < \bar{c}$ if and only if $N = 1$. Depending on parameters, the following are subgame-perfect Nash equilibria.

1. If $c \geq \bar{c}$, then $p = \mu - \alpha/(2 + N)$, $x = N/(2 + N)$, and $m = 0$. 

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2. If \( \xi \leq c \leq \bar{c} \), then \( p = \frac{\bar{m}}{N} \), \( x = N(\mu - p)/\alpha \), and \( m = 0 \), where
\[
p = \mu - \frac{\alpha(1 - c/\bar{m})}{N} + \frac{\bar{m}}{N} - \frac{\sqrt{2\alpha N(\bar{m} - c) + \bar{m}^2(N - 1)^2}}{N^2}.
\]

3. If \( \bar{m}(1 - \frac{N}{2 + N}) \leq c \leq \xi \), then \( p = \bar{p} \), \( x = 1 - c/\bar{m} \), and \( m = \bar{m} \), where
\[
\bar{p} = \mu + \bar{m} - \alpha(1 - c/\bar{m})/N.
\]

4. If \( c \leq \bar{m}(1 - \frac{N}{2 + N}) \), then \( p = \mu + \bar{m} - \alpha/(2 + N) \), \( x = N/(2 + N) \), and \( m = \bar{m} \).

The different types of Nash equilibria can be depicted graphically as in Figure 5, where the value of \( c \) changes for fixed \( \bar{m} \) and \( \alpha \). The numbers 1, 2, 3, and 4 refer to the numbering in Proposition 3. The solid line denotes the parameter region where the owner monitors, on the dashed line he does not.

![Figure 5](image-url)

Figure 5: For varying values of the monitoring cost \( c \) (for fixed \( \bar{m} \) and \( \alpha \)), four different types of subgame-perfect Nash equilibria exist. The numbers 1, 2, 3, and 4 refer to the numbering in Proposition 3. The solid line denotes the parameter region where the owner monitors, on the dashed line he does not. The constants \( \xi \) and \( \bar{c} \) are defined in Proposition 3.

At first sight the equilibria in Proposition 3 are very intuitive. If the cost of monitoring \( c \) is close to the benefit of monitoring \( \bar{m} \), there is no monitoring in equilibrium. The share price reflects this and is relatively low at \( p = \mu - \alpha/(2 + N) \), where \( \mu \) is the expected payoff of the firm without monitoring and the second term is a correction for risk. Note that from a social perspective it would be optimal to monitor, because \( c < \bar{m} \). The owner sells a fraction \( x = N/(2 + N) \) of his shares. On the other hand of the spectrum, where \( c \) is very small, the owner monitors in equilibrium. The stock price is therefore higher at \( p = \mu + \bar{m} - \alpha/(2 + N) \).

Interestingly, the fraction of the shares sold is the same as before. For intermediate values of \( c \) things are different. In the region where \( \bar{m}(1 - N/(2 + N)) \leq c \leq \xi \), the outside investors buy just enough shares to make the owner indifferent between monitoring and not monitoring. The reason why this particular equilibrium exists is the discrete monitoring choice. The risk sharing properties of this equilibrium are particularly bad, because \( x = 1 - c/\bar{m} \) is decreasing with \( c \). The welfare implications of distorted risk sharing are demonstrated in Section 3.2.1. Finally, in the region where \( \xi \leq c \leq \bar{c} \), it is optimal for the owner to charge a price \( \bar{p} \), which is at the discontinuity in Figure 4. At this price the investors are indifferent between high demand and low demand, while for the owner charging a slightly higher price makes no sense: he would have to monitor (and pay the costs) either way, plus he would be able to sell a smaller number of shares. Thus the owner exploits the discontinuity in the demand correspondence and sells a larger fraction of the firm at this point. This improves risk sharing between the owner and investors.
Corollary 2. As the number of outside investors \( N \) increases, the region where the owner strictly prefers to monitor contracts. In the limit as \( N \to \infty \) the owner never has a strict incentive to monitor. Also, the region where he monitors by indifference contracts for \( N \geq 2 \), but in the limit as \( N \to \infty \) this region does not collapse.

Informally the owner has a strict incentive to monitor if a slightly higher demand from the investors does not destroy his incentive to monitor. The Corollary shows that this particular outcome is not possible in the limit as \( N \to \infty \). Note that in traditional models of shareholder monitoring, such as Admati, Pfleiderer, and Zechner (1994), the owner always has a strict incentive to monitor. The reason for this discrepancy is that the present model assumes a discrete monitoring choice, whereas Admati, Pfleiderer, and Zechner (1994) assume a continuous monitoring level. Finally, the type of equilibrium which survives in the limit is the one with distortions in risk sharing between owner and investors.

Corollary 3. As the number of outside investors \( N \) increases, the price \( p \) at which the owner can sell his shares increases. The amount of shares sold \( x \) increases with \( N \), which improves risk sharing between the owner and investors.

The Corollaries says, intuitively, that the owner is better off when he faces more outside investors. He can sell more shares and at a higher price. This is good for risk sharing between the owner and investors, but at the same time the region where the owner strictly prefers to monitor contracts. Correspondingly the investors prefer \( N \) to be small.

Corollary 4. The number of shares sold in equilibrium is a non-monotonic function of the cost of monitoring.

The reason why Corollary 4 is interesting is that in traditional models of shareholder monitoring, such as Admati, Pfleiderer, and Zechner (1994), \( x \) is a strictly increasing function of \( c \). The previous Section assumes a continuous monitoring choice where \( m \in \mathbb{R}_+ \) is chosen by the owner and the cost of monitoring is quadratic in \( m \). There, \( x \) indeed increases with the cost of monitoring. This suggests that the non-monotonicity in the present model depends on the assumption that the monitoring choice is not continuous.

Corollary 5. As the risk of the firm or the risk aversion of the players increases, the region where the owner monitors contracts.

This relationship can also be found in Admati, Pfleiderer, and Zechner (1994), and it captures the trade-off between monitoring and risk sharing. As the firm’s riskiness or the players’ risk aversion increases, the part of the parameter space with monitoring shrinks. In the complement of this region, which corresponds to part 1 and 2 in Proposition 3, is where the owner sells a higher share of the firm, which improves risk sharing between owner and investors.

3.2.1 Special case with one outside investor

Now look at the special case of the model where \( N = 1 \). Figure 6 presents the fraction of the firm sold, \( x \), for different values of \( c \). The solid part of the discontinuous line is where the owner monitors, and in the dashed region he shirks in equilibrium. The discontinuous line can be divided to four regions which correspond to the regions in Proposition 3. The dotted line depicts the optimal sharing rule from the perspective of a social planner.
What is interesting in Figure 6 is the number of shares sold in the region where $x$ is decreasing in $c$, which corresponds to part three of Proposition 3. In this region, $x$ drops to a level that is particularly far from the first-best risk sharing. To illustrate the consequences of this distortion in risk sharing, Figure 7 depicts the welfare of the investor from equation (2). Again, the horizontal axis covers the range $0 < c < \bar{m}$. It can be seen very clearly how the investor’s utility deteriorates over the region $c \in [\bar{m}/3, \bar{c}]$. Also note that in a world where the owner does not have the possibility to work hard, the investor would receive the utility level in $[\bar{c}, \bar{m}]$ for all values of $c$. In equilibrium, however, the investor’s utility falls below that level for $c \in [2\bar{m}/3, \bar{c}]$.

Figure 8 depicts the welfare of the owner from equation (1). Again, the horizontal axis covers the range $0 < c < \bar{m}$. Analogously to the investor, the owner’s utility deteriorates in the interval $[2\bar{m}/3, \bar{c}]$. This is caused by the distortions in risk sharing apparent in Figure 6. In the interval $[\bar{c}, \bar{c}]$ the owner’s utility is low as well, but the reason for this is different. Here the owner tries to keep the investor at the discontinuity in Figure 4, where the investor is indifferent between high and low demand. In order to do that, the owner has to offer a particularly low price, which decreases his utility. Finally note that the owner’s welfare in the interval $[2\bar{m}/3, \bar{c}]$ is below the level that he could achieve in a world where monitoring is impossible. The last finding is accentuated by the following Corollary.

**Corollary 6.** For $N = 1$ there are parameter values such that both owner and investor are worse off than in a world where monitoring is impossible. The reason for that is distorted
Figure 7: The welfare of investor $i$ in equilibrium if $N = 1$, for different values of $c$. The symbols $c_L$ and $c_U$ stand for $c$ and $\bar{c}$, respectively. The graph is based on the parameter values $\bar{m} = 1$, $\mu = 10$, $\alpha = 10$.

Figure 8: The welfare of the owner in equilibrium if $N = 1$, for different values of $c$. The symbols $c_L$ and $c_U$ stand for $c$ and $\bar{c}$, respectively. The graph is based on the parameter values $\bar{m} = 1$, $\mu = 10$, $\alpha = 10$.

4 Conclusion

This paper provides an explanation for certain empirical regularities in the Law and Finance literature. Countries with weak protection of investor rights, i.e. where shareholder monitoring is most important, ownership concentration is high and stock market participation is
low. The formal model explains how these variables are connected. Limited stock market participation leads to an allocation of shares that investors who are the best monitors have large blocks of stock. This leads to a higher ownership concentration and more monitoring in equilibrium by a large activist investor. The intuition for this result is that if there are not too many passive investors, then they realize that by cutting back their demand for shares they can induce the activist shareholder to engage in monitoring, which increases the value of all shares. The model also provides a new explanation for the emergence of block holders. The existing literature focuses on private benefits of control and trading gains from asymmetric information as a motivation for block holders. Without these options to finance the monitoring costs, Admati, Pfleiderer, and Zechner (1994) show that large shareholders want to sell their shares on the market. The present paper, however, departs from Admati, Pfleiderer, and Zechner (1994) by assuming a finite number of strategic investors instead of a continuum.

The model predicts the emergence of a large activist shareholders and smaller holdings for passive investors. This allocation of cash flow rights is not peculiar to the stock market. Similar allocations are observable in syndicated loans, see Ivashina (2009), and in syndicated venture capital financing, Wright and Lockett (2003). Any lesson learned from the analysis of shareholder activism might be used to study syndication in bank loans or venture capital. The lead investor in a syndicate has a similar monitoring role as a large shareholder, with the same moral hazard problem as the lead investor’s share becomes small. The present paper might explain why empirically both types of syndicates are rather small groups. Another potential extension of the present framework is multiple trading rounds among shareholders. Admati, Pfleiderer, and Zechner (1994) show that in the case with a competitive stock market, i.e. with a continuum of passive investors, in the long run the large shareholder sells even more shares. If he cannot commit to a last trading round, the final allocation of shares is identical perfect risk sharing, i.e. there no large shareholder unless he is much less risk averse than passive investors. It would be interesting to examine what would happen in the present framework with multiple trading rounds.

References


Appendix

Proof of Proposition 1. For the case where $N \to \infty$, the statement follows from the observation that $p = 1/(\gamma(1 + \alpha \gamma))$ is decreasing in $\alpha$ and $\gamma$, $x = \alpha \gamma/(1 + \alpha \gamma)$ is increasing in $\alpha$ and $\gamma$, and $m = (1 - x)/\gamma = 1/(\gamma(1 + \alpha \gamma))$ is decreasing in $\alpha$ and $\gamma$.

For arbitrary values of $N$, i.e. where $N \geq 1$, the partial derivatives $\partial p/\partial \alpha < 0$, $\partial p/\partial \gamma < 0$, $\partial x/\partial \alpha > 0$, $\partial x/\partial \gamma > 0$ are based on the equilibrium prices and quantities in (4) and (5). The relationships $\partial m/\partial \alpha < 0$ and $\partial m/\partial \gamma < 0$ can be derived using chain rule and the fact that $m = (1 - x)/\gamma$.

The fact that more investors lead to a larger fraction of the firm being sold follows from $\partial x/\partial N > 0$. This leads to less monitoring, because the equilibrium monitoring intensity can be written as $m = (1 - x(N))/\gamma$. \qed
**Proof of Proposition 2.** The equilibrium price is calculated by substituting the aggregate demand function (6) into the owner’s payoff function $U_O$ and maximizing with respect to $p$. The equilibrium price together with the aggregate demand function provides the equilibrium quantity $x$. The optimal monitoring intensity follows from equilibrium demand and the relationship $m = (N - x)/(\gamma N)$. To see how equilibrium demand changes with $N$, it can be shown that $\partial x/\partial N > 0$. This leads to less monitoring, because equilibrium monitoring can be written as $m = (N - x(N))/(\gamma N)$.

**Proof of Corollary 1.** The owner’s absolute shareholdings after trade are

$$N - x(N) = N - \frac{\alpha \gamma N^3}{(2 + N)(1 + \alpha \gamma N)}.$$

The absolute shareholdings of a typical outside investor are

$$x(N)/N = \frac{\alpha \gamma N^2}{(2 + N)(1 + \alpha \gamma N)}.$$

Divide the former by the latter and simplify expressions.

**Proof of Lemma 1.** For the first part of the aggregate demand for stocks, take the first part of the best reply correspondence of investor $i$ in (8) and substitute his demand $x_i = (\mu - p)/\alpha$ for the demand of all other investors. Define $n = N - 1$, then the inequality in the first part of (8) becomes

$$(\mu + \bar{m} - p) \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right) - \frac{\alpha}{2} \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right)^2 \leq \frac{(\mu - p)^2}{2\alpha}.$$ 

At equality, this defines a quadratic equation in $p$. The smaller root of the equation, call it $p$, provides an upper bound for $p$. To see that $p$ is increasing in $N$, define the function

$$G(p, n) = (\mu + \bar{m} - p) \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right) - \frac{\alpha}{2} \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right)^2 - \frac{(\mu - p)^2}{2\alpha}.$$

The partial derivative

$$\frac{\partial G}{\partial p} = (n + 1) \left[ \frac{\mu - p}{\alpha} - \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right) \right] + \frac{n\bar{m}}{\alpha}$$

is positive if and only if the term in square brackets is positive. But this is obvious from case 1 in Figure 3, since the expression in parentheses is the vertical dashed line in the figure, while the term $(\mu - p)/\alpha$ is the critical point of the dashed parabola. The other partial derivative

$$\frac{\partial G}{\partial n} = - (\mu - p) \left[ \frac{\mu + \bar{m} - p}{\alpha} - \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right) \right]$$

is negative if and only if the expression in square brackets is positive. The previous expression in square brackets was positive and this one is larger, hence it must be positive as well. It follows from the Implicit Function Theorem that $\partial p/\partial n > 0$. To see that $p$ is increasing in $c$, consider the partial derivative

$$\frac{\partial G}{\partial c} = \frac{-\alpha}{\bar{m}} \left[ \frac{\mu + \bar{m} - p}{\alpha} - \left(1 - c/\bar{m} - n \frac{\mu - p}{\alpha}\right) \right].$$
which is negative by the same argument as before. The Implicit Function Theorem implies that $dp/dc > 0$.

In the second part of the best reply correspondence of investor $i$ it is clear that if each investor holds $x_i = 1 - c/\bar{m} - \sum_{j \neq i} x_j$, then aggregate demand is $x = 1 - c/\bar{m}$.

For the third part of the proof, assuming $x_i = (\mu + \bar{m} - p)/\alpha$ for every investor in the third part of (8) yields the inequality

$$\frac{\mu + \bar{m} - p}{\alpha} \leq 1 - c/\bar{m} - \frac{\mu + \bar{m} - p}{\alpha} \leq \frac{\mu + \bar{m} - p}{\alpha} \leq 1 - c/\bar{m} - \frac{\mu + \bar{m} - p}{\alpha} \leq 1 - c/\bar{m} - \frac{n}{n + 1} \equiv \bar{p}.$$

It follows that $\bar{p}$ is increasing in $N = n + 1$ and that $\lim_{N \to \infty} \bar{p} = \mu + \bar{m}$. To see that the demand correspondence is discontinuous at $p$, calculate $N(\mu - p)/\alpha$ and check that it is larger than $1 - c/\bar{m}$.

**Proof of Proposition 3.** The idea of the proof is to derive the optimal price for each segment of the demand correspondence in Lemma 1 separately, and then compare indirect utilities. Suppose the optimal price lies on the first segment of the demand correspondence. Within that segment the interior optimum is at $p = \mu - \alpha/(2 + N)$, which can be found by substituting $x = N(\mu - p)/\alpha$ and $m = 0$ to the owner’s utility in (1) and differentiating with respect to $p$. It follows that, at the optimum, $x = N(\mu - p)/\alpha = N/(2 + N)$. This price is feasible if $p \leq p$. Since $p$ is increasing in $c$, the feasibility constraint can be rephrased as a constraint on $c$, i.e. $c \geq c_1$, where

$$c_1 = \frac{2\alpha m - \bar{m}^2(2 + N) + \sqrt{\bar{m}^3(2 + N)(2\alpha + \bar{m}(2 + N))}}{\alpha(2 + N)}.$$

The indirect utility of the owner at the interior solution of the first segment is

$$U_O(\mu - \alpha/(2 + N), N/(2 + N), 0) = \mu - \frac{\alpha}{2 + N}.$$

Additionally, the first segment contains a corner solution, where $p = \bar{p}$, $x = N(\mu - p)/\alpha$, and $m = 0$. The indirect utility at the corner solution can be obtained by calculating

$$U_O(p, N(\mu - p)/\alpha, 0).$$

For the second part of the demand correspondence the demand is constant at $x = 1 - c/\bar{m}$, hence the owner chooses the largest feasible price $\bar{p} = \mu + \bar{m} - \alpha(1 - c/\bar{m})/N$. The corresponding indirect utility of the owner is

$$U_O(\mu + \bar{m} - \frac{\alpha(1 - c/\bar{m})}{N}, 1 - c/\bar{m}, \bar{m}) = \mu + \bar{m} - c - \frac{\alpha}{N} + \frac{2\alpha c}{Nm} - c^2 \left( \frac{\alpha}{Nm^2} + \frac{\alpha}{2m^2} \right).$$

Finally, consider the third part of the demand correspondence. Here the interior optimum is at $p = \mu + \bar{m} - \alpha/(2 + N)$, which follows from substituting $x = N(\mu + \bar{m} - p)/\alpha$ and $m = \bar{m}$.
to (1) and differentiating with respect to $p$. This results in $x = N/(2 + N)$ at the optimum. This price is feasible if

$$p \geq \bar{p}$$

$$\mu + \bar{m} - \frac{\alpha}{2 + N} \geq \mu + \bar{m} - \frac{\alpha(1 - c/\bar{m})}{N}$$

$$\bar{m} \left(1 - \frac{N}{2 + N}\right) \geq c.$$ 

The owner’s utility at the interior solution of the third segment is

$$U_O(\mu + \bar{m} - \frac{\alpha}{2 + N}, \frac{N}{2 + N}, \bar{m}) = \mu + \bar{m} - c - \frac{\alpha}{2 + N}. $$

This completes the derivation of optimal prices for all segments of the demand correspondence.

Now compare indirect utilities to find the globally optimal price for the owner. Note that the solution in segment 2 can be thought of as a corner solution of the problem in segment 3, so it can never be better than the interior solution in segment 3. Also, the indirect utility of segment 3 is larger than the indirect utility of the interior solution of segment 1, because by assumption $\bar{m} > c$. Therefore, the interior solution of segment 3 is the optimal price for the owner, whenever it is feasible. This is the case if $c < \bar{m} (1 - N/(2 + N))$. Now compare the interior solution of segment 1 to the solution of segment 2. The owner prefers segment 1 if and only if

$$\mu - \frac{\alpha}{2 + N} \geq \mu + \bar{m} - c - \frac{\alpha}{N} + \frac{2\alpha c}{\bar{m}} - c^2 \left(\frac{\alpha}{N\bar{m}^2} + \frac{\alpha}{2\bar{m}^2}\right).$$

At equality this defines a quadratic equation in $c$. Call the larger root of this equation $c_2$, then for $c \geq c_2$ the owner prefers the interior solution of segment 1 to the solution of segment 2, with

$$c_2 = \frac{2\alpha \bar{m} - \bar{m}^2 N + \sqrt{\bar{m}^3 N^2 (2\alpha + \bar{m})}}{\alpha (2 + N)}.$$

Define $\bar{c} = \max\{c_1, c_2\}$, then the owner prefers the interior solution of segment 1 for all $c \geq \bar{c}$. A comparison of $c_1$ and $c_2$ reveals that for $N = 1, c_1 > c_2$, while for $N \geq 2, c_2 > c_1$. It follows that for $N \geq 2$, for intermediate values of $c$, the owner chooses the solution of segment 2. For $N = 1$ the owner may prefer the corner solution of segment 1 over the solution of segment 2. Therefore, define the threshold $c_3$ as the value of $c$ where the owner is indifferent between the two. Thus $c_3$ is defined implicitly by the equation

$$U_O(p, N(\mu - p)/\alpha, 0) = U_O(\bar{p}, 1 - c_3/\bar{m}, \bar{m}).$$

Note that $c_3$ cannot lie below $c_2$, because the interior solution of segment 1 can never be worse than the corner solution. Finally define $\zeta = \min\{c_3, \bar{c}\}$. The interval $[\zeta, \bar{c}]$ has positive length if and only if $N = 1$, and in the interval the owner prefers the corner solution of segment 1 to segment 2. For $\bar{m} (1 - N/(2 + N)) \leq c \leq \zeta$ he prefers the solution of segment 2. This completes the different types of subgame-perfect Nash equilibrium. Finally note that, in equilibrium, the investors’ demand at $p$ is $(\mu - p)/\alpha$, that is at the upper end of the discontinuity in Figure 4. This is necessary otherwise for some parameter values the utility function of the owner does not have a maximum. □
Proof of Corollary 2. Part 4 of Proposition 3 presents the equilibrium where the owner strictly prefers to monitor, given his remaining shares \( 1 - x \), which happens if \( 0 < c \leq \bar{m} \left(1 - N/(2 + N)\right)\). This region contracts as \( N \) increases, and collapses as \( N \to \infty \). The region where the owner monitors if \( N \geq 2 \) is where \( 0 < c \leq \bar{c} \). This region contracts because \( \partial \bar{c}/\partial N = \partial c_2/\partial N < 0 \). To see that the parameter space in part 3 of Proposition 3 does not collapse, note that the owner prefers the solution of segment 2 of the demand correspondence to the interior solution of segment 1 if and only if

\[
\mu - \frac{\alpha}{2 + N} \leq \mu + \bar{m} - c - \frac{\alpha}{N} + 2\frac{\alpha c}{N\bar{m}} - c^2 \left(\frac{\alpha}{N\bar{m}^2} + \frac{\alpha}{2\bar{m}^2}\right).
\]

If \( N \) grows without bound this inequality simplifies to

\[
\bar{m} - c - \frac{c^2 \alpha}{2\bar{m}^2} \leq 0,
\]

which yields that the solution of segment 2 is preferred if and only if

\[
c \leq -\bar{m}^2 + \bar{m}\sqrt{2\alpha\bar{m} + \bar{m}^2}\alpha.
\]

The statement follows from the fact that this upper threshold on \( c \) is strictly positive.

Proof of Corollary 3. The statement follows directly from Proposition 3.

Proof of Corollary 4. For small values of \( c \), part 4 of Proposition 3, the quantity \( x \) is constant. In part 3, however, \( x = 1 - c/\bar{m} \) is decreasing in \( c \). At the point where the owner switches from part 3 to part 2, there is a discontinuity in \( x \), which follows from the discontinuity in the demand correspondence in Figure 4. After that \( x \) is again a decreasing function of \( c \), because \( x = N(\mu - p)/\alpha \) and \( p \) is increasing in \( c \). For large values of \( c \), the quantity \( x \) is constant again.

Proof of Corollary 5. For \( N = 1 \) the threshold \( \bar{c} \) determines the size of the parameter region where the owner monitors, with

\[
\bar{c} = \frac{2(3\alpha\bar{m} - 2\bar{m}^2 + \bar{m}\sqrt{6\alpha\bar{m} + 4\bar{m}^2})}{9\alpha}.
\]

The claim follows from the fact that \( \partial \bar{c}/\partial \alpha < 0 \). For \( N \geq 2 \), the threshold \( \bar{c} \) determines the region where the owner monitors. Again, the claim follows from the fact that \( \partial \bar{c}/\partial \alpha < 0 \).

Proof of Corollary 6. The statement follows from Proposition 3 with \( N = 1 \) and from Figures 7 and 8.