A FINANCIAL MODEL OF FOREIGN EXCHANGE EXPOSURE

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Abstract. The paper describes a model of foreign exchange exposure. This is defined as the sensitivity of a specific investment's value in reference currency to changes in exchange rate forecasts. This sensitivity may result because some share of the investment cash flows are denominated in foreign currency. Alternatively, a share of cash flows denominated in reference currency which are affected by future exchange rates can also generate sensitivity.

The model integrates a general corporate valuation framework with a theory of expectations and a general model of corporate macro-economic relationships. Its contribution is in generalizing previous models of these relationships. It also provides a link between empirical work, theoretical descriptions of the exchange rate/relative price relationship, and corporate valuation theory.

The model implies a fairly rich description of the corporate and economic characteristics which determine exposure. These descriptors may be used to explain differences in the responses to exchange rate changes of different companies, product lines, or industries.

Foreign exchange risk is one of the unique complications of financial management in an international environment. The values, in a firm's reference currency, of many of its cash flows depend on foreign exchange rates expected to be in effect at settlement. Therefore, expectations about future exchange rates are important determinants of the expected future values and thus the current values of such cash flows: further, variability in exchange rate forecasts may be a major source of variability in the current value. This variability is foreign exchange risk.

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The foreign exchange risk of any particular investment or project depends on both the variability of the exchange rate forecast and the specific sensitivity of the project's or investment's current value to changes in forecasts. In turn, the investment-specific sensitivity is determined by such factors as the physical location of the investment, the structure of its revenues and costs, the industrial structure of the markets in which it operates, and management's response to changing forecasts. This investment-specific sensitivity to changes in exchange rate forecasts is foreign exchange exposure.

This paper describes a model of foreign exchange exposure, that is, the relationship between changes in foreign exchange rate forecasts and changes in the value, in reference currency, of cash flows whose values are sensitive to currency movements. While the relevant flows obviously include those denominated in foreign currency, they also include future operating cash flows which are determined, among other things, by future exchange rates. The contribution of the model is in its application to the latter. Since the currency of settlement is irrelevant, future revenues and costs may be nominally denominated in either the reference or foreign currencies: the model easily recognizes both. To simplify exposition, the model here is applied only to changes in the values of investments which are claims to be settled in foreign currency. However, the model may be used to describe the sensitivity to changes in exchange rate forecasts of investments in riskless assets which are denominated in foreign currency, or simply to claims on reference currency flows whose amounts are influenced by currency movements.

Rather than being an ambitious attempt to describe the value of such investments, the model serves to describe the effects on a given value of changes in exchange rate forecasts. Nor does the paper attempt an extended discussion of the effects of foreign exchange risk on value. Such a digression would require at least an implicit model of valuation which might specify the effects of exchange risk on the discount rate or, through agency costs, on the expected values of the cash flows. Instead, the contribution here is a model of the sensitivity of value to changes in exchange rate forecasts, with an attendant description of specific determinants of sensitivity.

The implied description of the corporate and economic characteristics that determine exchange rate sensitivity is rather detailed. These characteristics may, in turn, explain differences in the responses of competing companies, of product lines, or of industries to changes in exchange rates.

The settlement of the claims for the investment under study will be in the currency of a single, small country. The value of these cash flows is measured in nominal, reference currency units and is assumed to be determined in perfect capital markets. Additional description of the capital market in which the investment is priced would be both extraneous to the model of investment sensitivity and beyond the scope of the paper. Rather, the values of the claims are taken in the context of a given but unspecified
capital asset pricing model. With value additivity, the aggregate investment value is segregated into components of value within that same capital asset pricing framework.

These components are the base case value of the claims if unlevered and unhedged, and the incremental values of outstanding foreign financing and forward foreign currency commitments. The model describes each component and shows that values depend on the exchange rate which initializes the stochastic generation of future rates. Comparative statics demonstrate foreign exchange exposure as the sensitivity of the investment’s value to changes in the initializing rate. The exposure or sensitivity of each component is described and assessed; aggregate exposure is the sum of the component exposures.

The research is by nature integrative. The model generalizes corporate valuation theory in order to incorporate claims whose reference-currency value is affected by changes in exchange rates; the relevance of expectations is maintained. Classical macroeconomic relationships between exchange rates and prices are imbedded in the model’s structure although it allows deviation from strict classical macroeconomics. As a result, one of the most important characteristics of the model is its consistency with a generally accepted macroeconomic structure.

The model is also consistent with and may serve as a framework for more specific models of foreign exchange exposure. These include Heckerman’s [9] description of the relationship between the level of the real profit stream and the real terms of trade when sales and production quantities are constant; Shapiro’s [17] extension of this paper by considering the effects of changes in the terms of trade on nominal profits when quantities are variable; and Levi’s [13] linkage of sales and financial effects in a multicurrency world based on modeling the effects of changes in the current spot exchange rate on the trading profits and portfolio returns of exporting firms.

Hodder [11] emphasized the domestic aspects of exposure by modeling the effects of changes in the current spot exchange rate on the real rate of return to a 2-country firm and found that the proportion of net worth exposed is the regression coefficient of real returns on exchange rate changes. He showed that this coefficient is determined by the probabilistic relationships among nominal exchange rates and domestic prices, real exchange rates, and the domestic inflation rate, the levels of assets and net worth, and the shares of net physical assets and initial net monetary liabilities denominated in foreign currency. He specified the important relationship between nominal and real exchange rates generally as a random disturbance.

The model described in this paper emphasizes the effects of changes in expected exchange rates on foreign investment values. However, because the underlying specification recognizes the existence of competitors who may be influenced by exchange rates, the model allows that purely domestic investments may be also affected by changes in exchange rate
forecasts. A more comprehensive specification of production and demand components enriches the more general approach of the earlier modeling exercises by identifying factors which determine company, product line, or industry-specific responses to exchange rate changes. Finally, the extension from the single period treatment of the earlier models to a multiperiod framework allows for the specification of the effects of changes in expectations or forecasts rather than simply the effects of changes in the current spot rate. Although not pursued here, the extension provides a framework for analyses of effects of changes in the term structure of expectations and the effects of lags imbedded in the price-exchange rate relationship over the term structure.

VALUATION FRAMEWORK

To model exposure—that is, the sensitivity of an asset’s value in reference currency to changes in exchange rate forecasts—we require an expression for the value of that asset. Here we consider the value of a single foreign currency investment, a claim to cash flows to be settled in the future in that foreign currency. The foreign country is “small” in the sense that the currency exchange rate and foreign economic conditions affect neither the economy nor required rates of returns in the reference country.

By assuming value additivity, we can disaggregate the value of the claim into components. The central component is the base case value, the value of the claims on future cash flows as if financed solely by equity and unhedged. This value is adjusted to reflect the incremental values of outstanding debt and forward foreign currency commitments.

\[
VE = VCF_u + (1 - \tau)VD + VF
\]

where

- \(VE\) = the present value, in the reference currency, of the flows to equity holders
- \(VCF_u\) = the base case value; the present value, in reference currency, of the firm’s future after-tax operating cash flows gross of payments to suppliers of capital or to forward contract commitments
- \(VD\) = the present value, in reference country currency, of the flows to existing bond holders
- \(\tau\) = the net effect of the corporate and personal tax structure on the gains to leverage

This paper is organized around these components of value. It specifies a model of each and reaggregates the components to describe the value of the asset. Without more specific assumptions about the underlying capital asset pricing mechanism, the valuation equations represent only incomplete descriptions of value. However, the absence of detailed specification does not preclude an analysis of the implied exposure, to be measured as the derivative of value with respect to changes in the vector of exchange rate forecasts. This exercise not only offers a model of exposure and its determinants, but also demonstrates the exposure-reduction potential of foreign currency debt and forward contracts.
THE BASE CASE VALUE

The assumption that the claims will be settled in a single foreign currency implies that investor expectations of the equivalents of these flows in reference currency depend on assessments both of the amount of the foreign currency flow and of the rate of exchange between the foreign and reference currencies at the settlement date. These expected values of the reference currency equivalents are discounted at a rate which reflects their systematic risk to obtain the reference currency value of the non-financial cash flows.\(^6\)

Investors discount the expected values of the base currency equivalence from the next period, \(t = 1\), to the liquidation period, \(T\). The implied rate of discount for the expected values of the base case cash flows reflects the unlevered and unhedged risk of a stream of reference currency flows. By prior assumption, this rate reflects the systematic components of both business and foreign exchange risk—that variability in the net present value of the cash flows resulting from changes in the expected values of exchange rates.

\[
VCF_u = \sum_{t=1}^{T} \frac{E(\xi_t \cdot c_t)}{(1 + r_0)^t}
\]

where:
- \(VCF_u\) = the current base-currency value of the after-tax, non-financial cash flows
- \(\xi_t\) = the variable spot base-currency price of the foreign currency at time \(t\)
- \(c_t\) = the variable after-tax foreign currency cash flows at time \(t\)
- \(r_0\) = the unlevered and unhedged base-currency cost of equity

The settlement values of the foreign currency cash flows, \(C_t\), may be described by a partial equilibrium model which assumes value-maximizing managerial behavior and an industry setting of competitive supply and demand for the firm's production.\(^7\) With fixed production function parameters and constant returns to scale, all variables within the model are shown to be functions of the foreign currency prices of traded goods and the exchange rate. Assuming international price parity for traded goods implies that foreign currency cash flows can be fully described by exchange rates and traded goods prices in the reference currency. That is, management can describe the firm's current and expected future output prices, input prices, profits, and cash flows conditional on a set of foreign exchange rate expectations.

The relationship between foreign cash flows and prevailing exchange rates derives from specific assumptions of production and demand function structures and input price elasticity. In particular, a Cobb-Douglas production function describes industry production. The model assumes that one factor of production is in perfectly elastic supply and is freely traded in
world markets. A second factor of production is inelastically supplied and is only traded locally. The output price, aggregate income, own price and income elasticity, and cross price elasticities for substitute and complementary products determine demand.

Other industries may use both internationally mobile and immobile inputs. The use of internationally mobile inputs is a common link among firms in the national economy when the relative prices of internationally mobile factors are assumed to be fixed. Under this assumption, the price of each traded factor is a linear function of the foreign currency index of traded goods prices.

It also holds that the foreign currency price of the internationally mobile factor of production is a single, exogenous determinant of the cash flows. The sensitivity of the firm's cash flows to changes in this price is strongly influenced by the share of that good in input costs as well as the degree of inelasticity with which the immobile factor of production is supplied. Finally, the price behavior of this factor links foreign exchange rate expectations and foreign cash flow expectations, because the factor mobility assumption implies effective international price arbitrage.

To begin development of the model we rely both on the assumption of market perfection, which implies dividend policy irrelevance, and the Cobb-Douglas production assumption, which implies constant factor cost shares. Together these imply that returns to capital are a constant proportion, $\alpha_k$, of total revenue.

$$\bar{C}_t = \alpha_k \bar{R}_t$$

where $\alpha_k$ = the share of total costs which is returned to capital

$$\bar{R}_t = \text{the variable total revenue in period } t$$

With production, factor supply, and demand assumptions outlined above, total revenue is a function of the index of traded goods prices in the foreign country, $P$.

$$\bar{R}_t = \beta \bar{P}_t^\eta$$

where $\beta$ = a constant parameter

$\eta$ = an aggregate elasticity of revenue with respect to changes in the foreign currency traded goods price index

$\bar{P}_t$ = the index of traded good prices at time $t$ stated in the foreign currency

The parameter, $\beta$, captures those determinants of total revenue which are insensitive to changes in traded goods prices. These include the levels of firm and industry technology, the share of the traded good input in the traded good price index, and the non-price determinants of demand for industry output. The demand determinants include aggregate income and the demand for substitutable and complementary products.

The revenue elasticity, $\eta$, relates proportionate changes in revenue to the index of foreign traded goods prices. This sensitivity is a function of the responsiveness of both quantity supplied and quantity demanded. The
output response reflects the underlying factor proportions and factor supply elasticities. The demand response reflects the effects of price changes on the joint price responses of substitutes and complements. The aggregate revenue elasticity is a function of these subsidiary elasticities.\textsuperscript{10}

Under these assumptions, the index of traded goods prices is the only variable element of firm cash flow.

\[
\zeta_t = \alpha_k \beta \overline{P}_t^n \eta
\]  

(5)

Constant parameters, \(\alpha_k\) and \(\beta\), capture the remaining determinants of cash flow through production and demand specifications which assume constant proportional relationships.\textsuperscript{11}

The macroeconomic relationship among exchange rates, the foreign traded goods price index, and the reference currency index of traded goods prices constitutes the remaining link to reference currency equivalents. This specification requires the prior assumption that the relative prices of world traded goods are constant and that internationally mobile factors of production are perfectly elastic. Together these imply perfect price parity for traded goods as well as for indices of traded goods prices.\textsuperscript{12}

\[
\overline{S}_t = \overline{P}_t^r
\]

(6)

where \(\overline{P}_t^r\) = the index of traded goods prices in the reference currency country.

The price parity relationship is an important transmission and translation mechanism. With the additional assumption that the base country price level is fixed, exchange rates completely determine variable foreign price levels. The restatement is substituted into equation 5 for a description of the foreign currency cash flow where the foreign exchange rate is the single variable determinant.

\[
\zeta_t = \alpha_k \beta \overline{P}_t^n \overline{S}_t^{-\eta}
\]

(7)

Further substitution of equation 7 into equation 2 yields an expression for the base currency value of the foreign currency cash flow.

\[
VCF_u = \sum_{t=1}^{T} \frac{E(\alpha_k \beta \overline{P}_t^n \overline{S}_t^{1-\eta})}{(1 + r_0)^t}
\]

(8)

Since all parameters are constant, equation 8 can be rewritten to emphasize the role of exchange rate expectations in the valuation process.

\[
VCF_u = \sum_{t=1}^{T} \frac{\alpha_k \cdot \beta \cdot \overline{P}_t^n \overline{S}_t^{1-\eta}}{(1 + r_0)^t}
\]

(9)

The important result is that the reference currency value of the base case cash flows depends on the expectations of future spot exchange rates. Expectations on this single variable vector suffice to determine value because they directly imply expectations for both the price levels of traded goods.
in the foreign country and cash flows. In turn, the expectations are a function of the stochastic process which generates exchange rate observations. In modeling this process, assume that exchange rates evolve as a random walk with drift $\mu$ and that differences in the natural logarithms of successive exchange rates are normally distributed.\textsuperscript{13}

$$\ln \tilde{\mathcal{S}}_{t+1} - \ln \tilde{\mathcal{S}}_t = \mu + \tilde{\varepsilon}_t$$

where $\tilde{\varepsilon}_t \sim N(0, \sigma^2)$

This assumption implies that exchange rate appreciations and depreciations will proceed at a constant rate. In other words, the ratio of successive exchange rates is lognormally distributed.\textsuperscript{14}

$$\tilde{\mathcal{S}}_{t+1}$$

$$\tilde{\mathcal{S}}_t = \ln \frac{\tilde{\mathcal{S}}_{t+1}}{\tilde{\mathcal{S}}_t}$$

$$\tilde{\mathcal{S}} \sim N(\mu, \sigma^2); \frac{\tilde{\mathcal{S}}_{t+1}}{\tilde{\mathcal{S}}_t} \sim \Lambda(\mu, \sigma^2)$$

By this assumption 3 factors determine exchange rates. The first is an initial rate, $S_0$. The second is a drift term, $\mu$, which reflects the anticipated constant differential between inflation in traded goods prices in the 2 countries. The last factor is the intervening random term and the intervening random change.

$$\tilde{\mathcal{S}}_t = S_0 \cdot \exp(\tilde{\mathcal{S}} \cdot t)$$ \hspace{1cm} (12)

where $S_0 = $ the initial exchange rate

The expectation of the variable exchange rate can now be rewritten as a function of the initial exchange rate, $S_0$, time, and the $t(1 + \eta)$ -th moment of the lognormal random variable.\textsuperscript{15}

$$E[\tilde{\mathcal{S}}_t(1 - \eta)] = S_0(1 - \eta) E[\exp \tilde{\mathcal{S}} \cdot (1 - \eta) \cdot t]$$ \hspace{1cm} (13)

With the substitution of equation 13 into equation 9 value is stated as a function of constant parameters, the price index of traded goods in the reference currency country, expected exchange rate "returns" and the initial exchange rate, $S_0$.\textsuperscript{16}

$$VCF_u = \sum_{t=1}^{T} \frac{\alpha_k \beta P_{1T}^{\eta} E[\exp \tilde{\mathcal{S}} \cdot (1 - \eta) \cdot t] S_0(1 - \eta)}{(1 + r_0)^t}$$ \hspace{1cm} (14)

In summary, fixed relationships among aggregate traded goods price levels, exchange rate expectations and cash flows determine base case value in reference currency terms. Exchange rates are stochastic, and independent drawings from a distribution with fixed parameters generate changes. Thus, exchange rate expectations and the base case value depend on the initializing or conditioning exchange rate.
THE FOREIGN EXCHANGE EXPOSURE OF THE BASE CASE VALUE

Foreign exchange exposure is the variation in reference currency value which results from exchange rate changes. When the base country price index and the parameters of the stochastic exchange rate process are fixed, the only source of change is the initializing or conditioning exchange rate.\(^1\)

This change is the adjustment of the expected levels of exchange rates, an adjustment which affects all future periods. In effect, the straight but potentially sloping yield curve is unilaterally raised or lowered by changes in the conditioning rate.

The effect of changes in the initial exchange rate is the derivative of the valuation equation with respect to changes in the conditioning rate.

\[
\frac{dVCF_u}{dS_0} = (1-\eta) \sum_{t=1}^{T} \frac{\alpha_k \beta^t \mathbb{E}[\exp^{-z(t)} \cdot t \cdot (1-\eta)] S_0^{(1-\eta)^{-1}}}{(1+r_0)^t}
\]  

(15)

With simple manipulation, exposure may be stated as either a coefficient or an elasticity.

\[
\frac{dVCF_u}{dS_0} = (1-\eta) VCF_u
\]  

(16)

\[
\frac{dVCF_u}{VCF_u} \bigg/ \frac{dS_0}{S_0} = (1-\eta)
\]  

(16')

The aggregate elasticity, \((-\eta)\), is the difference between unity and the elasticity of corporate revenue in foreign currency with respect to the prices of foreign traded goods. In turn, the revenue elasticity, \(\eta\), is a function of production functions, input elasticities and demand elasticities.

This formulation has the following intuitive content: the base currency values of foreign cash flows are exposed to the extent that changes in foreign cash flows fail to offset exactly changes in exchange rate expectations. For example, dollar values are exposed if foreign revenues cannot be raised by the full extent of any foreign inflation/currency devaluation. Values are also exposed if such foreign revenue response should exceed devaluation. However, in this case exposure is positive in that dollar values increase with foreign currency devaluation. Conversely, foreign revenues in a revaluing currency are exposed if the impact of foreign recession exceeds the coincident appreciation of the foreign currency.

Exposure as defined above is a sensitivity which is influenced by many relationships. These relationships can be captured as a single coefficient when the several important and restrictive assumptions of the model are valid. One of these assumptions is that the response of revenue and cash flow to changes in foreign price conditions is independent of the time horizon. Another is that shifts in exchange rate expectations are equivalent
across the time horizon. With these assumptions the determinants of sensitivity are identical for all future time periods and exposure may be characterized by a simple coefficient.

THE ADJUSTMENT FOR THE VALUE OF OUTSTANDING DEBT

Outstanding debt is a senior commitment which reduces the cash flow accruing to equity at any given asset level. This income drain may be mitigated by gains to leverage if the tax structure favors debt financing.18 The net incremental effect on the value of equity in each period is the value of debt service less the net value of tax shields. In determining the value of equity, the adjustment for the value of debt is deducted from the base case value. All flows are evaluated in reference currency terms.

\[
AVD = \sum_{t=1}^{T} \frac{[(1 - \tau)i_t + D_t]}{(1 + r_d)^t} E[\tilde{S}_t] \tag{17}
\]

where \( AVD \) = the adjustment for the value of debt
\( i_t \) = the tax deductible interest payment at time \( t \)
\( D_t \) = the principal repayment at time \( t \)
\( r_d \) = the pre-tax base currency return commensurate with the risk of the flows to bondholders

Equation (17) can be rewritten to recognize the earlier assumption that exchange rates are a stochastic process.

\[
AVD = \sum_{t=1}^{T} \frac{[(1 - \tau)i_t + D_t] S_0 E[\exp(\tilde{S} \cdot t)]}{(1 + r_d)^t} \tag{18}
\]

The effect of outstanding debt on foreign exchange exposure is simply the derivative of the debt adjustment with respect to changes in \( S_0 \).

\[
\frac{dAVD}{dS_0} = \sum_{t=1}^{T} \frac{[(1 - \tau)i_t + D_t] E[\exp(\tilde{S} \cdot t)]}{(1 + r_d)^t} \tag{19}
\]

The elasticity form emphasizes the nature of foreign currency debt as a completely exposed liability.

\[
\frac{dAVD/AVD}{dS_0/S_0} = 1 \tag{20}
\]

Alternatively, debt can be viewed as a special case of the more general valuation equation 14 and exposure equations 15 through 16' where the cash flow elasticity, \( \eta \), is zero.19
THE ADJUSTMENT FOR THE VALUE OF OUTSTANDING
FORWARD FOREIGN EXCHANGE CONTRACTS

Forward foreign exchange contracts serve as perfect hedges of value when contract amounts equal the level of 'real' exposure and the assumed constancy of the parameters is maintained. Though this result has been recognized elsewhere, the development specifies the determinants of the incremental values of such contracts.

Forward contracts are agreements to deliver a specified amount of foreign currency at a future time in exchange for a fixed amount of base country currency. Outstanding forward foreign exchange commitments reduce or increase the value of the firm as the values of the contracts are less than or greater than zero. At the initiation of forward contracts the value of the firm must be unaffected by fairly priced forward commitments. With the passage of time, however, existing contracts may gain or lose value with changes in the exchange rate expected to prevail at the corresponding future settlement date.

The contractual forward rate is determined at the initiation of the contract and is assumed to be determined in a frictionless market. When appropriately priced, the forward rate at time \( k \) for future period \( t \) is related to the expectation of the corresponding future spot rate by a risk premium. This premium reflects the risk of forward contracting as assessed in the base country capital markets:

\[
F_{k,t} = (1 + r_f)^{1-k} E_k(\tilde{S}_t) \tag{21}
\]

where

\[
F_{k,t} = \text{the forward rate at time } k \text{ for foreign exchange to be delivered at time } t.
\]

\[
E_k(\tilde{S}_t) = \text{the expectation at time } k \text{ of the spot rate at time } t.
\]

\[
r_f = \text{the per period return assessed by base country capital markets for risk incurred in forward contracting.}
\]

The potential for arbitrage among securities in the base country's capital market insures that the return to forward contracting is related to other reference country capital market returns. In particular, the value of any hedged cash flow must equal the combined values of an identical unhedged cash flow and the hedge itself. At initiation of the contract, the hedge has no value; the value of the hedged and unhedged flows must be identical. The relationship between a discount rate for hedged cash flows, the forward risk premium, and the rate for unhedged flows derives from this equality:

\[
(1 + r_h) = (1 + r_u)(1 + r_f) \tag{22}
\]

where

\[
r_u = \text{the simple, per period rate of return assessed by base country capital markets commensurate with the risk of foreign currency cash flows to be received with certainty but unhedged.}
\[ r_h = \text{the simple, per period rate of return assessed by base country capital markets commensurate with the risk of foreign currency cash flows to be received with certainty but hedged.} \]

With the passage of time forward contracts attain value since the claims on forward exchange are likely to be settled at rates different from the contractual rates. Thus, existing forward contracts enhance or detract from total corporate value when gains or losses are expected at the settlement date.

Thus, the values of outstanding claims on forward exchange reflect the required rates of return as well as the expected future values of the claims. Specifically, the claims are agreements to deliver \( G_t \) units of foreign currency at a future time \( t \). The seller receives base-country currency at the contractual exchange rate \( F_t \) in exchange. The value of outstanding forward contracts reflects the difference between the previously contracted rate of forward exchange and the rate at which the contract could be reversed. The latter is simply \( \tilde{F}_t \), the current forward rate for delivery at time \( t \).

\[
VF = \sum_{t=1}^{T} \frac{G_t (F_t - \tilde{F}_t)}{(1 + r_h)^t} \quad (23)
\]

where \( VF \) = the value, in reference currency, of outstanding forward foreign exchange contracts

\( F_t \) = the contractual forward rate on an existing commitment for settlement at time \( t \)

\( \tilde{F}_t \) = the current market rate for forward exchange for delivery at time \( t \).

\( G_t \) = the contract amount, in foreign currency units; assumed to be made with certainty

Like other elements of value, the forward contract is exposed. The degree of exposure is the derivative of the forward contract value with respect to changes in the initializing exchange rate.

Following earlier derivation, which recognizes the stochastic process determining spot and, by implication, forward exchange rates, the value of outstanding forward contracts may also be expressed as a function of the initializing exchange rate.

\[
VH = \sum_{t=1}^{T} \frac{G_t \left( F_{k, t} - S_0 \exp(\rho_t \cdot t) \mathbb{E}[\exp(\Sigma \cdot t)] \right)}{(1 + r_h)^t} \quad (24)
\]

The derivative of this value with respect to that exchange rate emphasizes the role of exchange rate expectations in determining both the current forward rate and the values of existing forward contracts.
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\[
\frac{dVF}{dS_0} = - \sum_{t=1}^{T} \frac{G_t \bar{F}_t / S_0}{(1 + r_h)^t} \quad (25)
\]

The formulation also can demonstrate the use of forward contracting as a hedging instrument. For example, consider the simple case of an unlevered cash flow matched by a forward contract commitment. Set the forward contract at \( G^* \) to match the level of foreign currency cash flow which is exposed in each period. (This particular example is distinguished by variables and parameters with asterisks (*).)

\[
G_t^* = (1 - \eta) E(\bar{C}_t^*) \quad (26)
\]

Applying equation 15, the exposure of the base case value is a function of the 'real' exposure.

\[
\frac{dVCF^*_u}{dS_0} = (1 - \eta) \sum_{t=1}^{T} \frac{E(\bar{C}_t^*)}{(1 + r_u^*)^t} \quad (27)
\]

The exposure of the forward contract follows from equation 25.

\[
\frac{dVF^*}{dS_0} = - \sum_{t=1}^{T} \frac{G_t^* \bar{F}_t / S_0}{(1 + r_h^*)^t} = - (1 - \eta) \sum_{t=1}^{T} \frac{E(\bar{C}_t^*)}{(1 + r_u^*)^t} \quad (28)
\]

In this case the changes in the base case value are exactly offset by changes in the value of the forward contract.

\[
\frac{dVE^*}{dS_0} = \frac{dVCF^*_u}{dS_0} + \frac{dVF^*}{dS_0} = 0 \quad (29)
\]

FOREIGN EXCHANGE EXPOSURE

The equity ownership value is the sum of the individual elements of value. The primary element is the base case value, the value of the flows as if unlevered and unhedged. This value is adjusted to reflect the incremental values of outstanding foreign financing and forward foreign currency commitments.

\[
VE = VFC_u - AVD + VF \quad (30)
\]

Each component described above assumes perfect markets for goods and capital and value maximizing managerial behavior. Aggregation from equations 30, 14, 18 and 24 yields a comprehensive description of equity value.

\[
VE = \sum_{t=1}^{T} \frac{\alpha_k \beta P_t \eta E[\exp(1 - \eta) \bar{F} - t] S_0 (1 - \eta)}{(1 + r_0)^t} \quad (31)
\]

\[
- \sum_{t=1}^{T} \frac{[(1 - \tau) i_t + D_t] E[\exp(F - t)] S_0}{(1 + r_d)^t}
\]
Foreign exchange exposure is the derivative of equity value with respect to changes in exchange rate expectations. These changes are generated by changes in the initializing exchange rate, \( S_0 \), when the parameters of the stochastic exchange rate process are assumed fixed.

Since the initializing rate is the only variable in each element of value, the derivation is straightforward. The derivation also follows from equations 30, 15, 19 and 25.

\[
\frac{dVE}{dS_0} = (1 - \eta) \sum_{t=1}^{T} \frac{a_k \beta P_t^{sT} E[\exp((1 - \eta)\bar{s} \cdot t)] S_0 (1 - \eta)^{-1}}{(1 + r_0)^t} - \sum_{t=1}^{T} \frac{[(1 - \tau)i_t + D_t] E[\exp(\bar{s} \cdot t)]}{(1 + r_d)^t} - \sum_{t=1}^{T} \frac{G_t \bar{F}_t S_0^{-1}}{(1 + r_h)^t}
\]  

(32)

With some manipulation, the equation can be rewritten in elasticity terms. The revision highlights the aggregative nature of the model in that elasticity of equity value with respect to exchange rates is a weighted sum of the components' elasticities.

\[
\frac{dVE/VE}{dS_0/S_0} = (1 - \eta) \frac{\text{VCF}_u}{VE} - \frac{\text{AVD}}{VE} - \sum_{t=1}^{T} \frac{G_t \bar{F}_t}{(1 + r_h)^t}
\]

(33)

The weights reflect the shares of each component in the total base currency value of equity. Specifically, these include 1) the share of the base case value in total equity value, 2) the share of the incremental adjustment for the value of debt in total equity value, and 3) the share of total equity value of the cash flows which have been hedged by forward commitment. (Note that the last component is not the value of existing forward contracts, but the value of the foreign currency under contract.)

The individual elasticities, as described in each section above, are respectively \((1 - \eta), -1,\) and \(-1\). In other words, the base case value may be less than fully exposed to the extent of local inflationary offset \((1 > \eta > 0)\). Foreign currency debt and forward contracts are fully exposed.

The formulation is appealing in its segregation of natural or operating exposure as represented by the exposure of the base case value. In essence, the elasticity \((1 - \eta)\) adjusts nominal cash flow to capture real exposure.
This natural exposure may be fully offset by local currency debt and forward contracts. Together, the exposure of these financial and hedging instruments captures the degree of exposure reduction.

CONCLUSION

Foreign exchange exposure is derived from a model of the base currency value of foreign cash flows. With some restrictions on the model's structure and underlying assumptions, the resulting measure of exposure is a simple and potentially estimable coefficient.

There are 3 important characteristics of the model. First, all elements of value are assumed to be functions of a single variable source—the vector of expected exchange rates. Second, the parameters of the stochastic exchange rate process are fixed, so that changes in the initializing exchange rate reflect evenly across all maturities. Finally, the assumption of perfect capital markets simplifies the structure of the model substantially.

The research might be expanded in several directions. The effects of variation in the parameters of the exchange rate generating process might be explored: for example, how do changes in expected average annual currency devaluations alter base currency values? A term structure in forward exchange rates and exchange rate expectations might be introduced. Such a generalization could be useful for introducing lags in the macroeconomic relationships among exchange rate, price and price level expectations. The assumption that the elasticities of managerial response are independent of time frame might be relaxed in consideration of lags in behavior or optimal adjustment paths. Finally, the extension to a multicurrency framework promises to increase the potential managerial relevance of the model.

NOTES

1. The single currency analysis simplifies the exposition and provides the framework for extension to a multi-currency model. Extension to more than a single currency would also require specifying the currency covariance structure. The small country assumption insulates reference-currency interest rates and economic conditions from foreign economic conditions and emphasizes the relationship between investment value and the foreign economic environment.

2. As stated above, claims to be settled in reference currency but which are affected by exchange rates, and claims on multiple currencies are excluded for simplicity. The basic framework is sufficiently general to accommodate either, though the treatment of multicurrency claims would require the specification of currency covariances.


4. In principle, the term describing the value of the forward contract may be generalized to include foreign currency borrowings as replicated by borrowings denominated in reference currency plus forward sales of foreign currency. The particular disaggregation scheme adopted here emphasizes forward contracting and foreign financing activities as separate instruments of managerial choice.

5. Miller (15) has described the \( \tau \) parameter as a function of the corporate tax rate, \( \tau_c \), the personal tax rate on income from stocks, \( \tau_{ps} \), and the personal tax rate income from bonds, \( \tau_{pb} \):

\[
\tau = 1 - \frac{(1 - \tau_c)(1 - \tau_{ps})}{1 - \tau_{pb}}
\]

6. This risk is evaluated relative to the home country capital market. Whether this market is best represented by a portfolio of domestic assets only or a portfolio of world assets is an empirical
question which is rather insignificant when the home country is the United States. The pricing of variability depends, of course, on the structure of global equilibrium.

The underlying assumptions of the model are consistent with the use of risk-adjusted discount rates. We assume investors' constant relative risk aversion here and show later that returns are independently distributed over time.

7. Only the implications of the underlying value maximization model are presented here. Important assumptions are recognized and the determinants of important parameters are described. The detailed model is described in Hekman [10].

8. For simplicity, the inputs are assumed to be either perfectly mobile or totally immobile. However, nothing in the model's logic or design precludes expansion to recognize intermediate mobility.

9. This is implied by the "small country" assumption since the prices of globally mobile factors are determined exogenously and are unaffected by events in the local economy. With this assumption the theoretical framework is consistent with Dornbusch's macroeconomic model [5] where: "The exchange rate now plays the key role of determining relative wages, relative prices, and therefore the allocation of resources."

10. In the following development $\beta$ and $\eta$ are constant parameters. They are independent of both the magnitude of change in $P_t$ and of time. Revenue responses to changed prices are complete in each period. These assumptions are not necessary for the model's development, but simplify both the presentation and the result. In fact, time-dependent parameters are a complication which would facilitate the analysis of lagged responses to changing economic conditions.

11. Constancy of these parameters is assumed for simplicity and in consistency with the article's focus on the effects of exchange rate changes. Since the model's construction insures that changes in the parameters are independent from changes in $P_t$, parameter variations could be introduced and treated additively.

12. For simplicity we also assume identical price index weights in both countries. In the absence of this assumption relative price levels would be related in constant proportionality to the exchange rates.

13. The assumption of lognormality is useful but not required for further development of this model. However, this specification is consistent with the Shapiro [18] and Adler and Dumas [13] reviews of theoretical and empirical work in this area. In particular, the empirical analyses described in Roll [16] and Adler and Lehmann [4] conclude that the deviations show no significant mean reversion tendencies for the great majority of country pairs and time periods examined.

14. The assumption of lognormality is useful but not required for further development of this model.

15. Moments of the log normal distribution are simple functions of the first and second moments of the normal distribution; see Lindgren [14], page 176. "The $k$th moment . . . is expressible in terms of the moment generating function of $X$:

$$E(X^k) = \exp \left( k\mu + \sigma^2 k^2/2 \right)$$

With this property equation 13 reduces to

$$E(S_t^{1-(1-\eta)}) = S_0^{1-(1-\eta)} \exp \left( (1-\eta)\mu + (1-\eta)^2 \epsilon^2/2 \right)$$

(13a)

16. Note that the assumption of lognormality reduces the expectation of exchange rate "returns" to a function of the fixed parameters of the underlying normal distribution. (See equation [13a].) Subsequent development ignores the passage of time and the generation of exchange rates. It should be clear that the assumed exchange rate process together with the reflection of errors, $\epsilon_t$, in revised cash flow expectations implies independent and identically distributed returns to equity.

17. Of course this change is only one of the several which we could consider. It is interesting because it implies the conditions under which generally accepted definitions of exposure are valid. The effects of changes in other determinants of exchange rate expectations might be explored in future research.

18. See Miller [15] for discussion. See also above. The development here is intentionally unspecific since tax treatment of foreign debt obligations and the valuation of such treatment is complex and perhaps indeterminant.

19. See Adler and Dumas [2], for a more general model of foreign bond valuation which relaxes the assumption that changes in the current exchange rate shift expectations on all future rates by an equivalent amount. In this case the cash flow elasticity of debt may not equal zero.

20. It should be carefully noted that financial futures provide imperfect hedges in the more realistic case of the fixed parameters being allowed to change.
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21. As above, capital market structure within which risk premia are determined is left unspecified. The issue of forward contract risk premia is addressed in Hansen and Hodrick [7], Frankel [6], and Stulz [19]. The approach taken below is to assume constancy of premia.

22. The potential for forward contract default may be incorporated without loss of generality.

REFERENCES


