The Real Effects of Foreign Exchange Rate Changes on a Competitive, Profit-Maximizing Firm

by

Christine R. Hekman
Peter F. Drucker Graduate Management Center
The Claremont Graduate School

July, 1988
ABSTRACT

The Real Effects of Foreign Exchange Rate Changes on a Competitive Profit-Maximizing Firm

The paper describes a model of the output and revenue of a profit-maximizing firm operating in a competitive industry and a small country. This setting implies that both output prices and the prices of traded inputs are determined by world prices and the foreign exchange rate, but are not affected by the firm's output or revenue levels. The constraining, variable input factor is both non-traded and inelastically supplied. This constraint determines the optimal output response to those changes in the price of the traded input which are induced by changes in either world prices or foreign exchange rates. It also determines the responsive behavior of output prices and factor employment to foreign exchange rate variability.

The model has important implications for the continuing controversies over the effects of foreign exchange rate variability on such real phenomena as the volume of world trade and employment. The descriptions of the parameters implied by the model may be useful in the identification of companies, industries, or countries most severely affected by exchange rate variability.
THE REAL EFFECTS OF FOREIGN EXCHANGE RATE CHANGES ON A COMPETITIVE, PROFIT-MAXIMIZING FIRM

INTRODUCTION

Popular families of international macroeconomic and international trade and general equilibrium models are distinguished by an emphasis on the relationship between exchange rates and the relative prices of traded and non-traded goods. These models describe a world in which the sensitivity of aggregate real variables to changes in exchange rates are a direct result of changes in the relative price of tradeables and non-tradeables. In Dornbusch's words, "The exchange rate now plays the key role of determining relative wages, relative prices, and therefore the allocation of resources."¹

This paper describes a model of the microeconomic basis of this exchange rate/relative price relationship. The analysis has both microeconomic and macroeconomic implications. At the microeconomic level, the model explains differences in industry and firm responses to exchange rate changes. Differences in the effects of exchange rate changes
on output, demand for inputs, industry price, and profits are examined for both the short and long run. The effects on industrial employment of factors is also examined.

The emphasis on microeconomic foundations complements the macroeconomic and general equilibrium literature referenced above. The extension is provided by the focus on differential sectoral effects of exchange rate changes. These effects can be reaggregated to explain effects at the macroeconomic level as well as to explain the strengths of the macroeconomic responses as represented by parameters of assumed aggregate relationships. Differential sectoral parameters can also explain differential effects of exchange rate changes on macroeconomic aggregates across countries or time and can illucidate the fundamental determinants of such differences. The analysis also supports the conclusion that real effects of exchange rate changes diminish over time and disappear in the long run.

The paper also extends the framework established in the foreign exchange exposure literature by Levi(1979). Here the contribution is a recognition of the effects of cross demand elasticities for competing products and the provision of a more detailed and realistic description of production.

The model assumes profit maximizing behavior in competitive industry settings. For simplicity, the analysis is limited to two countries; multilateral analysis is accom-
lished by straightforward extension. The framework is classical, but the important assumptions are constrained to meet standards of validity.
THE MODEL – AN OVERVIEW

The model describes equilibrium price, output and revenue levels for a profit-maximizing firm operating in a competitive environment. These are determined by the intersection of industry supply and demand curves. Since output is determined by a production function which is common to all firms in the industry and production functions are identical, each firm produces an equal share of industry output.

The common production function defines a short run supply curve which depends on the output price, the prices of variable inputs, and, in the short run, the fixed quantity of capital available.

\[ Q_s = Q_s(P, P_N, P_T, K) \]

where \( Q_s \) = the quantity of output produced and the functional form which describes quantity

\( P \) = the price per unit of output

\( P_N, P_T \) = the cost per unit of input factors, non-traded and traded, respectively

\( K \) = the quantity of fixed capital used in production.

Demand for the industry's output is a function of industry price and is determined by the industry demand

-4-
curve.

\[ Q_D = Q_D(P) \]

where \( Q_D \) is the quantity of output demanded and the functional form which describes demand.

In the short run, output is constrained by two factors. The first is the fixed quantity of capital available for production, \( K \).

\[ K = \bar{K} \]

The second constraint is the limited availability of one of the variable factors of production. This limitation is reflected in the dependence of the input price on the quantity of the factor, \( N \), used by the industry. Note that factor demand is determined, in turn, by the level of industry production.

\[ p_N = p_N(N) \]

The price of the remaining variable input is the determining factor in the system. This input is traded on world markets and from the industry perspective its supply is unlimited. As a traded commodity its price is set in world
markets and is independent of industry or firm production levels. In local currency terms the price is the world price, \( p_T^* \), after translation at the foreign exchange rate, \( X \).

\[
(5) \quad p_T = p_T^* X
\]

\[
(6) \quad p_T^* = \tilde{p}_T^*
\]

where \( p_T^* \) = the fixed world price of the traded input.

With the quantity of production determined by the price of the traded input and this input price determined by the exchange rate, the only exogenous variable in the system is the foreign exchange rate, \( X \). Firm and industry output, input and output prices, firm revenue, factor employment and factor returns may all be explained as functions of the exchange rate. The short run response of each may also be described as a derivative taken with respect to a change in the exchange rate.

In conclusion, the price of the traded input is determined exogenously as a world price translated by the exchange rate into a local currency price. The short run, profit-maximizing level of production is determined jointly by that traded input price and by the prices of the
non-traded input which correspond to each output level. Relative usage of the factors depends on the form of the production function.

In the short run, changes in the foreign exchange rate alter the system directly by changing the local currency price of the traded input. There is no direct effect on the price of the non-traded input. However, the consequent alteration in the level of production changes the quantity of the non-traded input which is demanded by the industry. The resulting short run equilibrium output is achieved at a new quantity-price combination for the non-traded input. The system's solution describes the effects on the endogenous variables of changes in the exchange rate.

The long run is the limiting case of the short-run equilibrium, because the constraints imposed by a fixed capital stock and an inelastic supply of the non-traded input disappear. In long run equilibrium the quantity supplied, $Q'_s$, depends on the price of output and the prices of inputs.

\[(7) \quad Q'_s = Q'_s (P', P'_N, P'_T, i)\]

where apostrophe's ('') indicate long run values of variables or long run functional forms

\[i = \text{the cost of financial capital.}\]

A long run demand curve describes the relationship between the quantity demanded, $Q'_D$, and the long run price of...
output.

(8) $Q'_D = Q'_D(P)$

Arbitrage opportunities are effective in the long run so that prices of both traded and non-traded inputs are determined in world markets. Finally, in the long run and from the industry's perspective the formerly non-traded input is available in unlimited quantities.

(9) $p'_T = p^*_T \cdot x$

(10) $p'_N = p^*_N \cdot x$

Like the short run system, the only exogenous variable in the long run system is the exchange rate. In contrast to the short run system, changes in the exchange rate have larger effects on prices and smaller effects on output in the long run. Changes in the exchange rate directly alter the local currency prices of both variable inputs, because arbitrage is fully effective in the long run. Further, no supply inelasticities mitigate the effect of the external shock. Finally, the flexibility of the capital stock exacerbates the price response and mitigates the volume response.

The long run equilibrium solution depends on the relationship between interest rates and exchange rates as well as on the long run characteristics of demand. However, it is
clear that the price effects of exchange rate shocks are larger and quantity effects smaller or even non-existent in the long run.
THE MODEL - COMPLETE SPECIFICATION

1. The Supply Function

A Cobb-Douglas function is assumed to describe the firm's production. There are two variable factors and a single fixed factor of production.

\[(11) \quad Q_s = A N^{\alpha_N} T^{\alpha_T} K^{\alpha_K}\]

where \(A\) = a function of technology

\(N\) = the quantity of the non-traded input used in production

\(T\) = the quantity of the traded input used in production

\(K\) = the quantity of fixed capital used in production

\(\alpha_N, \alpha_T, \alpha_K\) = the respective shares of cost of non-traded, traded, and capital inputs in total cost of production;

\(\alpha_N + \alpha_T + \alpha_K = 1\).

Maximum profits are generated by equating the prices of variable inputs with the values of the marginal products. In the short run, the supply of capital is fixed. The implied short-run supply curve emphasizes the effect of this capital constraint.

\[(12) \quad Q_s = \left[ \frac{(-\alpha_N)}{m \quad P_N} \frac{(-\alpha_T)}{P_T} \right]^{(1/\alpha_K)} K \quad \frac{v}{p^v}\]

where \(m = A \quad \alpha_N \quad \alpha_T\)

\(v = \frac{\alpha_N + \alpha_T}{\alpha_K}\).

-10-
Output is therefore determined by the prices of the variable inputs and by their cost shares relative to the cost share attributed to the fixed factor. Fixed technology, fixed production function parameters and the fixed supply of capital also determine the level of output. Finally the price elasticity of supply is the ratio of the variable cost shares to the fixed cost share. This ratio is subsequently termed \( v \). Note that \( v \) is the short-run aggregate elasticity of supply for a constant cost industry and is determined by the relative shares of variable or unconstrained inputs in total cost.

The dual of equation (12) describes the price of output as a function of the prices of the inputs and the quantity of production in the short run.

\[
(12a) \quad P = \left[ m \begin{pmatrix} -\alpha_N & -\alpha_T & (1/\alpha_K) - (-1/v) & (1/v) \\ p_N & p_T & 1 & 0 & 0 \end{pmatrix} \right] Q_S
\]

In the long run, fixed capital is supplied to the industry with complete elasticity. With this flexibility and Cobb-Douglas homogeneity, the supply price of output is
independent of the quantity produced. The price of output is determined by the prices of the inputs and is independent of the level of production.\(^3\)

\[(13) \quad p' = \frac{1}{m^T} p_N \cdot (\alpha'_N p_T^{\alpha'_i} (\alpha'_T) (\alpha'_K) i)
\]

A realistic complication is introduced with the assumption that the locally traded input is supplied to the industry with limited but positive elasticity in the short run.

\[(14) \quad N = B p_N^\psi
\]

where \(B\) = a constant

\[\psi = \text{the elasticity of supply of the non-traded input.}\]

When the value of the marginal product is equated to the cost of this non-traded input, the input price can be described as a function of industry price and output. Note that the exponent, \(\theta\), is positive and inversely related to the underlying elasticity of factor supply, \(\psi\).

\[(15) \quad p_N = F(Q_s \cdot p)^\theta
\]

where \(F = \left[\frac{\alpha_N}{B}\right]\)

\[\theta = \frac{1}{\psi+1}\]
The inelastic factor supply implies increasing costs to the industry and alters both the quantity and price forms of the short run supply equation. The revised forms are derived by substituting equation (15) into equations (12) and (12a).

\[ Q_s = \left[ m/(F' \begin{pmatrix} \alpha_N \\ p \end{pmatrix} \begin{pmatrix} \alpha_T \\ p_T \end{pmatrix}) \right] \frac{1}{(\alpha_K + \theta \alpha_N)} \]

\[ \cdot \left[ \frac{\alpha}/(\alpha_K + \theta \alpha_N) \right] \frac{1}{p} \]

where \( \mu = \frac{\alpha_T + \alpha_N (1-\theta)}{\alpha_K + \theta \alpha_N} \).

\[ P = \left[ m/(F' \begin{pmatrix} \alpha_N \\ p_T \end{pmatrix}) \right] \frac{1}{(\alpha_K + \theta \alpha_N)} (-1/\mu) \]

\[ \cdot \left[ \frac{\alpha_K}/(\alpha_K + \theta \alpha_N) \right] (-1/\mu) (1/\mu) Q_s \]

A comparison of equations (16) and (16a) to the constant cost forms, equations (12) and (12a), demonstrates the effect of increasing costs. Each elasticity in the more general equations reflect the inelasticity of factor supply. The net effect of this inelasticity is captured in terms which jointly reflect the degree of inelasticity, \( \theta \), and the...
relative cost weight of the non-traded factor, $\alpha_N$. In particular, the aggregate elasticity of supply, $u$, is a modification of the constant cost elasticity, $v$. Like the constant-cost case, this elasticity captures the share of the variable inputs in total cost. In the increasing cost case this cost share is modified to reflect the partial inflexibility of the non-traded input which is implied by its inelasticity of supply. The modification is in the allocation of only a portion of the cost share of the non-traded input, $(1-\theta)$, to the numerator of the exponent. The partial allocation reflects the fact that the supply of the input is neither totally fixed nor fully flexible. The more elastic or flexible the supply, the smaller is $\theta$ and the closer to a fully flexible or constant-cost case is the overall supply elasticity.

When the supply cost of the non-traded factor is highly elastic or the factor's cost share in small, the joint effect is miniscule. In this case $\theta$ and $\alpha_N$ approach zero. In the limit equations (16) and (16a) collapse to equations (12) and (12a), the constant cost cases.

Proceeding toward a description of short run equilibrium with constrained factor supply, we rearrange and simplify the relevant supply curve, equation (16). The determinants of supply can be aggregated into one of the following categories: fixed elements, elements related to the price of
the traded input, or elements related to the price of output. With this rearrangement, the quantity of production is described by the level of the supply curve. This level is, in turn, determined by technology, constant parameters and the supply of capital; by the price of output; and by the elasticity of supply with respect to price.

\[
Q_s = S p_T^{-z} \rho^\varepsilon
\]

where \( \varepsilon = \left[ \frac{\alpha_T + \alpha_N (1-\theta)}{\alpha_K + \theta \alpha_N} \right] = \mu \)

\[
S = \begin{bmatrix} (-\alpha_N) \\ (-\alpha_K) \end{bmatrix} \frac{1}{(\alpha_K + \theta \alpha_N)}
\]

\[
z = \frac{\alpha_T}{\alpha_K + \alpha_N \theta}.
\]

Note that \( \varepsilon \), the elasticity of supply, is large if the elasticity of supply of the non-traded factor is large. In this case, increases in output prices draw out larger quantities of the constraining factor. Also note that the output elasticity is lower when the cost share of the non-traded input in production costs is large, because a larger share of total costs accruing to the constraining factor exerts a greater dampening effect on the output response.

2. The Demand Function

The equilibrium price is jointly determined by industry
supply and aggregate demand. With identical output, each firm is a price-taker.

Demand for the industry's production is assumed to be a function of the output price, \( P \), aggregate income, \( Y \), and the price of a substitute product, \( P_c \). This implies a demand function of the following form:

\[
Q_D = P_c \eta_c \gamma \eta \quad (18)
\]

where \( P_c \) = the price per unit for a competing or substitutable product

\( \eta_c \) = the cross price elasticity of demand between own product and product \( c \)

\( \gamma \) = aggregate income

\( \eta \) = the elasticity of own demand with respect to aggregate income

\( \eta \) = the own price elasticity of demand.

Of course, the price of the traded input may be presumed to affect the price of the substitutable product. With a development similar to that followed above, the functional form includes a multiplicative factor \( \sigma \) and a constant elasticity.

\[
P_c = C P_T^\sigma \quad (19)
\]

where \( C \) = an aggregate function of technology, factor shares and the price of a non-traded input which is specific to the production of the substitute good

\( \sigma \) = the elasticity of price with respect to the cost of the traded input for good \( c \).
REFERENCES


A description of industry demand is obtained with the substitution of equation (19) into (18).

\[
Q_D = C_{\gamma} Y p_T \eta_c (-\eta)
\]

(20)

To simplify, the factors which determine demand may be rearranged into several groups.

\[
Q_D = D p_T \eta_c (-\eta)
\]

(21)

where \( D = C_{\gamma} Y \).

The shift term, \( D \), is a function of several fixed parameters including the level of aggregate income. The price of traded goods affects demand through its effect on the price of substitutable products. Finally, the shape of the demand curve is determined by price elasticity of demand, \( \eta \). For simplicity, the short run and long run demand curves are subsequently assumed to be coincident.

3. The Equilibrium Solution

With no change in inventory the short run solution to
this system is the intersection of the supply and demand curves described by equations (17) and (21). The short-run price and production equilibria are functions of the more aggregate short-run shift terms and of the short-run demand and supply elasticities.

\begin{equation}
(22) \quad P = \frac{(D/S)}{(1/(\varepsilon+\eta))} \quad (\sigma_{n_1} + z) \quad \frac{(1/(\varepsilon+\eta))}{(P_T)}
\end{equation}

\begin{equation}
(23) \quad Q = (S^\eta D^\varepsilon) \quad \frac{(1/(\varepsilon+\eta))}{(1/(\varepsilon+\eta))} \quad (\varepsilon \sigma_{n_1} - \eta z) \quad \frac{(1/(\varepsilon+\eta))}{(P_T)}
\end{equation}

In the short run equilibrium, industry revenue is simply the product of the solutions for short-run price and quantity.

\begin{equation}
(24) \quad R = P \cdot Q = \left( S^{\eta-1} D^\varepsilon + 1 \right) \quad \frac{(1/(\varepsilon+\eta))}{(1/(\varepsilon+\eta))} \quad \left( \frac{(1+\varepsilon) \sigma_{n_1} - (\eta-1) z}{(P_T)} \right) \quad \frac{(1/(\varepsilon+\eta))}{(1/(\varepsilon+\eta))}
\end{equation}

It is important to note that the price of the traded input determines the equilibrium values of all other variables as the single exogenous variable in the system. Thus, each variable price and quantity can be described as a
function of the price of the traded input. In fact, the exponent on the price of the traded input in each equation is effectively the elasticity of the left-sided variable with respect to changes in the traded input price. Anticipating the model's use in explanation of empirical results, note that the exponents are also multiple regression coefficients.

The long run equilibrium solution occurs at the intersection of the long run supply curve described by equation (13) and the long run form of equation (21). Together, they imply equilibrium industry output and revenue.

\[
Q' = D \left[ m'/(p_N^{\alpha_N'}) \right]^{\eta} \eta \left( \sigma \eta C - \eta \alpha_T' \right)
\]

\[
R' = D \left[ m'/(p_N^{\alpha_N'}) \right]^{\eta-1} \left( \sigma \eta C - \eta-1 \alpha_T' \right)
\]

4. Equilibrium and World Prices

The system is linked to the international economy when the price of the variable input is assumed to be determined in world market. A change in the price of this input is the
channel by which shocks in the international economy are transmitted to the local economy and to the firm.

The fundamental assumption is that price arbitrage is perfect for traded goods and imperfect for non-traded goods. As a result, the local price of traded inputs is simply the world price translated at the exchange rate.

\[(27) \quad p_T = p_T^* X\]

where \(p_T^*\) = the world price, in world currency terms, of the traded input

\(X\) = the local currency (LC) cost of the world currency (WC); \((LC/WC)\),

A second assumption, which is logically consistent with the earlier development of the model, is that the inelasticity supplied factor is immobile between countries in the short run. Thus, its price is determined solely in local markets as specified in equations (14) and (15). However, in the long run international arbitrage forces are effective and the prices of non-traded inputs conform to the Law of One Price.

\[(28) \quad p_N = p_N^{*'} X\]

where \(p_N^{*'}\) = the long-run world price, in world currency terms, of the non-traded input

These relationships complete the specification of the
model. When world prices are fixed, the system's single exogenous variable is the exchange rate. Equations (22) through (24), the equations for equilibrium output prices, production and revenue in the short run, can be rewritten as functions of the exchange rate.

\[
(29) \quad P = \left( \frac{D}{S} \right) \left[ \frac{1}{(\varepsilon+n)} \right] \left( \begin{array}{c}
\frac{\sigma}{c+z} \\
p_T^* \\
X
\end{array} \right) \left[ \frac{1}{(\varepsilon+n)} \right]
\]

\[
(30) \quad Q = \left( \frac{S^n_D}{D} \right) \left[ \frac{1}{(\varepsilon+n)} \right] \left( \begin{array}{c}
\varepsilon \sigma - \eta z \\
p_T^* \\
X
\end{array} \right) \left[ \frac{1}{(\varepsilon+n)} \right]
\]

\[
(31) \quad R = \left( S^n D^{1+\varepsilon} \right) \left[ \frac{1}{(\varepsilon+n)} \right] \\
\cdot \left( \begin{array}{c}
(1+\varepsilon) \sigma - (1-\eta) z \\
p_T^* \\
X
\end{array} \right) \left[ \frac{1}{(\varepsilon+n)} \right]
\]

The equations which describe long run equilibrium are also rewritten. Equations (13), (25), and (26) are revised to describe long run equilibrium values of price, output, and
revenue as functions of the exchange rate.

\[
P' = \frac{1}{m} \cdot p_N^* \cdot \left( \alpha_N' \right) \cdot \left( \alpha_T' \right) \cdot i \cdot X \cdot \left( \alpha_N' + \alpha_T' \right)
\]

\[
Q_D' = D \left[ \frac{1}{m} \cdot p_N^* \cdot \left( \alpha_N' \right) \cdot \left( \alpha_T' \right) \right]^{-\eta} \cdot \left( \sigma_n - \eta \alpha_T' \right) \cdot \left( \sigma_n - \eta \alpha_T' \right)
\]

\[
R' = D \left[ \frac{1}{m} \cdot \left( p_N^* \cdot \left( \alpha_N' \right) \right) \right]^{\eta - 1} \cdot \left[ p_T^* \cdot (\sigma_n - (\eta - 1)\alpha_T') \right]
\]

\[
\cdot \left[ X \cdot (\sigma_n - (\eta - 1)\alpha_T' + \alpha_N') \right]
\]

These equations complete the description of the effects of exchange rate changes on industries and firms. As tools of comparison, the expressions describe characteristics of firms or industries which exhibit relatively strong sensitivity to international shocks as transmitted by the exchange rate. These characteristics are reflected in the exponent on the exchange rate variable in each equation. For convenience these determinants of exchange rate sensitivity are
summarized in Table 1.

Table entries describe the determinants of the short run proportionate response of firm or industry variables to changes in the exchange rate. Responsive variables include prices, production quantities, revenues and factor employment. Industry-specific elasticities and production parameters determine the magnitude of the response.

The exchange rate sensitivity of all firm and industry variables is increased by the existence of easily substitutable or competing products, especially those whose prices are strongly affected by exchange rates.

Price sensitivity is always positive so that a devaluation of the currency always induces an increase in the industry price. Price sensitivity should be relatively large when the elasticities of both demand and the supply of the non-traded input are low.

The output response may be either positive or negative, its sign being determined by the characteristics of the competitive situation. The response is likely to be negative (production is contracted in response to local currency devaluation) when the elasticity of demand for the own product is high and the cross elasticity of demand for substitutes is low and/or the exchange rate response of the price of the substitute is low. In this case, price increases reduce the size of the original market with little
opportunity for expansion into the market for the substitutable product. In fact, producers of the substitutable good actually capture a share of the subject market. With a reversal of the elasticities described above, however, output can actually increase in response to a decline in the external value of the local currency.

This output response is also reflected in the response of the employment of the non-traded factor. The sensitivity of this variable is directly related to the elasticity of the non-traded factor.

Revenue sensitivity is a function of the underlying sensitivities of price and production. In fact, industries with similar revenue sensitivities may differ drastically in the extent to which that sensitivity is caused by responsive changes in output volume as opposed to changes in price.

The price response dominates the volume response in situations where $\alpha_k$ is large and $\theta$ is high because the non-traded factor is very inelastically supplied. The output response is relatively greater than the price response when the non-traded factor is inelastically supplied and when the effect of exchange rate changes on competing products exceeds the elasticity of own product demand.

In summary, the price response is always positive and the responses of both output and revenue may be either positive or negative. The revenue response is, always, less
negative than the output response because it is the product of both the price and volume responses. The employment of both the traded and non-traded inputs is less sensitive that revenue because the price response partially offsets the volume response.
CONCLUSION

The model provides a basis for several types of analysis. The analytical form may be theoretical, descriptive, or empirical.

The parameters of the sensitivity coefficients summarized on Table 1 may be useful in explaining different responses to exchange rate changes among firms, industries, sectors, or, with some extension, countries. Secular changes in the parameters may also explain differences in exchange rate sensitivities at different points in time.

The analytical capability provided by the microeconomic analysis may prove useful for several important empirical and theoretical controversies. Foreign exchange market volatility is frequently blamed for such ills as disruption of foreign trade, volatile financial markets, and inflation of capital costs for international companies. This description of the fundamental bases for sensitivity to exchange rate changes may provide the foundation for further understanding of the effects of such changes.
### TABLE 1

**SHORT RUN SENSITIVITY TO EXCHANGE RATE VARIABILITY**

<table>
<thead>
<tr>
<th>Industry Variable</th>
<th>Coefficient of Exchange Rate Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Industry Price (P)</strong></td>
<td>( \frac{\alpha_T}{\sigma_n} + \frac{\alpha_N}{\sigma_n} + \alpha_K ) [ \frac{\alpha_T + \alpha_N(1-\theta) + \eta(\alpha_K + \theta\alpha_N)}{\sigma_n} ]</td>
</tr>
<tr>
<td><strong>Industrial Output (Q)</strong></td>
<td>( \sigma_n ) [ \frac{\alpha_T(1-\frac{n}{\sigma_n}) + (1-\theta)\alpha_N}{\alpha_T + \alpha_N(1-\theta) + \eta(\alpha_K + \theta\alpha_N)} ]</td>
</tr>
<tr>
<td><strong>Industry Revenue (R = P·Q)</strong></td>
<td>( \sigma_n ) [ \frac{\alpha_T(1+\frac{(1-n)}{\sigma_n}) + \alpha_N + \alpha_K}{\alpha_T + \alpha_N(1-\theta) + \eta(\alpha_K + \theta\alpha_N)} ]</td>
</tr>
<tr>
<td><strong>Employment of the Non-Traded Input (N)</strong></td>
<td>( (1-\theta)\sigma_n ) [ \frac{\alpha_T [1+ \frac{(1-n)}{\sigma_n}] + \alpha_N + \alpha_K}{\alpha_T + \alpha_N(1-\theta) + \eta(\alpha_K + \theta\alpha_N)} ]</td>
</tr>
<tr>
<td><strong>Employment of the Traded Input (T)</strong></td>
<td>( \sigma_n ) [ \frac{\alpha_T + \frac{\left(1-\frac{n}{\sigma_n}\right)}{\alpha_T + \alpha_N(1-\theta) + \eta(\alpha_K + \theta\alpha_N)}}{\sigma_n} ] -1</td>
</tr>
</tbody>
</table>
FOOTNOTES


3. The long run and unconstrained system is left for development by the reader.

4. For convenience, a simple form is assumed for the relationship between the quantity of the non-traded good used by the industry and its price. A model of supply and demand in the input industry may be substituted quite easily.

5. The development of this relationship follows the development equation (12a). Reference to this equation confirms the industry and firm-specific parameters which determine the aggregate parameters $c$ and $\sigma$.

6. Aggregate income is assumed independent of the exchange rate and the price of traded goods. This obviously unrealistic assumption is made for convenience and because the alternative addition of a subsidiary model describing the determinations of aggregate income adds nothing to the development at hand. The addition of this complication is, however, easily accomplished and may be especially relevant if the model were used as the basis for empirical work.

7. When relative prices of traded goods are constant, a function of the aggregate price index for traded goods may be substituted for the price of the specific traded input. This substitution would greatly facilitate the empirical verification of the model. The assumption of constant relative prices would also support the earlier proposition that the price of the substitutue product is a function of the price of the traded input.
REFERENCES


