Calculating the pH of a solution consisting of $5.50 \times 10^{-2} \text{ M}$ of HI and $8.50 \times 10^{-3} \text{ M}$ of HF

Knowing that pH is calculated by the negative logarithm of the Hydrogen ion (proton) concentration in a given solution and expressed as:

$$\text{pH} = -\log[H^+]$$

Also, because of this relationship we know that:

$$[H^+] = 10^{-\text{pH}}$$

Starting with the simpler process for finding out the $H^+$ concentration of HI, it is important to realize that different approaches are needed for different acids of different strengths (i.e. strong or weak acids).

Assuming the HI, a strong acid, will dissociate completely in solution we can express the concentration of hydrogen ions, $[H^+]$, as follows, where the subscript ‘equilibrium’ indicates ‘post reaction’ or having reached the equilibrium point for the reaction or process in question:

$$[\text{HI}_{\text{initial}}] = [H^+_{\text{equilibrium}}] = [\text{I}^-_{\text{equilibrium}}]$$

From this we can calculate the hydrogen ion concentration to be:

$$[H^+_{\text{equilibrium}}] = 5.50 \times 10^{-2} \text{ M}$$

Now onto the more difficult part involving the hydrogen ion concentration of HF. HF is a weak acid, which means that it will not dissociate completely in solution and still have the bulk of the original compound in solution with the hydrogen and conjugate acid ions in solution after equilibrium.

It is appropriate to introduce (or recall) the concept of the acid dissociation constant, $K_a$, which is used to characterize the relative strengths of acids is expressed as such (using HF within the problem as an example):

$$K_a = \frac{[H^+_{\text{equilibrium}}][F^-_{\text{equilibrium}}]}{[\text{HF}_{\text{equilibrium}}]}$$

Where, if we express the equilibrium concentrations of $H^+$ and $F^-$ with the variable $x$, we can know that:

$$[\text{HF}_{\text{equilibrium}}] = [\text{HF}_{\text{initial}}] - x$$

This tells us that the concentration of HF that did dissociate has completely dissociated and because, $\text{HF} \rightleftharpoons H^+ + F^-$

the concentrations of the ions will be the same.

It is now possible to describe the $K_a$ of HF in solution for this problem as*,

$$K_a = \frac{x^2}{[\text{HF}_{\text{initial}}] - x}$$

However, in most cases $[\text{HF}_{\text{equilibrium}}] \approx [\text{HF}_{\text{initial}}] \gg x$ so it is practical to ‘drop’ the $x$ and describe $K_a$ as,

$$K_a = \frac{x^2}{[\text{HF}_{\text{initial}}]}$$
We can solve for $x$ and (using the above equation) get $x = 0.002474 = 2.474 \times 10^{-3}$ M.

Now we can calculate the pH of the combined solutions, assuming that the concentrations of protons and conjugate ions will \textit{not} affect the equilibria of the acids involved.

$$[H^+_{\text{total}}] = 5.50 \times 10^{-2} \text{ M} + 2.474 \times 10^{-3} \text{ M} = 5.747 \times 10^{-2} \text{ M}$$

$$\text{pH} = - \log(5.747 \times 10^{-2}) = 1.241$$

So, we have calculated the pH of the combined solution to be 1.241 which can be described as highly acidic.

\*\textit{for reference}: Calculating $x$ using,

$$K_a = \frac{x^2}{[HF_{\text{initial}}] - x}$$

yields $x = 0.00214$ M. Compare with $x = 0.002474$ M which was used to solve the problem.

Useful links and references:

http://ww2.chemistry.gatech.edu/class/peek/1310/notes/23-acids_bases.pdf
http://www.mpcfaculty.net/mark_bishop/weak_acid_equilibrium.htm
http://www.mpcfaculty.net/mark_bishop/pH_equilibrium.htm
http://en.wikiversity.org/wiki/Acid-base_chemistry
http://www.chem.purdue.edu/gchelp/howtosolveit/Equilibrium/Calculating_pHandpOH.htm